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FIRST COURSE IN ALGEBRA

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FIRST COURSE IN ALGEBRA

TEACHERS' EDITION

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PREFACE

The busy teacher of high-school algebra often finds that limitations of time forbid the actual working out of all the exercises and problems in a text. This teachers' edition is designed to meet such a condition. In its plan it is brief and schematic rather than complete and detailed. Less time should be required to prepare a lesson from this outline than from a text giving the solutions in full.

For those exercises involving not more than one process in their solution the answers only are given. If two or more processes are required in a solution, an outline is given which covers each step of the work. Moreover, after each indicated step of the work, the result obtained by taking that step is set down. This scheme affords an easy method of tracing a solution.

In the solution of problems the meaning of each letter representing an unknown is clearly stated, the equations representing the conditions of the problems in terms of these letters are given, and the schematic solution of the equation or system always follows.

The authors and publishers desire the coöperation of teachers and school authorities in limiting the circulation of this book to actual teachers of algebra.

FIRST COURSE IN ALGEBRA

TEACHERS' EDITION

Page 1

1. $4h + 3m = 4(3600) + 3(60) = 14{,}580$ seconds.

2. $5y + 4f = 5(36) + 4(12) = 228$ inches.

3. $4q + 6d = 4(25) + 6(10) = 160$ cents.

4. $4t + 6h = 4(2000) + 6(100) = 8600$ pounds.

5. $3x + 5x = 8x$.

6. $4x + 5x = 9x$.

7. $2x + 3x + 6x = 11x$.

8. $2x + 2 + 3x + 4 = 5x + 6$.

9. $x + x + 2 + x + x + 2 = 4x + 4$.

10. $n + n + 1 + n + 2 = 3n + 3$.

11. $5a + 18 - 3a - 7 = 2a + 11$.

12. $8x - 3 + 18 - 5x = 3x + 15$.

13. $4w - 8 + 3w + 20 = 7w + 12$.

14. $y + 5m = 17m$.

15. $2y + 7f = 13f$.

16. $4q + 3n = 23n$.

17. $2d + 15h = 63h$.

18. $15h + 50m = 950m$.

19. Area, $5 \cdot 5 \cdot 1$ square inch $= 25$ square inches; perimeter, $4 \cdot 5$ inches $= 20$ inches.

20. Perimeter, $4s$ inches; area, $s \cdot s$ square inches.

21. Area, $12 \cdot 4 \cdot 1$ square foot $= 48$ square feet; perimeter, $2 \cdot 12$ feet $+ 2 \cdot 4$ feet $= 32$ feet.

22. Area, ab square feet; perimeter, $2a$ feet $+ 2b$ feet $= (2a + 2b)$ feet.

23. (*a*) $2x$ inches; (*b*) $2 \cdot x + 2 \cdot 2x = 6x$ inches; (*c*) $2x \cdot x$ square inches.

24. $3 \cdot y$ years.

25. $(30 + y)$ years.

26. Length, $(12 + w)$ feet; perimeter, $w + w + 12 + w + 12 + w = (24 + 4w)$ feet.

27. Length, $(18 + w)$ feet; perimeter, $w + w + 18 + w + 18 + w = (36 + 4w)$ feet.

28. Let w feet = the width.

Then $2w + 4$ feet = the length,

and the perimeter $= w + w + 2w + 4 + 2w + 4,$

$= (6w + 8)$ feet.

Page 5

1. Greater number + less number = 160.

If we represent the less by l, then $4l$ must represent the greater, and the above statement becomes

$$4l + l = 160.$$

Whence $l = 32,$

and $4l = 128.$

Therefore the greater number is 128 and the less is 32.

2. The number + five times the number = 216.

If we represent the number by n, the above statement becomes

$$n + 5n = 216,$$

or $6n = 216.$

Whence $n = 36.$

3. Greater number + less number = 72.

If we represent the less by l, then $7l$ must represent the greater, and the above statement becomes

$$7l + l = 72.$$

Whence $l = 9,$

and $7l = 63.$

4. 1st number + 2d number + 3d number = 105.

If we let t represent the 3d number, $2t$ must represent the 1st number and $4t$ the 2d. Then the above statement becomes

$$2t + 4t + t = 105.$$

Whence $t = 15,$

$2t = 30,$

and $4t = 60.$

5. 1st number + 2d number + 3d number = 117.

If we represent the 1st number by n, $2n$ must represent the 2d number and $6n$ the 3d. Then the above statement becomes

$$n + 2n + 6n = 117.$$

Whence $n = 13,$

$2n = 26,$

and $6n = 78.$

6. 1st number + 2d number + 3d number = 192.

If we represent the 2d number by n, $2n$ must represent the 1st number and $3n$ the 3d. Then the above statement becomes

$$2n + n + 3n = 192.$$

Whence $n = 32,$

$2n = 64,$

and $3n = 96.$

7. 1st number + 2d number + 3d number = 324.

If we represent the 1st number by n, $5n$ must represent the 2d number and 30 the 3d. Then the above statement becomes

$$n + 5n + 30n = 324.$$

Whence $n = 9,$

$5n = 45,$

and $30n = 270.$

8. 1st number + 2d number + 3d number = 104.

If we represent the 1st number by n, $3n$ must represent the 2d number and $4n$ the 3d. Then the above statement becomes

$$n + 3n + 4n = 104.$$

Whence $n = 13,$

$3n = 39,$

and $4n = 52.$

10. From arithmetic, principal + interest = amount.

Principal + .06 principal = $265.

If p represents the principal, this last statement becomes

$$p + .06p = 265.$$

Whence $p = 250.$

Therefore the sum is $250.

11. Principal + .12 principal = $700.

If p represents the principal, the above statement becomes

$$p + .12p = 700.$$

Whence $p = 625.$

Therefore the required sum is $625.

12. $225 \times .06 \cdot$ (number of years) = 27.

If we represent the number of years by n, this statement becomes

$$225 \times .06 \cdot n = 27,$$

or $13.5n = 27.$

Whence $n = 2.$

13. $520 \times .065 \cdot$ (number of years) = 169.

If n represents the number of years, this statement becomes

$$520 \times .065 \cdot n = 169,$$

or $33.8n = 169.$

Whence $n = 5.$

15. Let x = the rate of interest.

Then $825$x$ = the interest for one year,

and $3300$x$ = the interest for four years.

But $\$165 =$ the interest for four years.
Whence $3300\,x = 165,$
and $x = \frac{1}{20}.$
Therefore the money is lent at 5%.

16. Let $x =$ the rate of interest.
Then $\$250\,x =$ the interest for one year,
and $\$1500\,x =$ the interest for six years.
But $\$317.50 - \$250$, or $\$67.50 =$ the interest for six years.
Whence $1500\,x = 67.50,$
and $x = .045.$
Therefore the money is lent at $4\frac{1}{2}\%$.

17. Let $x =$ the number of years.
Then $\$8\,x =$ the interest at 4%.
But $\$200 =$ the interest at 4%.
Whence $8\,x = 200,$
and $x = 25.$
Therefore the time required is 25 years.

18. $\$150 \times .05 \cdot$ (number of years) $= \$450 - \$150.$
If we represent the number of years by n, this statement becomes

$$3.50\,n = 300.$$

Whence $n = 40.$
Therefore the required time is 40 years.

19. Let $s =$ a side of the square in feet.
Then $4\,s =$ the perimeter, or 160 feet.
Whence $4\,s = 160,$
and $s = 40.$
Therefore the square is 40 feet on a side.

20. Let $s =$ the number of feet in the width of the rectangle.
Then $3\,s =$ the number of feet in the length of the rectangle,
and $2\,s + 2 \cdot 3\,s$, or $8\,s =$ the number of feet in the perimeter of the rectangle, or 256 feet.
Whence $8\,s = 256.$
Therefore $s = 32,$
and $3\,s = 96.$

21. Let $s =$ the side of either square in inches.
Then $2\,s =$ the length of the rectangle in inches,

and $2 \cdot s + 2 \cdot 2s$, or $6s =$ the perimeter of the rectangle, or 198 inches.
Therefore $6s = 198$.
Whence $s = 33$,
and $4s = 132$, the perimeter.

22. Let $x =$ the side of each square in inches.
Then $4x =$ the perimeter of each square, or 120 inches.
Therefore $4x = 120$.
Whence $x = 30$,
and $6x = 180$, the perimeter.

Page 9

1. $3a + 5b$.
2. $5b - 3a$.
3. $b^2 - a^2$.
4. $3a^2 - 2a^2$.
5. $\frac{a}{b}$.
6. $4a^2b$.
7. $\frac{a+b}{a-b}$, or $\frac{a+b}{b-a}$.
8. $a(2b - c)$.
9. $a(b + c)$.
10. $7x - (a - b)$.
11. $(a + b)(a - b)$, or $(a+b)(b-a)$.
12. $\sqrt{5a} + \sqrt[3]{7b}$.
13. $\sqrt{7x}(x - y)$.
14. $(a + b)^2$.
15. $(a-b)^2$, or $(b-a)^2$.
16. $\frac{3ab^2}{4ca^3}$.
17. $\frac{a}{3x} + \frac{4y}{c}$.

Page 10

1. $20 - 5 + 6 - 10 = 15 + 6 - 10 = 21 - 10 = 11$.
2. $16 - (8 - 2) = 16 - 6 = 10$.
3. $14 - (16 - 8) + (12 - 4) = 14 - 8 + 8 = 14$.
4. $6 \div 3 - 2 = 2 - 2 = 0$.
5. $8 \cdot 6 \div 3 - 10 = 48 \div 3 - 10 = 16 - 10 = 6$.
6. $18 \div (2 \cdot 3) = 18 \div 6 = 3$.
7. $(6 - 3)(17 - 2 \cdot 5) = 3(17 - 10) = 21$.
8. $23 - 2 \cdot 6 - 4 \div 2 + 16 = 23 - 12 - 2 + 16 = 11 + 14 = 25$.
9. $18 \div (9 - 3) = 18 \div 6 = 3$.
10. $(10 - 3)(16 - 3 \cdot 2 + 8 \div 4) = 7(16 - 6 + 2) = 7 \cdot 12 = 84$.
11. $14 - 3 \cdot (16 - 2 \cdot 5) \div 6 + 8 \cdot 2 = 14 - (16 - 10) \div 2 + 16 = 30 - 3 = 27$.
12. $(18-2) - 6 \div (4 + 2 \cdot 8 - 18 \div 9) = 16 - 6 \div (4 + 16 - 2) = 16 - 6 \div 18 = 15\frac{2}{3}$.
13. $(16 - 6)(18 - 8) \div 100 \cdot 5 - 5 = 10 \cdot 10 \div 100 \cdot 5 - 5 = 5 - 5 = 0$.
14. $(5 + 3)(5 - 3) \div 5 - 3 = 8 \cdot 2 \div 5 - 3 = 3\frac{1}{5} - 3 = \frac{1}{5}$.

Page 11

If $a = 3$, $b = 1$, $c = 5$, $d = 7$, and $e = 2$:

1. $4a + 3d = 4 \cdot 3 + 3 \cdot 7 = 33$.
2. $a^2 + 2c = 3^2 + 2 \cdot 5 = 19$.
3. $ab + cd = 3 \cdot 1 + 5 \cdot 7 = 38$.
4. $c^2 - 5ab = 25 - 5 \cdot 3 \cdot 1 = 10$.

5. $abcd - 4e^2 = 3 \cdot 1 \cdot 5 \cdot 7 - 4 \cdot 4 = 89.$

6. $\dfrac{d+c}{2} = \dfrac{7+5}{2} = 6.$

7. $\dfrac{cd\cancel{e}}{5\cancel{e}} + \dfrac{\cancel{d}ce}{2\cancel{d}} = \dfrac{5 \cdot 7}{5} + \dfrac{5 \cdot 2}{2} = 12.$

8. $\dfrac{10b}{e} - \dfrac{c}{b} = \dfrac{10 \cdot 1}{2} - \dfrac{5}{1} = 0.$

9. $\dfrac{1}{c} + \dfrac{1}{d} + \dfrac{1}{e} = \dfrac{1}{5} + \dfrac{1}{7} + \dfrac{1}{2} = \dfrac{14}{70} + \dfrac{10}{70} + \dfrac{35}{70} = \dfrac{59}{70}.$

10. $\dfrac{e^2}{ac} + \dfrac{4cd}{ae} = \dfrac{4}{3 \cdot 5} + \dfrac{4 \cdot 5 \cdot 7}{3 \cdot 2} = \dfrac{4}{15} + \dfrac{70}{3} = \dfrac{354}{15} = 23\tfrac{3}{5}.$

11. $\dfrac{cd + ae + ce}{a - b + c} = \dfrac{35 + 6 + 10}{3 - 1 + 5} = \dfrac{51}{7} = 7\tfrac{2}{7}.$

12. $\dfrac{a^2 + b^2 + c^2 + d^2}{a + b + c + d} = \dfrac{9 + 1 + 25 + 49}{3 + 1 + 5 + 7} = \dfrac{84}{16} = 5\tfrac{1}{4}.$

13. $\dfrac{a^3 - ce^2 + 3cd}{d - a + c} = \dfrac{27 - 5 \cdot 4 + 3 \cdot 35}{7 - 3 + 5} = \dfrac{112}{9} = 12\tfrac{4}{9}.$

14. $5b^3 + 4b^2 - 2b - 5 = 5 \cdot 1 + 4 \cdot 1 - 2 \cdot 1 - 5 = 9 - 7 = 2.$

15. $3b^5 - 14b^4 + 11b^3 + 11b^2 + 13b - 20 = 3 - 14 + 11 + 11 + 13 - 20 = 4.$

16. $a^e = 3^2 = 9.$

17. $c^a = 5^3 = 125.$

18. $d^e + e^a = 7^2 + 2^3 = 57.$

19. $b^c + c^a = 1^5 + 5^3 = 126.$

20. $d^e - c^2 + b^3 = 7^2 - 25 + 1 = 25.$

21. $e^2 \cdot c^a = 4 \cdot 5^3 = 500.$

22. $\dfrac{d + e^c}{2c + a} = \dfrac{7 + 2^5}{2 \cdot 5 + 3} = \dfrac{7 + 32}{13} = 3.$

23. $\dfrac{a^a + 3b}{c^e - d - e - b} = \dfrac{3^3 + 3 \cdot 1}{5^2 - 7 - 2 - 1} = \dfrac{30}{25 - 10} = 2.$

If $a = 4$, $b = 0$, $c = 5$, $d = 7$, and $e = 8$:

24. $\dfrac{4a + 3b + 2d}{c + e + 2} = \dfrac{4 \cdot 4 + 3 \cdot 0 + 2 \cdot 7}{5 + 8 + 2} = \dfrac{16 + 14}{15} = 2.$

25. $\dfrac{b}{a + c + d} = \dfrac{0}{4 + 5 + 7} = 0.$

26. $abc + acd - be = 4 \cdot 0 \cdot 5 + 4 \cdot 5 \cdot 7 - 0 \cdot 8 = 140.$

27. $\dfrac{ab}{c} + \dfrac{bd}{a} + \dfrac{be}{cd} = \dfrac{0}{5} + \dfrac{0}{4} + \dfrac{0}{35} = 0.$

28. $\dfrac{a^2 - b^2 + c^3}{3d - 2e + c} = \dfrac{16 - 0 + 125}{21 - 16 + 5} = 14\tfrac{1}{10}.$

29. $\sqrt{a} + \sqrt{2e} = \sqrt{4} + \sqrt{16} = 6.$

30. $2\sqrt{a} + \sqrt[3]{e} = 2\sqrt{4} + \sqrt[3]{8} = 6.$

31. $b\sqrt{a} + cd + \sqrt[3]{e} \cdot \sqrt{a} = 0 + 35 + 2 \cdot 2 = 39.$

32. $c\sqrt[3]{2ae} + d\sqrt[3]{2ae} = 5\sqrt[3]{2\cdot 4\cdot 8} + 7\sqrt[3]{2\cdot 4\cdot 8} = 40 + 28 = 68.$

33. $\sqrt{b^2 + c^2 + d + a} = \sqrt{0 + 25 + 7 + 4} = 6.$

34. $\sqrt[3]{3d + ec + a - 1} = \sqrt[3]{21 + 40 + 4 - 1} = 4.$

35. $de + ac\sqrt{ac^2 + 2e + c} = 56 + 20\sqrt{100 + 16 + 5} = 56 + 220 = 276.$

36. $(a + c)c = (4 + 5)5 = 45.$

37. $(a + e)(c + d) = (4 + 8)(5 + 7) = 144.$

38. $ab(a + b) = 4\cdot 0(4 + 0) = 0.$

39. $acd(a + c + d) = 4\cdot 5\cdot 7(4 + 5 + 7) = 140\cdot 16 = 2240.$

40. $ac(e - c + b + d) = 4\cdot 5(8 - 5 + 0 + 7) = 20\cdot 10 = 200.$

41. $(a + d)^2 = (4 + 7)^2 = 121.$

42. $(e - a)^2 = (8 - 4)^2 = 16.$

43. $(d - a)^3 = (7 - 4)^3 = 27.$

44. $\dfrac{3ad(3a - 2c)^2(e - c)^3}{6ed} = \dfrac{\cancel{3}\cdot\cancel{4}\cdot\cancel{7}(12 - 10)^2(8 - 5)^3}{\cancel{6}\cdot\underset{4}{\cancel{8}}\cdot\cancel{7}} = \dfrac{4\cdot 27}{4} = 27.$

45. $\dfrac{ab(c + d)(a + b)}{(e - a)} = \dfrac{4\cdot 0(5 + 7)(4 + 0)}{(8 - 4)} = 0.$

46. $c^3 - 3c^2a + 3ca^2 - a^3 = 125 - 3\cdot 25\cdot 4 + 3\cdot 5\cdot 16 - 64 = 1.$

47. If $x = 2$ and $y = 3$, $3x + 5y = 21$ becomes $3\cdot 2 + 5\cdot 3 = 21$. Yes.

48. If $x = 8$, $7x - 9 = 3x + 25$ becomes $7\cdot 8 - 9 = 3\cdot 8 + 25$, or $47 = 49$, which is false. No.

49. If $x = 2$, $x^2 + 5x + 6 = 0$ becomes $4 + 10 + 6 = 0$, which is false. No.
If $x = 3$, it becomes $9 + 15 + 6 = 0$, which is false. No.
If $x = 4$, it becomes $16 + 20 + 6 = 0$, which is false. No.

50. If $x = 5$, $3x^2 - 14x - 5 = 0$ becomes $3\cdot 25 - 14\cdot 5 - 5 = 0$. Yes.
If $x = \frac{1}{3}$, it becomes $3\cdot\frac{1}{9} - \frac{14}{3} - 5 = 0$, which is false. No.
If $x = 6$, it becomes $3\cdot 36 - 14\cdot 6 - 5 = 0$, which is false. No.
If $x = 4$, it becomes $3\cdot 16 - 14\cdot 4 - 5 = 0$, which is false. No.

Page 14

1. $4 + 2 = 6.$
2. $-2 + 4 = 2.$
3. $-3 + 5 = 2.$
4. $-3 + 3 = 0.$
5. $-4 + 2 = -2.$
6. $-7 + 5 = -2.$
7. $5 - 2 = 3.$
8. $2 - 5 = -3.$
9. $4 - 6 = -2.$
10. $-2 - 4 = -6.$
11. $-4 - 3 = -7.$
12. $-3 - 2 = -5.$

Page 15

1. (*a*) $10° - 5° = 5°$; (*b*) $10° - 10° = 0°$; (*c*) $10° - 18° = -8°$.

2. (*a*) $-12° + 7° = -5°$;
(*b*) $-12° + 12° = 0°$;
(*c*) $-12° + 25° = +13°$.

3. (*a*) $-12° - 7° = -19°$;
(*b*) $-12° - 12° = -24°$;
(*c*) $-12° - 25° = -37°$.

4. $13^\circ + 7^\circ = 20^\circ$ or 69 miles $\cdot\, 20 = 1380$ miles.

5. $20^\circ - 4^\circ = 16^\circ$, at end of first day.
$20^\circ - 2 \cdot 4^\circ = 12^\circ$, at end of second day.
$20^\circ - 3 \cdot 4^\circ = 8^\circ$, at end of third day.
$20^\circ - 4 \cdot 4^\circ = 4^\circ$, at end of fourth day.
$20^\circ - 5 \cdot 4^\circ = 0^\circ$, at end of fifth day.
$20^\circ - 6 \cdot 4^\circ = -4^\circ$, at end of sixth day.

$$\frac{20 + 16}{4} = 9 \text{ days.}$$

6. (*a*) Property $= +\$5200$.
Debts $= -\$2300$.
(*b*) Standing $= \$5200 - \$2300 = +\$2900$.

7. (*a*) Property $= +\$2300$.
Debts $= -\$5200$.
(*b*) Standing $= \$2300 - \$5200 = -\$2900$.

8. $-12^\circ + 3^\circ \cdot 3 = -3^\circ$ at 9.00 A.M.
$-12^\circ + 3^\circ \cdot 4 = 0^\circ$ at 10.00 A.M.
$-12^\circ + 3^\circ \cdot 7 = +9^\circ$ at 1.00 P.M.

Page 16

1. $+3 + (+2) = +5$.
2. $-3 + (-2) = -5$.
3. $+3 + (-2) = +1$.
4. $-3 + (+2) = -1$.
5. $7 + (-4) = +3$.
6. $-7 + (+5) = -2$.
7. $-6 + (-4) = -10$.
8. $6 + (-5) = +1$.
9. $4 + (-7) = -3$.
10. $-3 + (-6) = -9$.

Page 17

1. $+4 + (+7) = +11$.
2. $-4 + (-7) = -11$.
3. $+4 + (-7) = -3$.
4. $-4 + (+7) = +3$.
5. $+8 + (-5) = +3$.
6. $-8 + (+5) = -3$.
7. $-12 + (-9) = -21$.
8. $-6 + 6 = 0$.
9. $-6 + (-6) = -12$.
10. $-4 + (+3) + (+6) = +5$.
11. $3 + (-7) + (5) + (-4) = -3$.
12. $8 + (-2) + (-4) + (+6) = +8$.
13. $6 + (+3) = 9$.
14. $6 + (-4) = 2$.
15. $8 + (+4) = 12$.
16. $8 + (-4) = 4$.
17. $-8 + (-2) = -10$.
18. $-8 + (+2) = -6$.
19. $-10 + (-6) = -16$.
20. $-10 + (+17) = 7$.
21. $12 + (-8) = 4$.
22. $-12 + (+16) = 4$.

Page 18

Subtracting:	Changing signs and adding:
1. $8 - 5 = 3.$	**11.** $8 + (-5) = 3.$
2. $13 - 9 = 4.$	$13 + (-9) = 4.$
3. $5 - 8 = -3.$	$5 + (-8) = -3.$
4. $9 - 13 = -4.$	$9 + (-13) = -4.$
5. $-8 - (-5) = -3.$	$-8 + (+5) = -3.$
6. $8 - (-5) = 13.$	$8 + (+5) = 13.$
7. $-10 - 5 = -15.$	$-10 + (-5) = -15.$
8. $-4 - (-6) = 2.$	$-4 + (+6) = 2.$
9. $-18 - 12 = -30.$	$-18 + (-12) = -30.$
10. $+13 - 25 = -12.$	$13 + (-25) = -12.$

Yes. The answers are the same as those obtained before.

Page 19

1. $8 - (+5) = 8 + (-5) = 3.$
2. $+8 - (-5) = 8 + (+5) = 13.$
3. $+5 - (-8) = 5 + (+8) = 13.$
4. $+5 - (+8) = 5 + (-8) = -3.$
5. $12 - (+9) = 12 + (-9) = 3.$
6. $12 - (-9) = 12 + (+9) = 21.$
7. $-12 - (9) = -12 + (-9) = -21.$
8. $-12 - (-9) = -12 + (9) = -3.$
9. $+6 - (+6) = 6 + (-6) = 0.$
10. $-6 - (-6) = -6 + (+6) = 0.$
11. $6 - (-6) = 6 + (6) = 12.$
12. $-6 - (+6) = -6 + (-6) = -12.$
13. $-14 - (-19) = -14 + (19) = 5.$
14. $0 - (+1) = 0 + (-1) = -1.$
15. $-0 - (-1) = +(+1) = 1.$
16. $1 - (-2) = 1 + (+2) = 3.$
17. $12 - (+3) - (+2) = 12 + (-3) + (-2) = 7.$
18. $-12 - (+3) - (+2) = -12 + (-3) + (-2) = -17.$
19. $-12 - (-3) - (-2) = -12 + (3) + (2) = -7.$
20. $18 - (-5) - (7) = 18 + (5) + (-7) = 16.$
21. $+6 + \mathbf{4} = 10.$
22. $-6 + (-\mathbf{4}) = -10.$
23. $-3 + \mathbf{3} = 0.$
24. $+6 + (-\mathbf{6}) = 0.$
25. $+6 + (-\mathbf{2}) = 4.$
26. $-8 + \mathbf{5} = -3.$
27. $-8 + \mathbf{3} = -5.$
28. $-7 + \mathbf{14} = 7.$
29. $9 - \mathbf{6} = 3.$
30. $-7 - (-\mathbf{2}) = -5.$
31. $+5 - \mathbf{11} = -6.$
32. $-7 - (-\mathbf{11}) = 4.$
33. $-5 - (-\mathbf{5}) = 0.$
34. $2 - \mathbf{2} = 0.$
35. $4 - (-\mathbf{14}) = 18.$
36. $12 + (3) - (5) = 12 + (3) + (-5) = 10.$
37. $12 - (-3) + 6 = 12 + (3) + 6 = 21.$
38. $12 - (-4) + (-6) = 12 + (4) + (-6) = 10.$
39. $18 + (-6) - (+7) = 12 + (-7) = 5.$
40. $-16 + (-10) - (+11) = -26 + (-11) = -37.$
41. $-13 - (8) + (-14) = -27 + (-8) = -35.$

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1. $+3 \cdot +4 = +12$.
2. $+4 \cdot +12 = +48$.
3. $-5 \cdot +4 = -20$.
4. $+6 \cdot -6 = -36$.
5. $-7 \cdot +8 = -56$.
6. $-7 \cdot -3 = +21$.
7. $-12 \cdot +9 = -108$.
8. $+6 \cdot -4 = -24$.
9. $-6 \cdot -6 = +36$.
10. $+5 \cdot -10 = -50$.
11. $+0 \cdot +4 = 0$.
12. $-7 \cdot 0 = 0$.
13. $+4 \cdot -5 \cdot +6 = -120$.
14. $+4 \cdot -5 \cdot -6 = +120$.
15. $-4 \cdot -5 \cdot -6 = -120$.
16. $12 \cdot +0 \cdot -5 = 0$.
17. $9 \cdot -10 \cdot -0 = 0$.
18. $-4 \cdot +3 \cdot -6 = +72$.
19. $-3 \cdot -2 \cdot -5 = -30$.
20. $2 \cdot -3 \cdot +5 = -30$.

Page 23 (First set)

1. $+10 \div +2 = +5$.
2. $-10 \div -2 = +5$.
3. $-15 \div +3 = -5$.
4. $+14 \div -7 = -2$.
5. $-18 \div -2 = +9$.
6. $-7 \div +7 = -1$.
7. $0 \div +5 = 0$.
8. $0 \div -5 = 0$.
9. $+18 \div +3 \div -2 = -3$.
10. $+45 \div -5 \div -3 = 3$.
11. $-64 \div +8 \div -2 = +4$.
12. $+96 \div -6 \div +4 = -4$.
13. $72 \div +9 \div -4 = -2$.
14. $60 \div -5 \div -12 = 1$.
15. $48 \div +3 \div -4 = -4$.
16. $\dfrac{-10}{-5} = 2$.
17. $\dfrac{-12}{6} = -2$.
18. $\dfrac{+16}{+2} = 8$.

Page 23 (Second set)

1. $(7) + (5) = 12$.
2. $(7) - (5) = 2$.
3. $(7) + (-5) = 2$.
4. $(7) - (-5) = 12$.
5. $(-7) + (5) = -2$.
6. $-7 + 5 = -2$.
7. $(-9) - (4) = -13$.
8. $-9 - 4 = -13$.
9. $-11 + (-13) = -24$.
10. $-6 - (-10) = 4$.
11. $-6 + 10 = 4$.
12. $8 + (-10) = -2$.
13. $12 - 18 = -6$.
14. $-18 - 12 = -30$.
15. $15 - 14 = 1$.
16. $+7 - 0 = 7$.
17. $-0 - 3 = -3$.
18. $(-3)(6) = -18$.
19. $(-5)6 = -30$.
20. $(7)(-5) = -35$.
21. $8(-3) = -24$.
22. $(-5)(-12) = 60$.
23. $-3(-8) = 24$.
24. $-5 \cdot 4 = -20$.
25. $5 \cdot 0 = 0$.
26. $0 \cdot (-9) = 0$.
27. $4 \cdot 8 = 32$.
28. $-3 \cdot 6 = -18$.
29. $12 \div (-2) = -6$.
30. $-12 \div 2 = -6$.

31. $-39 \div (-3) = 13.$

32. $45 \div (-15) = -3.$

33. $0 \div (-6) = 0.$

34. $0 \div 3 = 0.$

35. $-27 \div 9 = -3.$

36. $3 - 5 + 6 = 4.$

37. $-4 + 6 - 2 + 1 = 1.$

38. $-4 + 6 + 2 - 1 = 3.$

39. $2 - 3 + 4 - 5 - 6 = -8.$

40. $\begin{array}{r} 7 \\ -2 \\ 3 \\ \underline{-5} \\ 3 \end{array}$ **41.** $\begin{array}{r} 6 \\ -2 \\ -3 \\ \underline{4} \\ 5 \end{array}$ **42.** $\begin{array}{r} -8 \\ 6 \\ 2 \\ \underline{-5} \\ -5 \end{array}$ **43.** $\begin{array}{r} 4 \\ -9 \\ -3 \\ \underline{6} \\ -2 \end{array}$

44. $3 \cdot 6 \div 3 = 6.$

45. $-4(7) \div (-2) = 14.$

46. $3(-6) \div 2 = -9.$

47. $4 \cdot 6(-8) \div (-16) = 12.$

48. $18 \div (-3) \cdot 6 \div 4 = -9.$

49. $3^2 = 9.$

50. $(-3)^2 = 9.$

51. $2^3 = 8.$

52. $(-2)^3 = -8.$

53. $(-4)^3 + 4^2 = -64 + 16 = -48.$

54. $(-1)^2 + (-1)^3 + (-2)^2 + (-2)^3 = 1 - 1 + 4 - 8 = -4.$

55. $6 + 3 \cdot 2 + 18 \div (-3) = 6 + 6 - 6 = 6.$

56. $5^2 - 4 \div (-2) + 6(-3) = 25 + 2 - 18 = 9.$

57. $3 \cdot 6 \div 9 - 2 \cdot 6 \div 4 + (-3)^2 = 2 - 3 + 9 = 8.$

58. $6 + 6 \cdot 3^2 - 5^2 \cdot 2 \div 10 = 6 + 54 - 5 = 55.$

If $x = 3$ and $y = -2$:

59. $y^2 = 4.$

60. $y^3 = -8.$

61. $y^4 = 16.$

62. $y^5 = -32.$

63. $2y^2 = 8.$

64. $2y^3 = -16.$

65. $5x^2y^2 = 5 \cdot 9 \cdot 4 = 180.$

66. $4x^2y^4 = 4 \cdot 9 \cdot 16 = 576.$

67. $x^2 - y^2 = 9 - 4 = 5.$

68. $x^3 - y^3 = 27 - (-8) = 35.$

69. $x^2 + 2xy + y^2 = 9 + 2 \cdot 3(-2) + 4 = 1.$

70. $x^2 - 2xy + y^2 = 9 - 6(-2) + 4 = 25.$

71. $(x + y)(x - y) = 1 \cdot 5 = 5.$

72. $x^3 + 3x^2y + 3xy^2 + y^3 = 27 + 3 \cdot 9(-2) + 3 \cdot 3 \cdot 4 - 8 = 1.$

73. $y^3 - 3xy^2 + 3x^2y - x^3 = -8 - 3 \cdot 3 \cdot 4 + 3 \cdot 9(-2) - 27 = -125.$

74. If $x = 5$, $4x - 2 = 2x + 8$ becomes $4 \cdot 5 - 2 = 2 \cdot 5 + 8$, or $18 = 18$. Yes.

75. If $x = -9$, $3x - 5 = 2x + 8$ becomes $-9 \cdot 3 - 5 = -9 \cdot 2 + 8$, but $-32 \neq -10$. No.

76. If $x = 4$, $x^2 - x - 12 = 0$ becomes $16 - 4 - 12 = 0$, or $0 = 0$. Yes.
If $x = -8$, it becomes $64 + 8 - 12 = 0$, but $60 \neq 0$. No.
If $x = -4$, it becomes $16 + 4 - 12 = 0$, but $8 \neq 0$. No.

77. If $x = \frac{2}{3}$, $3x^2 + 19x = 14$ becomes $3 \cdot \frac{4}{9} + \frac{38}{3} = 14$, or $14 = 14$. Yes.
If $x = 2$, it becomes $3 \cdot 4 + 2 \cdot 19 = 14$, but $50 \neq 14$. No.
If $x = -7$, it becomes $3 \cdot 49 - 7 \cdot 19 = 14$, or $14 = 14$. Yes.

78. $15^\circ - 3^\circ \cdot 2 = +9^\circ$, at 9.00 A.M.
$15^\circ - 3^\circ \cdot 5 = 0^\circ$, at noon.
$15^\circ - 3^\circ \cdot 8 = -9^\circ$, at 3.00 P.M.

79. Average height $= \dfrac{30 + (-7) + 18 + (-10) + 16}{5}$, or $9\frac{2}{5}$ feet.

The average height was $9\frac{2}{5}$ feet above the assumed level.

80. Mean height $= \dfrac{9\frac{2}{5} \cdot 5 + (-38)}{6}$, or $1\frac{1}{2}$ feet above the level.

81. Let $x =$ the height of the seventh point.

Then $\dfrac{1\frac{1}{2} \cdot 6 + x}{7} = 0$, or $x = -9$.

The last point was -9 feet high.

82. Euclid lived about -300; Sir Isaac Newton died in $+1727$.

83. The date -450 means 450 B.C.; the date $+1910$ means 1910 A.D. The difference, $1910 - (-450)$, is 2360 years.

84. Rate upstream, $(8 - 1\frac{1}{2})$ miles per hour; rate downstream, $(8 + 1\frac{1}{2})$ miles per hour.

85. Rate toward bow, $(12 + 3)$ miles per hour; rate toward stern, $(12 - 3)$ miles per hour.

86. Balloon, -500 pounds; men, 1500 pounds; both, $[1500 + (-500)]$, or 1000 pounds.

87. (*a*) $(2 + 1\frac{1}{2})$ miles per hour; (*b*) $(2 - 1\frac{1}{2})$, or $-\frac{1}{2}$ mile per hour; i.e. he is being carried downstream $\frac{1}{2}$ mile per hour.

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1. $18 - 5 + 6 = 19$.
2. $18a - 5a + 6a = 19a$.
3. $12 - 7 + 3 + 4 = 12$.
4. $12a^2 - 7a^2 + 3a^2 + 4a^2 = 12a^2$.
5. $6 - 4 - 17 + 20 = 5$.
6. $6ab - 4ab - 17ab + 20ab = 5ab$.
7. $15 - 17 + 8 - 12 - 25 = -31$.
8. $8abc - 17abc - 4abc + 15abc - 12abc = -10abc$.
9. $11 + 5 - 9 - 3 + 16 - 25 = -5$.
10. $16ac - 9ac + 5ac - 2ac - 3ac + 11ac = 18ac$.
11. $7x + 4x - 15x - 8x + 3x = -9x$.
12. $12y - 17y + 10y + 20y - 25y = 0$.
13. $4xy - 8xy - 12xy + 13xy - xy = -4xy$.
14. $14x^2 - 13x^2 + x^2 - 5x^2 + 4x^2 = x^2$.
15. $5y^2 - 2y^2 - 11y^2 + y^2 - 7y^2 = -14y^2$.
16. $7a^2b - 5a^2b + 8a^2b - a^2b + 9a^2b = 18a^2b$.

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1. The sum of a, $3b$, and $-c = a + 3b - c$.
2. The sum of $4x$, $-2b$, $3y$, and $10 = 4x - 2b + 3y + 10$.
3. The sum of $3ab^2$, $2bx$, $-cy$, and $4a^2b = 3ab^2 + 2bx - cy + 4a^2b$.
4. The sum of $5x^3y$, $-5xy^3$, c^3y, and $-2cy^3$
   $= 5x^3y - 5xy^3 + c^3y - 2cy^3$.
5. The sum of $4x$, $-3a$, $2b$, $-5x$, and $3y = -x - 3a + 2b + 3y$.
6. The sum of $5a$, $-4b$, $+3c$, $6b$, and $-2c^2 = 5a + 2b + 3c - 2c^2$.
7. The sum of $3a^2$, $+2b^2$, $-5c^2$, $-4b^2$, and $5a^2 = 8a^2 - 2b^2 - 5c^2$.
8. The sum of $4a^3b$, $-4ab^3$, $-3a^2b$, $3ab^2$, $4a^3b$, and $2ab^3$
   $= 8a^3b - 2ab^3 - 3a^2b + 3ab^2$.
9. $-4a^2b + 6a^2b^2 - 15a^2b^2 + 3a^2b^2 + 0a^2b^2 = -4a^2b - 6a^2b^2$.
10. $-b^3 - 23b^3 + 17b^3 + b^3 - 0b^3 + 13b^3 = 7b^3$.
11. $12a^3b + 6a^3b - a^3b + 16a^3b - 13a^3b - 25a^3b = -5a^3b$.
12. $11\sqrt{a} - 14\sqrt{a} + 21\sqrt{a} - \sqrt{a} = 17\sqrt{a}$.
13. $3\sqrt{x-y} - \sqrt{x-y} + 9\sqrt{x-y} - 7\sqrt{x-y} = 4\sqrt{x-y}$.
14. $3(a+b) - 2(a+b) + 8(a+b) = 9(a+b)$.
15. $-7(a-2b) + (a-2b) + 12(a-2b) = 6(a-2b)$.
16. $8(a+b) + 3(a-b) - 4(a+b) - 2(a-b) = 4(a+b) + (a-b)$.
17. $4(x-y) - 3(x+3) - 6(x-y) + 5(x-3)$
    $= -2(x-y) - 3(x+3) + 5(x-3)$.
18. $(2x-y)^2 - 3(2x-y)^2 + 4(2x-y)^2 - 7(2x-y)^2 = -5(2x-y)^2$.

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1.
$$\begin{array}{r} x + y + z \\ x - 2y + 3z \\ 3x + 4y - 7z \\ \hline 5x + 3y - 3z \end{array}$$

2.
$$\begin{array}{r} 2x + 5y - z \\ 3x - 8y + 6z \\ x - y - z \\ \hline 6x - 4y + 4z \end{array}$$

3.
$$\begin{array}{r} 3x + 5y \phantom{{}+6z} \\ 4x - 7y + 6z \\ 3x - 3y - 3z \\ \hline 10x - 5y + 3z \end{array}$$

4.
$$\begin{array}{r} 7x - y + 3z \\ 5x \phantom{{}+6y} - 4z \\ 2x + 6y - 5z \\ \hline 14x + 5y - 6z \end{array}$$

5.
$$\begin{array}{r} 4x - 3y - 5z \\ 6y - 2z \\ 7x - 6y - 4z \\ \hline 11x - 3y - 11z \end{array}$$

6.
$$\begin{array}{r} x + 3y + 2z \\ x + y - 3z \\ -2x - 4y + z \\ \hline 0 + 0 + 0 \end{array}$$

7.
$$\begin{array}{r} 5x - y + 3z \\ x + 2y - 11z \\ -7y + 9z \\ \hline 6x - 6y + z \end{array}$$

8.
$$\begin{array}{r} 8a - 7b - 6c \\ -4a - 3b + 5c \\ 3b + 7c \\ \hline 4a - 7b + 6c \end{array}$$

9.
$$\begin{array}{r} 9ac - bc \phantom{{}-12ab} \\ -4ac \phantom{{}-bc} + 8ab \\ -ac \phantom{{}-bc} - 12ab \\ \hline 4ac - bc - 4ab \end{array}$$

10.
$$\begin{array}{r} a^2 - 4a + 10 \\ -6a^2 + 5a + 4 \\ 2a^2 + 3a - 16 \\ \hline -3a^2 + 4a - 2 \end{array}$$

11.
$$\begin{array}{r} -a^2 + a + 9 \\ 6a^2 - 4a - 7 \\ 5a^2 - a \phantom{{}+9} \\ \hline 10a^2 - 4a + 2 \end{array}$$

12.
$$\begin{array}{r} \frac{1}{2}x - \frac{2}{3}y + \frac{2}{5}z \\ x - y + 2z \\ \frac{1}{3}x - \frac{1}{2}y + \frac{1}{10}z \\ \hline 1\frac{5}{6}x - 2\frac{1}{6}y + 2\frac{1}{2}z \end{array}$$

13.
$$\begin{array}{r} \frac{1}{2}x \qquad\qquad - \frac{3}{20}z \\ \frac{1}{3}x + \frac{4}{9}y \qquad\qquad \\ - \frac{5}{3}y + \frac{3}{5}z \\ \hline \frac{5}{6}x - 1\frac{2}{9}y + \frac{9}{20}z \end{array}$$

14.
$$\begin{array}{r} a - 3(x-y) + z \qquad \\ -10a + 4(x-y) \qquad + 5 \\ -2(x-y) \qquad + 6 \\ \hline -9a - (x-y) + z + 11 \end{array}$$

15.
$$\begin{array}{r} a + 2(b-c) + d \\ -12a + 7(b-c) + 6d \\ 11a - 5(b-c) \qquad \\ \hline 4(b-c) + 7d \end{array}$$

16.
$$\begin{array}{r} 7a^2 - 13b^2 + 12c^2 \\ -a^2 + 15b^2 - 7c^2 \\ 3b^2 + 5c^2 \\ \hline 6a^2 + 5b^2 + 10c^2 \end{array}$$

17.
$$\begin{array}{r} x^2 - 2xy + y^2 \qquad \\ -4x^2 - 4xy - y^2 \qquad \\ -16x^2 - 8xy + y^2 + y^3 \\ \hline -19x^2 - 14xy + y^2 + y^3 \end{array}$$

18.
$$\begin{array}{r} -a^2 + 5ab + b^2 \\ 5a^2 - 4ab - 9b^2 \\ -2ab + 2b^2 \\ \hline 4a^2 - ab - 6b^2 \end{array}$$

19.
$$\begin{array}{r} 3x^2 - 6x + 11 \\ -4x - 8 \\ -3x^2 + 5x - 16 \\ +13x \qquad \\ \hline 8x - 13 \end{array}$$

20.
$$\begin{array}{r} 8b^2 - 10bc + 12c^2 \qquad \\ -6b^2 - bc - c^2 \qquad \\ -11c^2 + c^3 \\ \hline 2b^2 - 11bc \qquad\qquad + c^3 \end{array}$$

21.
$$\begin{array}{r} 4x^2y - xy + y^2 \qquad\qquad \\ -2x^2y - 3xy \qquad\qquad + 4xy^2 \\ x^2y - 3xy + 4y^2 \qquad\qquad \\ \hline 3x^2y - 7xy + 5y^2 + 4xy^2 \end{array}$$

22.
$$\begin{array}{r} \frac{1}{2}a + \frac{1}{3}b - \frac{1}{4}c \qquad \\ -\frac{2}{3}a + \frac{1}{2}b - c \qquad \\ -b + \frac{1}{2}c \qquad \\ -\frac{3}{4}a \qquad\qquad\qquad + 7 \\ \hline -\frac{11}{12}a - \frac{1}{6}b - \frac{3}{4}c + 7 \end{array}$$

23. $ax + bx + cx = (a + b + c)x$.

24. $2ax - 3x + bx = (2a - 3 + b)x$.

25. $3ax - 4cx + x = (3a - 4c + 1)x$.

26. $3ax^2 - bx^2 - x^2 + a^2x^2 = (3a - b - 1 + a^2)x^2$.

27. $by - 4cy - y - 4by = -4cy - y - 3by = (-4c - 1 - 3b)y$.

28. $a(b + c) + 3(b + c) = (a + 3)(b + c)$.

29. $4(a - x) - 3b(a - x) = (4 - 3b)(a - x)$.

30. $8a(a + 3b) - 1(a + 3b) = (8a - 1)(a + 3b)$.

31. $7b(x^2 + y^2) - a(x^2 + y^2) + (x^2 + y^2) = (7b - a + 1)(x^2 + y^2)$.

Page 35 (First set)

1. $x + 5 = 11$.

Subtracting 5 from each member, $x = 6$. Ax. II

2. $x - 4 = 12$.

Adding 4 to each member, $x = 16$. Ax. I

3. $3x + 10 = 28$.

Subtracting 10 from each member, $3x = 18$. Ax. II

Dividing each member by 3, $x = 6$. Ax. IV

4. $5x-6=19.$
Adding 6 to each member,
$5x=25.$
Whence $x=5.$

5. $9x-12=6.$
Adding 12 to each member,
$9x=18.$
Whence $x=2.$

6. $4x=12+x.$
Whence $x=4.$

7. $6x=20+2x.$
Whence $x=5.$

8. $9n=40-n.$
Whence $n=4.$

9. $13n=-5n+36.$
Whence $n=2.$

10. $-4n=-13n+27.$
Whence $n=3.$

11. $3y+2=y+8.$
Whence $y=3.$

12. $5+4y=3y+20.$
Whence $y=15.$

13. $2y-3=17-y.$
Whence $y=6\frac{2}{3}.$

14. $8x-15=6x-15.$
Adding 15 to each member,
$8x=6x.$
Whence $x=0.$

15. $-7h+19=25-9h.$
Whence $h=3.$

16. $-7x+18=4x+18.$
Whence $x=0.$

17. $5k-4=3k+18.$
Whence $k=11.$

18. $8-6x+12+10x=26.$
Whence $x=1\frac{1}{2}.$

19. $x+2x+18+x+2x+18$
$=116.$
Whence $x=13\frac{1}{3}.$

20. $4x-3+8x-17=40+6x-54.$
Collecting, $12x-20=6x-14.$
Whence $x=1.$

Page 35 (Second set)

1. Length, $3x$; perimeter, $2\cdot x+2\cdot 3x$, or $8x$.
2. Length, $x+10$ feet; perimeter, $2\cdot x+2(x+10)$, or $4x+20$ feet.
3. Width, $x-18$ feet; perimeter, $2\cdot x+2(x-18)$, or $4x-36$ feet.
4. Width, x feet; length, $2x+4$ feet; perimeter, $2x+2(2x+4)$, or $6x+8$ feet.
5. Greater by 1.
6. $n+1$.
7. $n+1$ and $n+2$; sum is $2n+3$.
8. $n+1$, $n+2$, $n+3$; the sum of the four $=4n+6$.
9. Greater by 2.
10. $n+2$; $n-2$.
11. n, $n+2$, $n+4$; their sum $=3n+6$.
12. n, $n+2$, $n+4$, $n+6$; their sum $=4n+12$.

13. First, n; second, $2n$; third, $6n$.

14. First, n; second, $n + 10$; third, $n + 10 - 7$, or $n + 3$.

15. $x - 4$; $x + 6$.

16. (*a*) B, x years; A, $2x$ years. (*b*) B, $x - 7$ years; A, $2x - 7$ years. (*c*) B, $x + 12$ years; A, $2x + 12$ years.

17. $8 + x = 5$.
18. $x = 10 - 2$.
19. $x = 3 + 5$.
20. $x = 5 + y$.
21. $3x = 21$.
22. $4a = 18 + 2$.
23. $8 + x = 34 - x$.
24. $2x - 9 = 14 + x$.
25. $2x + 3x = 48 + x$.
26. $3x - 12 = 50 + x$.

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1. Greater number + less number = 135.

If we represent the less number by n, $2n$ must represent the greater, and the above statement becomes

$$2n + n = 135.$$

Whence $$n = 45,$$

and $$2n = 90.$$

2. Greater number + less number = 105.

If we represent the less number by n, $4n$ must represent the greater, and the above statement becomes

$$4n + n = 105.$$

Whence $$n = 21,$$

and $$4n = 84.$$

3. Greater number − less number = 52.

If we let n represent the less number, $5n$ must represent the greater, and the above statement becomes

$$5n - n = 52.$$

Whence $$n = 13,$$

and $$5n = 65.$$

4. Greater number + less number = 129.

If we let n represent the less number, $n + 5$ must represent the greater, and the above statement becomes

$$n + 5 + n = 129.$$

Whence $$n = 62,$$

and $$n + 5 = 67.$$

5. Greater number + less number = 168.

If we represent the less number by n, $n + 18$ must represent the greater, and the above statement becomes

$$n + 18 + n = 168.$$

Whence $n = 75,$

and $n + 18 = 93.$

6. Twice the width + twice the length = 156 feet.

If we let w represent the width in feet, $5w$ must represent the length, and the above statement becomes

$$2 \cdot w + 2 \cdot 5w = 156.$$

Whence $w = 13,$

and $5w = 65.$

7. 2 · length + 2 · width = 96 feet.

If we represent the width in feet by w, $12 + w$ must represent the length, and the above statement becomes

$$24 + 2w + 2w = 96.$$

Whence $w = 18,$

and $12 + w = 30.$

8. 2 · length + 2 · width = 98 feet.

If we represent the width in feet by w, $2w + 4$ must represent the length, and the above statement becomes

$$4w + 8 + 2w = 98.$$

Whence $w = 15,$

and $2w + 4 = 34.$

9. 2 · length + 2 · width = 88 feet.

If we let w represent the width in feet, $3w - 20$ must represent the length, and the above statement becomes

$$6w - 40 + 2w = 88.$$

Whence $w = 16,$

and $3w - 20 = 28.$

10. 1st number + 2d number + 3d number = 45.

If we represent the first number by n, $n + 1$ and $n + 2$ must represent the next two numbers greater, and the above statement becomes

$$(n) + (n + 1) + (n + 2) = 45.$$

Whence $n = 14,$

$n + 1 = 15,$

and $n + 2 = 16.$

11. 1st number + 2d number + 3d number + 4th number = 106.

If we represent the first number by n, $n + 1$, $n + 2$, and $n + 3$ must represent the three consecutive numbers greater, and the above statement becomes

$$(n) + (n + 1) + (n + 2) + (n + 3) = 106.$$

Whence $n = 25,$

$$n + 1 = 26,$$
$$n + 2 = 27,$$
and $$n + 3 = 28.$$

12. The sum of five consecutive numbers = 85.

If we represent the first number by n, $n + 1$, $n + 2$, $n + 3$, $n + 4$ must represent the next four numbers greater, and the statement above becomes $(n)+(n+1)+(n+2)+(n+3)+(n+4)= 85.$

Whence $$n = 15,$$
$$n + 1 = 16,$$
$$n + 2 = 17,$$
$$n + 3 = 18,$$
and $$n + 4 = 19.$$

13. 1st odd number + next odd number + next odd number = 291.

If we represent the first number by n, the others must be represented by $n + 2$ and $n + 4$ respectively, and the above statement becomes

$$(n) + (n + 2) + (n + 4) = 291.$$

Whence $$n = 95,$$
$$n + 2 = 97,$$
and $$n + 4 = 99.$$

14. 1st even number + next even number + next even number = 66.

If we represent the first number by n, $n + 2$ and $n + 4$ must represent the other numbers, and the above statement becomes

$$(n) + (n + 2) + (n + 4) = 66.$$

Whence $$n = 20,$$
$$n + 2 = 22,$$
and $$n + 4 = 24.$$

15. The sum of five consecutive odd numbers = 315.

Representing the first number by n, we must then represent the others by $n+2$, $n+4$, $n+6$, $n+8$ respectively, and the above statement becomes $(n)+(n+2)+(n+4)+(n+6)+(n+8)= 315.$

Whence $$n = 59,$$
$$n + 2 = 61,$$
$$n + 4 = 63,$$
$$n + 6 = 65,$$
and $$n + 8 = 67.$$

16. A's age + B's age = 43 years.

If we represent B's age in years by x, $x - 2$ must represent A's age, and the above statement becomes

$$x - 2 + x = 43.$$

Whence $$x = 22\tfrac{1}{2},$$
and $$x - 2 = 20\tfrac{1}{2}.$$

17. A's age + 6 years + B's age + 6 years = 54 years.

If we represent A's age in years by x, $2x$ must represent B's age, and the above statement becomes

$$x + 6 + 2x + 6 = 54.$$

Whence $x = 14$,

and $2x = 28$.

18. Mileage of United States + mileage of Europe = 402,000 miles.

If we represent the mileage of Europe by m, $m + 52{,}000$ must represent that of United States, and the above statement becomes

$$m + 52{,}000 + m = 402{,}000.$$

Whence $m = 175{,}000$,

and $m + 52{,}000 = 227{,}000$.

19. Length of Nile + length of Amazon + length of Mississippi = 11,500 miles.

If we represent the length of the Amazon in miles by l, $l + 500$ must represent the length of the Nile, and $l + 500 + 300$, or $l + 800$, the length of the Mississippi. The above statement becomes

$$(l + 500) + (l) + (l + 800) = 11{,}500.$$

Whence $l = 3400$,

$l + 500 = 3900$,

and $l + 800 = 4200$.

20. Height of Mount McKinley + height of Pike's Peak = 34,607 feet.

If we represent the height of Pike's Peak in feet by h, $h + 6313$ must represent the height of Mount McKinley, and the statement above becomes

$$(h + 6313) + (h) = 34{,}607.$$

Whence $h = 14{,}147$,

and $h + 6313 = 20{,}460$.

The height of Mount Everest = 34,607 feet − 5605 feet = 29,002 feet.

21. Horse power of a freight engine + that of a passenger engine + that of a tractor = 11,200 H.P.

If we represent the H.P. of the freight engine by x, $x - 1800$ must represent the H.P. of the tractor and $x + 1000$ that of the passenger engine. The above statement becomes

$$(x) + (x + 1000) + (x - 1800) = 11{,}200.$$

Whence $x = 4000$,

$x + 1000 = 5000$,

and $x - 1800 = 2200$.

Page 40 (First set)

1. $3x - 2x = x.$
$2x - 3x = -x.$

2. $3x - 4x = -x.$
$4x - 3x = x.$

3. $-3x - (-2x) = -x.$
$-2x - (-3x) = x.$

4. $-3x - (-5x) = 2x.$
$-5x - (-3x) = -2x.$

5. $4x - (-x) = 5x.$
$-x - 4x = -5x.$

6. $-3x - (-x) = -2x.$
$-x - (-3x) = 2x.$

7. $5c - (-c) = 6c.$
$-c - 5c = -6c.$

8. $-5ac - (-ac) = -4ac.$
$-ac - (-5ac) = 4ac.$

9. $-11a^2c - 8a^2c = -19a^2c.$
$8a^2c - (-11a^2c) = 19a^2c.$

10. $-6x^2y^2 - 6x^2y^2 = -12x^2y^2.$
$6x^2y^2 - (-6x^2y^2) = 12x^2y^2.$

11. $0 - 3x = -3x.$
$3x - 0 = 3x.$

12. $0 - (-4ab) = 4ab.$
$-4ab - 0 = -4ab.$

13. $b - a.$
$a - b.$

14. $2x - c.$
$c - 2x.$

15. $-4y - x.$
$x - (-4y) = x + 4y.$

16. $2b - (-3a) = 2b + 3a.$
$-3a - (+2b) = -3a - 2b.$

17. $-5b - (-2a) = -5b + 2a.$
$-2a - (-5b) = -2a + 5b.$

18. $-2a - (-2a) = 0.$
$-2a - (-2a) = 0.$

Page 40 (Second set)

1. $\begin{array}{r} a+3 \\ a+2 \\ \hline 1 \end{array} \quad \begin{array}{r} a+2 \\ a+3 \\ \hline -1 \end{array}$

2. $\begin{array}{r} a-2 \\ a-4 \\ \hline 2 \end{array} \quad \begin{array}{r} a-4 \\ a-2 \\ \hline -2 \end{array}$

3. $\begin{array}{r} 3a+5 \\ 2a-3 \\ \hline a+8 \end{array} \quad \begin{array}{r} 2a-3 \\ 3a+5 \\ \hline -a-8 \end{array}$

4. $\begin{array}{r} 9-4a \\ 7+3a \\ \hline 2-7a \end{array} \quad \begin{array}{r} 7+3a \\ 9-4a \\ \hline -2+7a \end{array}$

5. $\begin{array}{rr} 4a & -3 \\ 4a & \\ \hline & -3 \end{array} \quad \begin{array}{rr} 4a & \\ 4a & -3 \\ \hline & 3 \end{array}$

6. $\begin{array}{rr} 5c & \\ -5c & +2ab \\ \hline 10c & -2ab \end{array} \quad \begin{array}{rr} -5c & +2ab \\ 5c & \\ \hline -10c & +2ab \end{array}$

7. $\begin{array}{rr} 5xy & \\ 3xy & -a \\ \hline 2xy & +a \end{array} \quad \begin{array}{rr} 3xy & -a \\ 5xy & \\ \hline -2xy & -a \end{array}$

8. $\begin{array}{r} x+3 \\ 0 \\ \hline x+3 \end{array} \quad \begin{array}{r} 0 \\ x+3 \\ \hline -x-3 \end{array}$

9. $\begin{array}{r} 0 \\ 2x-3 \\ \hline -2x+3 \end{array} \quad \begin{array}{r} 2x-3 \\ 0 \\ \hline 2x-3 \end{array}$

10. $\begin{array}{rrr} 2x & -2y & +z \\ x & -2y & +3z \\ \hline x & & -2z \end{array}$

11. $\begin{array}{rrr} 4x & -5y & +3z \\ 4x & -8y & -2z \\ \hline & 3y & +5z \end{array}$

12. $\begin{array}{rrr} 4a & -b & +2c \\ 3a & -2b & \\ \hline a & +b & +2c \end{array}$

13. $\begin{array}{rrr} 3a & -5b & \\ 5a & -4b & -3c \\ \hline -2a & -b & +3c \end{array}$

14. $\begin{array}{rrrr} & b & -x & +y \\ a & & -x & -y \\ \hline b & -a & & +2y \end{array}$

15. $\begin{array}{rrrr} 4a & -b & +5c & \\ 3a & -2b & -c & +6 \\ \hline a & +b & +6c & -6 \end{array}$

16. $$\begin{array}{r} a-3b \\ 2a-2b-2c+4 \\ \hline -a-b+2c-4 \end{array}$$

17. $$\begin{array}{l} c-d+e \\ -c\qquad\qquad\;+a+b \\ \hline 2c-d+e-a-b \end{array}$$

18. $$\begin{array}{l} 2x+5y-3z \\ x-2y+z \\ \hline x+7y-4z \end{array}$$

19. $$\begin{array}{r} 5x-2y-4z \\ 7x-9y+3z \\ \hline -2x+7y-7z \end{array}$$

20. $$\begin{array}{l} 6a+3b \\ 5a-4b-6c \\ \hline a+7b+6c \end{array}$$

21. $$\begin{array}{l} 5 \\ +a+b-2c \\ \hline 5-a-b+2c \end{array}$$

22. $$\begin{array}{l} 3xy+z \\ +3ab+c \\ \hline 3xy+z-3ab-c \end{array}$$

23. $$\begin{array}{r} 0 \\ 2x-4y-z \\ \hline -2x+4y+z \end{array}$$

24. $$\begin{array}{r} xy^2+x^2y+z \\ -3xy^2+2x^2y \\ \hline 4xy^2-x^2y+z \end{array}$$

25. $$\begin{array}{r} x-2y+z \\ 3x+2y-z \\ \hline -2x-4y+2z \end{array}$$

26. $$\begin{array}{r} x^2-7x+10 \\ 3x^2+14x-8 \\ \hline -2x^2-21x+18 \end{array}$$

27. $$\begin{array}{r} x-y+z \\ 5x+3y-8z \\ \hline -4x-4y+9z \end{array}$$

28. $$\begin{array}{r} -8x^2+4 \\ x^2-5x+6 \\ \hline -9x^2+5x-2 \end{array}$$

29. $$\begin{array}{l} 3a-5b+c \\ 0 \\ \hline 3a-5b+c \end{array}$$

30. $$\begin{array}{r} 4a-6b+8y \\ +x-12 \\ \hline 4a-6b+8y-x+12 \end{array}$$

31. $$\begin{array}{rrl} & a^2-2ab+b^2 & (1) \\ & a^2-12ab+20 & (2) \\ \cline{2-2} (1)+(2), & 2a^2-14ab+b^2+20 & (3) \end{array} \qquad \begin{array}{rrl} & a^2-13a+30 & (4) \\ & 2a^2+20-14ab+b^2 & (3) \\ \cline{2-2} (4)-(3), & -a^2-13a+10+14ab-b^2 & \end{array}$$

32. $$\begin{array}{rrl} & a-3b+c & (1) \\ & 4a+5b-6c+4 & (2) \\ \cline{2-2} (1)+(2), & 5a+2b-5c+4 & (3) \end{array} \qquad \begin{array}{rrl} & a-b+c-x & (4) \\ & 5a+2b-5c+4 & (3) \\ \cline{2-2} (4)-(3), & -4a-3b+6c-x-4 & \end{array}$$

33. $$\begin{array}{rrl} & 5x+3x^2y-15xy^2 & (1) \\ & -6x+7x^2y-12xy^2 & (2) \\ \cline{2-2} (1)+(2), & -x+10x^2y-27xy^2 & (3) \\ & 11x-5x^2y+7xy^2 & (4) \\ \cline{2-2} (3)-(4), & -12x+15x^2y-34xy^2, & \text{or } 15x^2y-12x-34xy^2. \end{array}$$

34. $$\begin{array}{rrl} & 4abc^2-3ab^2c+2a^2bc & (1) \\ & 6abc^2-5ab^2c-4a^2b^2c & (2) \\ \cline{2-2} (1)+(2), & 10abc^2-8ab^2c+2a^2bc-4a^2b^2c & (3) \\ & 2abc^2+7ab^2c-3a^2bc & (4) \\ \cline{2-2} (3)-(4), & 8abc^2-15ab^2c+5a^2bc-4a^2b^2c & \end{array}$$

35.

$$\begin{array}{rrl} & 3x-4xy-\ 2z & (1)\\ & \underline{-3x+7xy-\ 4z} & (2)\\ & 3xy-\ 6z & (3)\\ & \underline{9x-2xy+\ 4z-7a^2bc} & (6)\\ (3)-(6), & -9x+5xy-10z+7a^2bc & \end{array}$$

$$\begin{array}{rrl} & 5z-2xy-a^2bc & (4)\\ & \underline{-\ z\qquad -6a^2bc+9x} & (5)\\ (4)+(5), & 4z-2xy-7a^2bc+9x & (6) \end{array}$$

36.

$$\begin{array}{rrl} & 4x-3y+\ 6 & (1)\\ & \underline{3x+5y-10} & (2)\\ (1)+(2), & 7x+2y-\ 4 & \end{array}$$

37.

$$\begin{array}{rrl} & 7c+5d-\ e & (1)\\ & \underline{4c+5d-9e} & (2)\\ (1)-(2), & 3c\qquad +8e & \end{array}$$

38.

$$\begin{array}{rrl} & x^2+2x+\ 5 & (1)\\ & \underline{2x^2+\ x-10} & (2)\\ (1)+(2), & 3x^2+3x-\ 5 & (3)\\ & \underline{x^2-5x+\ 3} & (4)\\ (3)-(4), & 2x^2+8x-\ 8 & \end{array}$$

39.

$$\begin{array}{rrl} & x+3y-2z & (1)\\ & \underline{4x-5y+3z} & (2)\\ (1)+(2), & 5x-2y+\ z & (3)\\ & \underline{3x-2y-6z} & (4)\\ (3)-(4), & 2x\qquad +7z & \end{array}$$

40.

$$\begin{array}{rrl} & 5x+3y-\ z & (1)\\ & \underline{+4y+7z} & (2)\\ (1)+(2), & 5x+7y+6z & (3)\\ & \underline{x-\ y+3z} & (4)\\ (3)-(4), & 4x+8y+3z & \end{array}$$

41.

$$\begin{array}{rrl} & 4x-3y+\ 7 & (1)\\ & \underline{2x-5y-\ 4} & (2)\\ (1)-(2), & 2x+2y+11 & (3)\\ & \underline{4x\qquad -\ 8} & (4)\\ (3)+(4), & 6x+2y+\ 3 & \end{array}$$

42.

$$\begin{array}{rrl} & 3c-\ 5d-\ \ e & (1)\\ & \underline{+\ 6d+11e+5} & (2)\\ (1)-(2), & 3c-11d-12e-5 & (3)\\ & \underline{5c\qquad +\ 4e\quad} & (4)\\ (3)-(4), & -2c-11d-16e-5 & \end{array}$$

43.

$$\begin{array}{rrl} & 3a+3b-4c & (1)\\ & \underline{-3b-3c-4} & (2)\\ (1)-(2), & 3a+6b-\ c+4 & (3)\\ & \underline{4a\qquad -8c\qquad +x} & (4)\\ (3)-(4), & -\ a+6b+7c+4-x & \end{array}$$

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1. $8x-2=6x+6.$

Transposing, $8x-6x=2+6.$

Whence $x=4.$

2. $4x-5=2x+10.$

Transposing,

$4x-2x=5+10.$

Whence $x=7\frac{1}{2}.$

3. $6y-5=9y+2.$

Transposing, $6y-9y=5+2.$

Whence $y=-2\frac{1}{3}.$

4. $7y+3=10+8y.$

Transposing,

$7y-8y=10-3.$

Whence $y=-7.$

5. $5n-3+21=18+4n.$

Combining,

$5n+18=18+4n.$

Transposing,

$5n-4n=18-18.$

Whence $n=0.$

6. $6 + 4n - 15 = 15 - n.$
Transposing,
$4n + n = 15 + 9.$
Whence $n = 4\frac{4}{5}.$

7. $5n + 3 - 2n = 7 - 4.$
Combining, $3n + 3 = 3.$
Whence $n = 0.$

8. $3k + 9 + 5k + 31 = 0.$
Combining, $8k + 40 = 0.$
Whence $k = -5.$

9. $6k + 3 - 2k = 27.$
Whence $k = 6.$

10. $3x - 6 = 34 + 8x.$
Transposing,
$-5x = 40.$
Whence $x = -8.$

11. $2x - 14 - 5x + 4 = 0.$
Whence $x = -3\frac{1}{3}.$

12. $x + 12 - 11x = -15x + 22.$
Transposing,
$15x - 10x = 22 - 12.$
Whence $x = 2.$

13. $5y + 3 = 17 + 3y + 8.$
Transposing,
$5y - 3y = 25 - 3.$
Whence $y = 11.$

14. $3y + 5 + 8y + \frac{1}{2} = 0.$
Whence $y = -\frac{1}{2}.$

15. $2 - 4h = 3 - 8h + 8.$
Transposing,
$8h - 4h = 11 - 2.$
Whence $h = 2\frac{1}{4}.$

16. $3 - 5h + 2 = 7h + 5.$
Whence $h = 0.$

17. $3h - 25 + 8h - 20 = 0.$
Whence $h = 4\frac{1}{11}.$

18. $14x - 6x = 22 + 17x - 11x.$
Transposing,
$8x - 6x = 22.$
Whence $x = 11.$

19. $7x - 13 + 8 = x - 27 - 5x.$
Transposing,
$7x + 4x = 5 - 27.$
Whence $x = -2.$

20. $4x - 15 - 11x - 18 + 16x - 17 = 0.$
Whence $x = 5\frac{5}{9}.$

21. $5y - 6 + 3y + 18 - 2y - 25 + 1 = 0.$
Whence $y = 2.$

22. $0 = 9x - 3 - 4x + 27 + 16x + 18.$
Whence $x = -2.$

23. $7n - 5 - 4n + 8 = 3n + 18 - 2n - 3.$
Collecting, $3n + 3 = n + 15.$
Whence $n = 6.$

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1. $n + 5.$
2. $n + a.$
3. $n - 3.$
4. $n - b.$
5. $4n.$
6. $a + b + 3.$
7. $a + b + c.$
8. $2a - b - 5.$
9. $2a - b - c.$
10. $2n - b.$
11. $5n + a.$
12. $4n - 7.$
13. $3n + 8.$
14. $10 - 6$, or $4.$
15. $x - 4.$
16. $12 - y.$
17. $x - a.$
18. $a - x.$
19. $x + y - z.$
20. $18 - 7$, or $11.$

21. $18 - n$. **23.** $n - 18$. **25.** $a - 7$.
22. $34 - 18$, or 16. **24.** $30 - b$. **26.** $a - x$.

27. $6 - d$, if 6 is the greater number. $6 + d$, if 6 is the less number.

28. $n - d$, if n is the greater number. $n + d$, if n is the less number.

29. $10 - 4 = 6$; $10 - x$.

30. $25 - 9 = 16$; $25 - y$; $16 - (a + b)$.

31. $40 - 27 = 13$; $40 - 40 = 0$; $40 - a$; $a - b$.

32. $36 - 22 = 14$; $a - 14$; $y - x$.

33. $a + 6 - (a - 6) = 12$; $a + 6 - (b - 6) = a - b + 12$;
$4x - 3 - (3x - 4) = x + 1$.

34. $n + 4$ years; $n + x$ years; $n - 3$ years.

35. $2n - 3 + 10$ years $= 2n + 7$ years; $2n - 3 + a$ years $= 2n - 3 + a$ years;
$2n - 3 - 8$ years $= 2n - 11$ years; $2n - 3 - a$ years.

36. A, $x - 4$ dollars; B, $x + 4$ dollars.

37. A, $x + 50 + y$ dollars; B, $x + 50 - y$ dollars.

38. (*a*) $x + 30 + 3x - 4 = 200$. (*c*) $x + 30 + 10 = 3x - 4$.
(*b*) $x + 30 = 3x - 4$. (*d*) $x + 30 + 100 = 3x - 4 - 50$.

39. (*a*) A, $x + 5$ years; B, $2x + 12$ years; C, $3x - 3$ years.
(*b*) A, $x - 3$ years; B, $2x + 4$ years; C, $3x - 11$ years.
(*c*) $x + 4 + 2x + 7 + 4 = 40$.
(*d*) $(3x - 8 - 6) - (x - 6) = 24$, if C is older than A.
(*e*) $x + 10 = 2x + 7$.
(*f*) $3x - 8 - 4 = 2x + 7 + 10$.
(*g*) $2x + 7 + x = 40$.
(*h*) $x + 2 + 2x + 7 + 2 + 3x - 8 + 2 = 100$.

40. A number less 2 is 8.

41. The sum of a number and 3 is 5.

42. Three times a number is 27.

43. Four times a number less 2 is 16.

44. Subtracting a number from 18 gives the same result as subtracting 4 from the number.

45. Subtracting 4 from three times a number gives the same result as adding 8 to twice the number.

46. The sum of a number and a second number is 20.

47. The result of subtracting a second number from a number is 2.

48. Twice one number has the same value as the sum of a second number and 6.

49. Three times a number less twice a second number is 8.

50. The sum of a number and a second number is a.

51. A number less a second number is b.

52. The sum of a number and a is a second number.

53. A number less b is equal to a second number.

54. Three times a number is equal to twice a second number.

55. A number is equal to twice a second number less 6.

56. The sum of three times a number and 4 is equal to the sum of twice a second number and 4.

57. Eighty less a certain number is equal to the sum of a second number and 30.

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1. Let $n =$ the unknown number.
Then $n + 22 = 50.$
Whence $n = 28.$

2. Let $n =$ the unknown number.
Then $n - 15 = 47.$
Whence $n = 62.$

3. Let $n =$ the unknown number.
Then $n + 9 = 28.$
Whence $n = 19.$

4. Let $n =$ the unknown number.
Then $n - 17 = 35.$
Whence $n = 52.$

5. Let $n =$ the unknown number.
Then $2n - 27 = 49.$
Whence $n = 38.$

6. Let $n =$ the unknown number.
Then $3n - 36 = 2n.$
Whence $n = 36.$

7. Let $n =$ the unknown number.
Then $3n - 17 = 2n - 1.$
Whence $n = 16.$

8. Let $n =$ the unknown number.
Then $5n + 6 = 2n + 15.$
Whence $n = 3.$

9. Let $n =$ the unknown number.
Then $4n + 9 = 7n - 33.$
Whence $n = 14.$

10. Let $n =$ the unknown number.
Then $n + 9 = 71 - n.$
Whence $n = 31.$

11. Let $n =$ the unknown number.
Then $2n + 6 = 4n - 10.$
Whence $n = 8.$

13. Let $n =$ the less number.
Then $n + 12 =$ the greater number.
Therefore $n + n + 12 = 74.$
Whence $n = 31,$
and $n + 12 = 43.$

14. Let $n =$ the less number.
Then $n + 3 =$ the greater number.
Therefore $n + n + 3 = 45.$
Whence $n = 21,$
and $n + 3 = 24.$

15. Let $n =$ the less number.
Then $n + 8 =$ the greater number.
Therefore $n + n + 8 = 44.$
Whence $n = 18,$
and $n + 8 = 26.$

16. Let $n =$ the first number.
Then $n - 4 =$ the second number,
and $n + 9 =$ the third number.
Therefore $n + n - 4 + n + 9 = 83.$
Whence $n = 26,$
$n - 4 = 22,$
and $n + 9 = 35.$

17. Let $n =$ the first number.
Then $n - 3 =$ the second number,
and $n - 3 + 18$, or $n + 15 =$ the third number.
Therefore $n + n - 3 + n + 15 = 66.$
Whence $n = 18,$
$n - 3 = 15,$
and $n + 15 = 33.$

18. Let $n =$ one number.
Then $n + 1 =$ the other.
Therefore $n + n + 1 = 37.$
Whence $n = 18,$
and $n + 1 = 19.$

19. Let $n =$ the first number.
Then $n + 1$ and $n + 2 =$ the next two numbers.

Therefore $n + n + 1 + n + 2 = 39.$
Whence $n = 12,$
$n + 1 = 13,$
and $n + 2 = 14.$

20. Let $n =$ the first number.
Then $n + 1$, $n + 2$, and $n + 3 =$ the next three numbers.
Therefore $n + n + 1 + n + 2 + n + 3 = 90.$
Whence $n = 21,$
$n + 1 = 22,$
$n + 2 = 23,$
and $n + 3 = 24.$

21. Let $n =$ the first even number.
Then $n + 2 =$ the next even number.
Therefore $n + n + 2 = 30.$
Whence $n = 14,$
and $n + 2 = 16.$

22. Let $n =$ the first odd number.
Then $n + 2$ and $n + 4 =$ the next two odd numbers.
Therefore $n + n + 2 + n + 4 = 87.$
Whence $n = 27,$
$n + 2 = 29,$
and $n + 4 = 31.$

23. Let $n =$ the first even number.
Then $n + 2$, $n + 4$, and $n + 6 =$ the next three even numbers.
Therefore $n + n + 2 + n + 4 + n + 6 = 100.$
Whence $n = 22,$
$n + 2 = 24,$
$n + 4 = 26,$
and $n + 6 = 28.$

24. Let $w =$ the width in feet.
Then $w + 3 =$ the length in feet.
Therefore $2w + 2w + 6 = 38.$
Whence $w = 8,$
and $w + 3 = 11.$

25. Let $w =$ the width in feet.
Then $w + 16 =$ the length in feet.
Therefore $2w + 2w + 32 = 128.$
Whence $w = 24,$
and $w + 16 = 40.$

26. Let $w =$ the width in feet.
Then $2w + 7 =$ the length in feet.
Therefore $2w + 4w + 14 = 104.$
Whence $w = 15,$
and $2w + 7 = 37.$

27. Let $w =$ the width in feet.
Then $4w + 5 =$ the length in feet.
Therefore $2w + 8w + 10 = 90.$
Whence $w = 8,$
and $4w + 5 = 37.$

28. Let $x =$ the length of the night in hours.
Then $x + 6\frac{1}{10} =$ the length of the day in hours.
Therefore $x + x + 6\frac{1}{10} = 24.$
Whence $x = 8\frac{19}{20}$ hours, or 8 hours and 57 minutes,
and $x + 6\frac{1}{10} = 15\frac{1}{20}$ hours, or 15 hours and 3 minutes.

29. Let $x =$ A's age in years.
Then $\frac{x}{2} =$ B's age in years,
and $x + 7 =$ C's age in years.
Therefore $x + \frac{x}{2} + x + 7 = 67.$
Whence $x = 24,$
$\frac{x}{2} = 12,$
and $x + 7 = 31.$

30. Let $x =$ B's age in years.
Then $3x =$ A's age in years,
and $x + 10 =$ C's age in years.
Therefore $3x + 5 + x + 5 + x + 10 + 5 = 60.$
Whence $x = 7,$
$3x = 21,$
and $x + 10 = 17.$

31. Let $x =$ B's age in years.
Then $x + 10 =$ A's age in years,
and $x - 6 =$ C's age in years.
Therefore $x + 10 - 4 + x - 4 + x - 6 - 4 = 46.$
Whence $x = 18,$
$x + 10 = 28,$
and $x - 6 = 12.$

32. Let $x =$ B's age in years.
Then $2x + 2 =$ A's age in years,
and $2x - 5 =$ C's age in years.
Therefore $2x + 2 + 6 + x + 6 + 2x - 5 + 6 = 70.$
Whence $x = 11,$
$2x + 2 = 24,$
and $2x - 5 = 17.$

33. Let $x =$ the number of million bushels of oats.
Then $x + 1838 =$ the number of million bushels of corn,
and $x - 120 =$ the number of million bushels of wheat.
Therefore $x + x + 1838 + x - 120 = 3981.$
Whence $x = 754\frac{1}{3},$
$x + 1838 = 2592\frac{1}{3},$
and $x - 120 = 634\frac{1}{3}.$

34. Here $47 =$ the number of degrees in the width of the torrid zone,
and $\frac{47}{2}$, or $23\frac{1}{2} =$ the number of degrees in the width of either frigid zone.
Let $x =$ the number of degrees in the width of either temperate zone.
Therefore $47 + 23\frac{1}{2} + 23\frac{1}{2} + x + x = 180.$
Whence $x = 43.$

35. Let $x =$ the per cent in commerce.
Then $x + 8 =$ the per cent in industries,
and $x + 20 =$ the per cent in agriculture.
Therefore $x + x + 8 + x + 20 = 100 - 24.$
Whence $x = 16,$
$x + 8 = 24,$
and $x + 20 = 36.$

36. Let $x =$ the number of million pounds produced by Montana.
Then $x - 110 =$ the number of million pounds produced by Michigan,
and $x - 139 =$ the number of million pounds produced by Arizona.
Therefore $x + x - 110 + x - 139 = 514.$
Whence $x = 254\frac{1}{3},$
$x - 110 = 144\frac{1}{3},$
and $x - 139 = 115\frac{1}{3}.$

37. Let $x =$ the height in feet of the Washington Monument.
Then $2x - 120 =$ the height in feet of the Eiffel Tower,
$x - 105 =$ the height in feet of the Great Pyramid,
and $x - 107 =$ the height in feet of St. Peter's.

Therefore $x + 2x - 120 + x - 105 + x - 107 = 2443.$
Whence $x = 555,$
$2x - 120 = 990,$
$x - 105 = 450,$
and $x - 107 = 448.$

38. Let $x =$ the area in square miles of the coal fields of all countries not named.
Then $2x + 38{,}400 =$ the area in square miles of the fields of the United States,
and $2x + 44{,}400 =$ the area in square miles of the fields of China and Japan.
Therefore $x + 2x + 38{,}400 + 2x + 44{,}400 = 471{,}800.$
Whence $x = 77{,}800,$
$2x + 38{,}400 = 194{,}000,$
and $2x + 44{,}400 = 200{,}000.$

39. Let $x =$ the output of Japan in tons.
Then $5x - 4615 =$ the output of other countries in tons,
$2x - 808 =$ the output of Spain and Portugal in tons,
and $5(2x - 808) - 5573,$
or $10x - 9613 =$ the output of the United States in tons.
Therefore $x + 5x - 4615 + 2x - 808 + 10x - 9613 = 486{,}084.$
Whence $x = 27{,}840,$
$5x - 4615 = 134{,}585,$
$2x - 808 = 54{,}872,$
and $10x - 9613 = 268{,}787.$

40. Let $x =$ the number of square miles in North America.
Then $2x + 1{,}818{,}000 =$ the number of square miles in Asia,
$x + 391{,}000 =$ the number of square miles in South America,
and $(x + 391{,}000) - 3{,}282{,}000,$
or $x - 2{,}891{,}000 =$ the area of Europe in square miles.
Therefore $x + 2x + 1{,}818{,}000 + x + 391{,}000 + x - 2{,}891{,}000$
$= 31{,}548{,}000.$
Whence $x = 6{,}446{,}000,$
$2x + 1{,}818{,}000 = 14{,}710{,}000,$
$x + 391{,}000 = 6{,}837{,}000,$
and $x - 2{,}891{,}000 = 3{,}555{,}000.$

41. Let $x =$ the number in the Revolution.
Then $9x - 15{,}621 =$ the number in the Civil War,
and $x + 266{,}841 =$ the number in the War of 1812.
Therefore $x + 9x - 15{,}621 + 266{,}841 + x = 3{,}658{,}811.$

Whence $x = 309{,}781,$
$9x - 15{,}621 = 2{,}772{,}408,$
and $x + 266{,}841 = 576{,}622.$

42. Let $x =$ the capacity of St. Peter's.
Then $x - 29{,}000 =$ the capacity of St. Paul's (London),
$x - 17{,}000 =$ the capacity of Cathedral (Milan),
and $x - 22{,}000 =$ the capacity of St. Paul's (Rome).
Therefore $x + x - 29{,}000 + x - 17{,}000 + x - 22{,}000 = 148{,}000.$
Whence $x = 54{,}000,$
$x - 29{,}000 = 25{,}000,$
$x - 17{,}000 = 37{,}000,$
and $x - 22{,}000 = 32{,}000.$

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1. $14 - (6 - 3) - 5 = 14 - 3 - 5 = 6.$
2. $10 + (7 - 4) - (9 - 7) = 10 + 3 - 2 = 11.$
3. $(7 - 3 + 2) - (6 - 4) + 11 = 6 - 2 + 11 = 15.$
4. $11a - (4a - 9a) + (6a - a) = 11a - (-5a) + 5a = 21a.$
5. $(2a - 5a) - (4a - a - 7a) = -3a - (-4a) = a.$
6. $a - (b - c) + (2b - 3c) = a - b + c + 2b - 3c = a + b - c.$
7. $a - b - (c - d) + (a + b) - (b - c)$
$= a - b - c + d + a + b - b + c = 2a - b + d.$
8. $(x - y) - (2y - 3x) + (x - 4y)$
$= x - y - 2y + 3x + x - 4y = 5x - 7y.$
9. $x - (x - y + 2z) - (3z - y + 4) + (x - 6)$
$= x - x + y - 2z - 3z + y - 4 + x - 6$
$= x + 2y - 5z - 10.$
10. $7 - [8 - (3 - 10)] - (13 - 25) = 7 - [8 + 7] - (-12) = 19 - 15 = 4.$
11. $a + [2a - (3a - 2b)] + (3b - 2a)$
$= a + [2b - a] + 3b - 2a = 5b - 2a.$
12. $(5x - 6y) - [-2x - (4z - y) - 2z]$
$= 5x - 6y - [-2x - 6z + y] = 7x - 7y + 6z.$
13. $[3x - (2y - z)] - [-(3y - 2x) - 5x]$
$= 3x - 2y + z - [-3y - 3x] = 6x + y + z.$
14. $[(a + 3) - (x - 5)] - [a + 3 + (x - 5)]$
$= (a + 3 - x + 5) - (a + 3 + x - 5) = 10 - 2x.$
15. $7 - [-6 - \{-4 + (6 - 10)\} + 11] = 7 - [-6 - \{-8\} + 11]$
$= 7 - [5 + 8] = -6.$
16. $-5x + [+10x - \{+11x - (2x - 7x + 4) - 3x\} - 22]$
$= -5x + [+10x - \{+8x + 5x - 4\} - 22]$
$= -5x + [-3x - 18] = -8x - 18.$

17. $\{4a - [2a - (3a - 2b) + 4a] - (4b - 6)\}$
$= 4a - [6a - 3a + 2b] - 4b + 6$
$= 4a - 3a - 2b - 4b + 6 = a - 6b + 6.$

18. $2x - 3y - [\{+3z - 7x - (y + 4z) - 9x\} + z]$
$= 2x - 3y - [3z - 16x - y - 4z + z]$
$= 2x - 3y + 16x + y = 18x - 2y.$

19. $(4y - 7x) - \{3x - [4x + (7y - 4x) - (3y - 3x)]\}$
$= 4y - 7x - 3x + [4x + 7y - 4x - 3y + 3x]$
$= 4y - 10x + [4y + 3x] = 8y - 7x.$

20. $[(a + b) + c] = [a + b + c].$
$[(a + b) - c] = [a + b - c].$

21. $[4x + (3z - 5y)] = [4x + 3z - 5y].$
$[4x - (3z - 5y)] = [4x - 3z + 5y].$

22. $[(a - 2b) + (3c - d)] = [a - 2b + 3c - d].$
$[(a - 2b) - (3c - d)] = [a - 2b - 3c + d].$

23. $[(4x - 3) + (5y - 7)] = [4x - 3 + 5y - 7] = [4x + 5y - 10].$
$[(4x - 3) - (5y - 7)] = [4x - 3 - 5y + 7] = [4x - 5y + 4].$

24. $[(x^2 - a^2) + (y^2 - 2a^2)] = [x^2 - a^2 + y^2 - 2a^2] = [x^2 + y^2 - 3a^2].$
$[(x^2 - a^2) - (y^2 - 2a^2)] = [x^2 - a^2 - y^2 + 2a^2] = [x^2 - y^2 + a^2].$

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1. $x^2 - a^2 - 2ab - b^2 = (x^2) - (a^2 + 2ab + b^2).$
2. $12ab + x^2 - 9b^2 - 4a^2 = (x^2) - (4a^2 - 12ab + 9b^2).$
3. $y^2 - 4b^2 + 4ab - a^2 = (y^2) - (4b^2 - 4ab + a^2).$
4. $10ab + x^2 - a^2 - 25b^2 = (x^2) - (a^2 - 10ab + 25b^2).$
5. $x^2 - b^2 - 4a^2 + 4y^2 - 4ab - 4xy$
$= (x^2 - 4xy + 4y^2) - (4a^2 + 4ab + b^2).$
6. $4ab + x^2 - 4b^2 + y^2 - a^2 - 2xy$
$= (x^2 - 2xy + y^2) - (a^2 - 4ab + 4b^2).$
7. $16x^2 - a^2 - 16xy - b^2 + 2ab + 4y^2$
$= (16x^2 - 16xy + 4y^2) - (a^2 - 2ab + b^2).$
8. $x^2 - b^2 - 10xy + 12ab - 36a^2 + 25y^2$
$= (x^2 - 10xy + 25y^2) - (36a^2 - 12ab + b^2).$

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1. $(3)(-8) = -24.$
2. $(-2)(5) = -10.$
3. $(-7)(-3) = 21.$
4. $(-4x)(3) = -12x.$
5. $(-4x)^2 = (-4x)(-4x) = 16x^2.$
6. $(7)(-5a) = -35a.$
7. $(3a)(-6) = -18a.$
8. $(-2y)^2 = (-2y)(-2y) = 4y^2.$
9. $(-9a)(-10) = 90a.$
10. $(-3ax)^2 = (-3ax)(-3ax)$
$= 9a^2x^2.$
11. $(4a)(-2a) = -8a^2.$

12. $(6\,abc)^2 = (6\,abc)(6\,abc)$
$= 36\,a^2b^2c^2.$
13. $(-11\,x)(2\,x) = -22\,x^2.$
14. $(7\,x)(-3\,x) = -21\,x^2.$
15. $(-2\,a)^3 = (-2\,a)(-2\,a)(-2\,a)$
$= -8\,a^3.$
16. $(-2\,a)(-3\,a^2) = 6\,a^3.$
17. $(5\,a^4)(7\,a^3) = 35\,a^7.$
18. $(-4\,x)^3 = -64\,x^3.$
19. $(a^8)(-20\,a) = -20\,a^9.$
20. $(-4\,a^5)(-6\,a^2) = 24\,a.^7$
21. $(+6\,y)^3 = 216\,y^3.$
22. $(4\,x)(5\,y) = 20\,xy.$
23. $(-3\,a^2x)^2 = 9\,a^4x^2.$
24. $(3\,x^2)(-y) = -3\,x^2y.$
25. $(5\,x^2y)(-2\,x^3) = -10\,x^5y.$
26. $(-6\,x^3y^2)^2 = 36\,x^6y^4.$
27. $(-x^4y)(-x^2y^4) = x^6y^5.$
28. $(2\,ax^2)^3 = (2\,ax^2)(2\,ax^2)(2\,ax^2)$
$= 8\,a^3x^6.$
29. $(5\,a^3)(-4\,a^2)(-3\,a) = 60\,a^6.$
30. $(3ax)(-4a^2x)(-2ax^3) = 24\,a^4x^5.$

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1. $\begin{array}{l} x+3 \\ \underline{2\,x\qquad\quad} \\ 2\,x^2+6\,x \end{array}$

2. $\begin{array}{l} 7\,x^2-5 \\ \underline{3\,x^3\qquad\qquad} \\ 21\,x^5-15\,x^3 \end{array}$

3. $\begin{array}{l} 5\,x^2-2\,x \\ \underline{-4\,x^2\qquad\qquad} \\ -20\,x^4+8\,x^3 \end{array}$

4. $\begin{array}{l} 7\,xy-z \\ \underline{3\,xy\qquad\qquad\quad} \\ 21\,x^2y^2-3\,xyz \end{array}$

5. $\begin{array}{l} -4\,x^2+5\,x-6 \\ \underline{6\,x^3\qquad\qquad\qquad\qquad\quad} \\ -24\,x^5+30\,x^4-36\,x^3 \end{array}$

6. $\begin{array}{l} x^3-3\,x^2+4 \\ \underline{-5\,x^4\qquad\qquad\qquad\qquad\quad} \\ -5\,x^7+15\,x^6-20\,x^4 \end{array}$

7. $\begin{array}{l} x^2-2\,xy+y^2 \\ \underline{-3\,xy\qquad\qquad\qquad\qquad\qquad} \\ -3\,x^3y+6\,x^2y^2-3\,xy^3 \end{array}$

8. $\begin{array}{l} a^4-a^2b^2+b^4 \\ \underline{-a^2b^2\qquad\qquad\qquad\qquad} \\ -a^6b^2+a^4b^4-a^2b^6 \end{array}$

9. $\begin{array}{l} -a^2x^2+2\,ax-7\,b^2 \\ \underline{-4\,abx\qquad\qquad\qquad\qquad\qquad\quad} \\ 4\,a^3bx^3-8\,a^2bx^2+28\,ab^3x \end{array}$

10. $\begin{array}{l} 7\,x^3-8\,x^2-12\,x+6 \\ \underline{-\frac{3}{4}\,x^3\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\ -5\frac{1}{4}\,x^6+6\,x^5+9\,x^4-4\frac{1}{2}\,x^3 \end{array}$

11. $\begin{array}{l} -9\,a^2-12\,ax+42\,x^2 \\ \underline{\frac{7}{3}\,ax^3\qquad\qquad\qquad\qquad\qquad\qquad} \\ -21\,a^3x^3-28\,a^2x^4+98\,ax^5 \end{array}$

12. $4(2\,x-3) = 8\,x-12.$
13. $2\,x(x-y) = 2\,x^2-2\,xy.$
14. $-8(3\,x-7) = -24\,x+56.$
15. $-9(-4\,a+b) = 36\,a-9\,b.$
16. $-3\,x(2\,x-7) = -6\,x^2+21\,x.$
17. $-3(x^2-2\,x-6) = -3\,x^2+6\,x+18.$
18. $5\,xy(x^2-6\,x+9) = 5\,x^3y-30\,x^2y+45\,xy.$
19. $-3\,x(ax-bx+3\,cx^2) = -3\,ax^2+3\,bx^2-9\,cx^3.$
20. $-7\,ab(ax^2+bx+c) = -7\,a^2bx^2-7\,ab^2x-7\,abc.$

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$$\begin{array}{rll}
1. & x + 4 & (1)\\
 & \underline{x + 3} & (2)\\
 & x^2 + 4x & \\
 & \underline{\qquad + 3x + 12} & \\
(1)\cdot(2), & x^2 + 7x + 12 &
\end{array}$$

$$\begin{array}{rll}
2. & 2x + 3 & (1)\\
 & \underline{\;\; x + 3} & (2)\\
 & 2x^2 + 3x & \\
 & \underline{\qquad + 6x + 9} & \\
(1)\cdot(2), & 2x^2 + 9x + 9 &
\end{array}$$

$$\begin{array}{rll}
3. & 4x + 7 & (1)\\
 & \underline{3x + 2} & (2)\\
 & 12x^2 + 21x & \\
 & \underline{\qquad\quad + \;\; 8x + 14} & \\
(1)\cdot(2), & 12x^2 + 29x + 14 &
\end{array}$$

$$\begin{array}{rll}
4. & 3x - 5 & (1)\\
 & \underline{3x + 8} & (2)\\
 & 9x^2 - 15x & \\
 & \underline{\qquad + 24x - 40} & \\
(1)\cdot(2), & 9x^2 + \;\; 9x - 40 &
\end{array}$$

$$\begin{array}{rll}
5. & 3x - 2 & (1)\\
 & \underline{2x + 3} & (2)\\
 & 6x^2 - 4x & \\
 & \underline{\qquad + 9x - 6} & \\
(1)\cdot(2), & 6x^2 + 5x - 6 &
\end{array}$$

$$\begin{array}{rll}
6. & -4a + 6 & (1)\\
 & \underline{\;\; 5a - 7} & (2)\\
 & -20a^2 + 30a & \\
 & \underline{\qquad\qquad + 28a - 42} & \\
(1)\cdot(2), & -20a^2 + 58a - 42 &
\end{array}$$

$$\begin{array}{rll}
7. & 2x + y & (1)\\
 & \underline{\;\; x + 3y} & (2)\\
 & 2x^2 + \;\; xy & \\
 & \underline{\qquad + 6xy + 3y^2} & \\
(1)\cdot(2), & 2x^2 + 7xy + 3y^2 &
\end{array}$$

$$\begin{array}{rll}
8. & 2x - 3y & (1)\\
 & \underline{3x - 2y} & (2)\\
 & 6x^2 - \;\; 9xy & \\
 & \underline{\qquad - \;\; 4xy + 6y^2} & \\
(1)\cdot(2), & 6x^2 - 13xy + 6y^2 &
\end{array}$$

$$\begin{array}{rll}
9. & 3x - \frac{1}{2} & (1)\\
 & \underline{2x - \frac{1}{3}} & (2)\\
 & 6x^2 - \;\; x & \\
 & \underline{\qquad - \;\; x + \frac{1}{6}} & \\
(1)\cdot(2), & 6x^2 - 2x + \frac{1}{6} &
\end{array}$$

$$\begin{array}{rll}
10. & -3x + 11a & (1)\\
 & \underline{\quad 5x - a} & (2)\\
 & -15x^2 + 55xa & \\
 & \underline{\qquad\qquad + \;\; 3xa - 11a^2} & \\
(1)\cdot(2), & -15x^2 + 58xa - 11a^2 &
\end{array}$$

$$\begin{array}{rll}
11. & ax - bx & (1)\\
 & \underline{cx + dx} & (2)\\
 & acx^2 - bcx^2 & \\
 & \underline{\qquad\qquad + adx^2 - bdx^2} & \\
(1)\cdot(2), & acx^2 - bcx^2 + adx^2 - bdx^2 &
\end{array}$$

$$\begin{array}{rll}
12. & -cx + d & (1)\\
 & \underline{\quad bx - cx^2} & (2)\\
 & -bcx^2 + bdx & \\
 & \underline{\qquad\qquad + c^2x^3 - cdx^2} & \\
(1)\cdot(2), & -bcx^2 + bdx + c^2x^3 - cdx^2 &
\end{array}$$

$$\begin{array}{rll}
13. & 4x - \frac{1}{3} & (1)\\
 & \underline{6x + \frac{2}{5}} & (2)\\
 & 24x^2 - 2\;\; x & \\
 & \underline{\qquad + 1\frac{3}{5}x - \frac{2}{15}} & \\
(1)\cdot(2), & 24x^2 - \;\; \frac{2}{5}x - \frac{2}{15} &
\end{array}$$

$$\begin{array}{rll}
14. & x^2 - 5x + 6 & (1)\\
 & \underline{x - 3} & (2)\\
 & x^3 - 5x^2 + \;\; 6x & \\
 & \underline{\qquad - 3x^2 + 15x - 18} & \\
(1)\cdot(2), & x^3 - 8x^2 + 21x - 18 &
\end{array}$$

15.

$$\begin{array}{lll}
 & 3x^2 - 3x - 7 & (1)\\
 & \underline{2x + 4\qquad\qquad} & (2)\\
 & 6x^3 - 6x^2 - 14x & \\
 & \underline{\qquad + 12x^2 - 12x - 28} & \\
(1)\cdot(2), & 6x^3 + 6x^2 - 26x - 28 &
\end{array}$$

17.

$$\begin{array}{lll}
 & a^2x^2 - 2a^2x + 4a^2 & (1)\\
 & \underline{ax + 2a\qquad\qquad\qquad} & (2)\\
 & a^3x^3 - 2a^3x^2 + 4a^3x & \\
 & \underline{\qquad + 2a^3x^2 - 4a^3x + 8a^3} & \\
(1)\cdot(2), & a^3x^3 \qquad\qquad\qquad + 8a^3 &
\end{array}$$

16.

$$\begin{array}{lll}
 & x^2 - xy + y^2 & (1)\\
 & \underline{x + y\qquad\qquad} & (2)\\
 & x^3 - x^2y + xy^2 & \\
 & \underline{\qquad + x^2y - xy^2 + y^3} & \\
(1)\cdot(2), & x^3 \qquad\qquad\qquad + y^3 &
\end{array}$$

18.

$$\begin{array}{lll}
 & 3x^3 - x^2 - 5x & (1)\\
 & \underline{2x^3 - 5x^2\qquad\qquad} & (2)\\
 & 6x^6 - 2x^5 - 10x^4 & \\
 & \underline{\qquad - 15x^5 + 5x^4 + 25x^3} & \\
(1)\cdot(2), & 6x^6 - 17x^5 - 5x^4 + 25x^3 &
\end{array}$$

19.

$$\begin{array}{lll}
 & 2x^2 - 7x + 12 & (1)\\
 & \underline{x^2 - 3x - 5\qquad\qquad} & (2)\\
 & 2x^4 - 7x^3 + 12x^2 & \\
 & \qquad - 6x^3 + 21x^2 - 36x & \\
 & \underline{\qquad\qquad - 10x^2 + 35x - 60} & \\
(1)\cdot(2), & 2x^4 - 13x^3 + 23x^2 - x - 60 &
\end{array}$$

20.

$$\begin{array}{lll}
 & a^2 - \frac{1}{2}a + \frac{1}{4} & (1)\\
 & \underline{a^2 - a + \frac{1}{3}\qquad\qquad} & (2)\\
 & a^4 - \frac{1}{2}a^3 + \frac{1}{4}a^2 & \\
 & \qquad - a^3 + \frac{1}{2}a^2 - \frac{1}{4}a & \\
 & \underline{\qquad\qquad + \frac{1}{3}a^2 - \frac{1}{6}a + \frac{1}{12}} & \\
(1)\cdot(2), & a^4 - 1\frac{1}{2}a^3 + 1\frac{1}{12}a^2 - \frac{5}{12}a + \frac{1}{12} &
\end{array}$$

21.

$$\begin{array}{lll}
 & x^2 - xy + y^2 & (1)\\
 & \underline{x^2 + xy + y^2\qquad} & (2)\\
 & x^4 - x^3y + x^2y^2 & \\
 & \qquad + x^3y - x^2y^2 + xy^3 & \\
 & \underline{\qquad\qquad + x^2y^2 - xy^3 + y^4} & \\
(1)\cdot(2), & x^4 \qquad + x^2y^2 \qquad + y^4 &
\end{array}$$

22.

$$\begin{array}{lll}
 & 3x^3 + 5x^2 - x + 2 & (1)\\
 & \underline{x^2 - 2x + 1\qquad\qquad\qquad} & (2)\\
 & 3x^5 + 5x^4 - x^3 + 2x^2 & \\
 & \qquad - 6x^4 - 10x^3 + 2x^2 - 4x & \\
 & \underline{\qquad\qquad + 3x^3 + 5x^2 - x + 2} & \\
(1)\cdot(2), & 3x^5 - x^4 - 8x^3 + 9x^2 - 5x + 2 &
\end{array}$$

23.

$$\begin{array}{lll}
 & x^3 - x - 5 & (1)\\
 & \underline{2x^2 - 3x - 4\qquad\qquad\qquad} & (2)\\
 & 2x^5 \qquad - 2x^3 - 10x^2 & \\
 & \qquad - 3x^4 \qquad + 3x^2 + 15x & \\
 & \underline{\qquad\qquad - 4x^3 \qquad + 4x + 20} & \\
(1)\cdot(2), & 2x^5 - 3x^4 - 6x^3 - 7x^2 + 19x + 20 &
\end{array}$$

24.

$$\begin{array}{llr}
 & -x^3+x^2+3x-6 & (1)\\
 & \underline{-x^2-3x+5} & (2)\\
 & x^5-x^4-3x^3+6x^2 & \\
 & \quad +3x^4-3x^3-9x^2+18x & \\
 & \quad\quad \underline{-5x^3+5x^2+15x-30} & \\
(1)\cdot(2), & x^5+2x^4-11x^3+2x^2+33x-30 &
\end{array}$$

25.

$$\begin{array}{llr}
 & a^3-5a^2+4a+7 & (1)\\
 & \underline{a^3+a^2-a+6} & (2)\\
 & a^6-5a^5+4a^4+7a^3 & \\
 & \quad +a^5-5a^4+4a^3+7a^2 & \\
 & \quad\quad -a^4+5a^3-4a^2-7a & \\
 & \quad\quad\quad \underline{+6a^3-30a^2+24a+42} & \\
(1)\cdot(2), & a^6-4a^5-2a^4+22a^3-27a^2+17a+42 &
\end{array}$$

26.

$$\begin{array}{llr}
 & 2x^3-5x^2-9x+8 & (1)\\
 & \underline{8x^3+5x-4} & (2)\\
 & 16x^6-40x^5-72x^4+64x^3 & \\
 & \quad +10x^4-25x^3-45x^2+40x & \\
 & \quad\quad \underline{-8x^3+20x^2+36x-32} & \\
(1)\cdot(2), & 16x^6-40x^5-62x^4+31x^3-25x^2+76x-32 &
\end{array}$$

27.

$$\begin{array}{llr}
 & x^2y-y^2x & (1)\\
 & \underline{-5x^2y+4xy} & (2)\\
 & -5x^4y^2+5x^3y^3 & \\
 & \quad \underline{+4x^3y^2-4x^2y^3} & \\
(1)\cdot(2), & -5x^4y^2+5x^3y^3+4x^3y^2-4x^2y^3 & (3)\\
 & \underline{3x^2y-7xy^2} & (4)\\
 & -15x^6y^3+15x^5y^4+12x^5y^3-12x^4y^4 & \\
 & \quad \underline{+35x^5y^4-28x^4y^4-35x^4y^5+28x^3y^5} & \\
(3)\cdot(4), & -15x^6y^3+50x^5y^4+12x^5y^3-40x^4y^4-35x^4y^5+28x^3y^5 &
\end{array}$$

28.

$$\begin{array}{llr}
 & x^2-xy+y^2-xz-yz+z^2 & (1)\\
 & \underline{x+y+z} & (2)\\
 & x^3-x^2y+xy^2-x^2z-xyz+xz^2 & \\
 & \quad +x^2y-xy^2-xyz+y^3-y^2z+yz^2 & \\
 & \quad\quad \underline{+x^2z-xyz-xz^2+y^2z-yz^2+z^3} & \\
(1)\cdot(2), & x^3-3xyz+y^3+z^3 &
\end{array}$$

29.

$$\begin{array}{llr}
 & a+b+c & (1)\\
 & \underline{a+b+c} & (2)\\
 & a^2+ab+ac & \\
 & \quad +ab+b^2+bc & \\
 & \quad\quad \underline{+ac+bc+c^2} & \\
(1)\cdot(2), & a^2+2ab+b^2+2ac+2bc+c^2 &
\end{array}$$

$$\begin{array}{lll}
\textbf{30.} & c+d-\frac{1}{2} & (1)\\
 & \underline{c+d-\frac{1}{2}\qquad\qquad} & (2)\\
 & c^2+cd\qquad -\frac{1}{2}c & \\
 & \quad +cd+d^2\qquad -\frac{1}{2}d & \\
 & \underline{\qquad\qquad\qquad -\frac{1}{2}c-\frac{1}{2}d+\frac{1}{4}} & \\
(1)\cdot(2), & c^2+2cd+d^2-c-d+\frac{1}{4} &
\end{array}$$

$$\begin{array}{lll}
\textbf{31.} & a-2b+3c-4d & (1)\\
 & \underline{a-2b+3c-4d\qquad} & (2)\\
 & a^2-2ab+3ac-4ad & \\
 & \quad -2ab\qquad\qquad +4b^2-6bc+8bd & \\
 & \qquad\quad +3ac\qquad\qquad -6bc\qquad +9c^2-12cd & \\
 & \underline{\qquad\qquad\quad -4ad\qquad\qquad +8bd\qquad -12cd+16d^2} & \\
(1)\cdot(2), & a^2-4ab+6ac-8ad+4b^2-12bc+16bd+9c^2-24cd+16d^2 &
\end{array}$$

$$\begin{array}{lll}
\textbf{32.} & x+y+z & (1)\\
 & \underline{x+y+z\qquad} & (2)\\
 & x^2+xy+xz & \\
 & \quad +xy\qquad +y^2+yz & \\
 & \underline{\qquad\quad +xz\qquad +yz+z^2} & \\
(1)\cdot(2), & x^2+2xy+2xz+y^2+2yz+z^2 & (3)\\
 & \underline{x+y+z\qquad\qquad\qquad\qquad\qquad\qquad} & (4)\\
 & x^3+2x^2y+2x^2z+xy^2+2xyz+xz^2 & \\
 & \quad +x^2y\qquad +2xy^2+2xyz\qquad +y^3+2y^2z+yz^2 & \\
 & \underline{\qquad\quad +x^2z\qquad +2xyz+2xz^2\qquad +y^2z+2yz^2+z^3} & \\
(3)\cdot(4), & x^3+3x^2y+3x^2z+3xy^2+6xyz+3xz^2+y^3+3y^2z+3yz^2+z^3 &
\end{array}$$

$$\begin{array}{llllll}
\textbf{33.} & x+y & (1) & & x-y & (6)\\
 & \underline{x+y} & (2) & & \underline{x-y} & (7)\\
 & x^2+xy & & & x^2-xy & \\
 & \underline{\qquad xy+y^2} & & & \underline{\quad -xy+y^2} & \\
(1)\cdot(2), & x^2+2xy+y^2 & (3) & (6)\cdot(7), & x^2-2xy+y^2 & (8)\\
 & \underline{x+y\qquad\qquad} & (4) & & \underline{x-y\qquad\qquad\qquad} & (9)\\
 & x^3+2x^2y+xy^2 & & & x^3-2x^2y+xy^2 & \\
 & \underline{\qquad x^2y+2xy^2+y^3} & & & \underline{\quad -x^2y+2xy^2-y^3} & \\
(3)\cdot(4), & x^3+3x^2y+3xy^2+y^3 & (5) & (8)\cdot(9), & x^3-3x^2y+3xy^2-y^3 & (10)\\
 & \underline{x^3-3x^2y+3xy^2-y^3} & (10) & & & \\
(5)+(10), & 2x^3\qquad +6xy^2 & & & &
\end{array}$$

$$\begin{array}{lll}
\textbf{34.} & 2a-b+3 & (1)\\
 & \underline{2a-b+3\qquad\qquad} & (2)\\
 & 4a^2-2ab\qquad +6a & \\
 & \quad -2ab+b^2\qquad -3b & \\
 & \underline{\qquad\qquad\quad +6a-3b+9} & \\
(1)\cdot(2), & 4a^2-4ab+b^2+12a-6b+9 &
\end{array}$$

$$
\begin{array}{lll}
\textbf{35.} & 2x - 3ay & (1) \\
 & \underline{2x - 3ay} & (2) \\
 & 4x^2 - 6axy & \\
 & \underline{\quad - 6axy + 9a^2y^2} & \\
(1)\cdot(2), & 4x^2 - 12axy + 9a^2y^2 & (3) \\
 & \underline{2x - 3ay} & (4) \\
 & 8x^3 - 24ax^2y + 18a^2xy^2 & \\
 & \underline{\quad - 12ax^2y + 36a^2xy^2 - 27a^3y^3} & \\
(3)\cdot(4), & 8x^3 - 36ax^2y + 54a^2xy^2 - 27a^3y^3 &
\end{array}
$$

$$
\begin{array}{llllll}
\textbf{36.} & x + 2y & (1) & & x - 2y & (4) \\
 & \underline{x + 2y} & (2) & & \underline{x - 2y} & (5) \\
 & x^2 + 2xy & & & x^2 - 2xy & \\
 & \underline{\quad + 2xy + 4y^2} & & & \underline{\quad - 2xy + 4y^2} & \\
(1)\cdot(2) & x^2 + 4xy + 4y^2 & (3) & (4)\cdot(5), & x^2 - 4xy + 4y^2 & (6) \\
 & \underline{x^2 - 4xy + 4y^2} & (6) & & & \\
(3)-(6), & \quad 8xy & & & &
\end{array}
$$

$$
\begin{array}{llllll}
\textbf{37.} & 4x - 3y & (1) & & 3x + 4y & (4) \\
 & \underline{4x - 3y} & (2) & & \underline{3x + 4y} & (5) \\
 & 16x^2 - 12xy & & & 9x^2 + 12xy & \\
 & \underline{\quad - 12xy + 9y^2} & & & \underline{\quad 12xy + 16y^2} & \\
(1)\cdot(2), & 16x^2 - 24xy + 9y^2 & (3) & (4)\cdot(5), & 9x^2 + 24xy + 16y^2 & (6) \\
 & \underline{9x^2 + 24xy + 16y^2} & (6) & & & \\
(3)-(6), & 7x^2 - 48xy - 7y^2 & & & &
\end{array}
$$

$$
\begin{array}{llllll}
\textbf{38.} & x - 3 & (a), & & 2x - 1 & (b) \\
 & \underline{x - 3} & & & \underline{2x - 1} & \\
 & x^2 - 3x & & & 4x^2 - 2x & \\
 & \underline{\quad - 3x + 9} & & & \underline{\quad - 2x + 1} & \\
 & x^2 - 6x + 9 & & (b)^2, & 4x^2 - 4x + 1 & (2) \\
 & \underline{x - 3} & & & & \\
 & x^3 - 6x^2 + 9x & & & & \\
 & \underline{\quad - 3x^2 + 18x - 27} & & & & \\
(a)^3, & x^3 - 9x^2 + 27x - 27 & (1) & & & \\
(b)^2, & \underline{\quad + 4x^2 - 4x + 1} & (2) & & & \\
(1)-(2), & x^3 - 13x^2 + 31x - 28 & & & &
\end{array}
$$

$$
\begin{array}{llllll}
\textbf{39.} & x^a - 3 & (1) & \textbf{40.} & x^{2a} + 5 & (1) \\
 & \underline{x^a + 4} & (2) & & \underline{x^{2a} + 5} & (2) \\
 & x^{2a} - 3x^a & & & x^{4a} + 5x^{2a} & \\
 & \underline{\quad + 4x^a - 12} & & & \underline{\quad + 5x^{2a} + 25} & \\
(1)\cdot(2), & x^{2a} + x^a - 12 & & (1)\cdot(2), & x^{4a} + 10x^{2a} + 25 &
\end{array}
$$

41.

$$\begin{array}{lr}
2x^a-3 & (a)\\
\underline{2x^a-3} & \\
4x^{2a}-6x^a & \\
\underline{\phantom{4x^{2a}}-6x^a+9} & \\
4x^{2a}-12x^a+9 & \\
\underline{2x^a-3} & \\
8x^{3a}-24x^{2a}+18x^a & \\
\underline{\phantom{8x^{3a}}-12x^{2a}+36x^a-27} & \\
8x^{3a}-36x^{2a}+54x^a-27 & (a)^3,
\end{array}$$

42.

$$\begin{array}{lr}
2x^{2a}-3x & (1)\\
\underline{2x^{2a}-3x} & (2)\\
4x^{4a}-6x^{2a+1} & \\
\underline{\phantom{4x^{4a}}-6x^{2a+1}+9x^2} & \\
4x^{4a}-12x^{2a+1}+9x^2 & (1)\cdot(2),
\end{array}$$

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1. $5(x-1)=30.$

Removing parentheses,

$$5x-5=30.$$

Whence $x=7.$

2. $3+2(x-3)=1.$

$$3+2x-6=1.$$

Whence $x=2.$

3. $7(3x-2)+11=60.$

$$21x-14=49.$$

Whence $x=3.$

4. $4(2x-5)+15=3(x+10).$

$$8x-20+15=3x+30.$$

Whence $x=7.$

5. $12y-2(4y-7)-16$

$$=0.$$

$$12y-8y+14-16=0.$$

Whence $y=\frac{1}{2}.$

6. $9y-3(2y-4)$

$$=2(5-4y)+2.$$

$$9y-6y+12=10-8y+2.$$

Whence $y=0.$

7. $4-2(4y-3)=3(y-5).$

Simplifying,

$$4-8y+6=3y-15.$$

Whence $y=2\frac{3}{11}.$

8.

$$7(y-3)-2(4+y)=9.$$

$$7y-21-8-2y=9.$$

Whence $y=7\frac{3}{5}.$

9.

$$5(n-7)+24+4n=0.$$

$$5n-35+24+4n=0.$$

Whence $n=1\frac{2}{9}.$

10. $5n-9(2n+4)=2(n-9).$

$$5n-18n-36=2n-18.$$

Whence $n=-1\frac{1}{5}.$

11. $7n-12-2(n-5)=n-19.$

$$6n-2n+10=-7.$$

Whence $n=-4\frac{1}{4}.$

12. $4(2n-7)-3(4n-8)+4$

$$=2n-3.$$

$$8n-28-12n+24+4=2n-3.$$

Whence $n=\frac{1}{2}.$

13. $3h-2(4h+8)=3h-24.$

$$-8h-16=-24.$$

Whence $h=1.$

14. $5(3h+1)-7h=3(h-7)+4.$

$$15h+5-7h=3h-21+4.$$

Whence $h=-4\frac{2}{5}.$

15. $(h-2)(h-5)=(h+3)(h+2).$

Simplifying, $h^2-7h+10=h^2+5h+6.$

Whence $h=\frac{1}{3}.$

16. $(h+4)(h+3)-(h+2)(h+1)-42=0.$
$h^2+7h+12-h^2-3h-2-42=0.$
Whence $h=8.$

17. $(x+4)(x+6)=(x+18)(x+13).$
$x^2+10x+24=x^2+31x+234.$
Whence $x=-10.$

18. $(k-7)(5+k)-(k-5)(k+7)+5=0.$
$(k^2-2k-35)-(k^2+2k-35)+5=0.$
Simplifying, $-2k-2k+5=0.$
Whence $k=1\frac{1}{4}.$

19. $(2x-5)(4x-7)=8x^2+52.$
Simplifying, $8x^2-34x+35=8x^2+52.$
Whence $x=-\frac{1}{2}.$

20. $(3y+5)(4y+7)-(2y+3)(6y+11)-2=0.$
Simplifying, $12y^2+41y+35-(12y^2+40y+33)-2=0.$
Whence $y=0.$

21. $(n+3)(6n+5)-(2n+4)(3n-8)=38.$
Simplifying, $6n^2+23n+15-(6n^2-4n-32)=38.$
Whence $n=-\frac{1}{3}.$

22. $(x+3)^2-(x+5)^2=-40.$
Expanding, $x^2+6x+9-x^2-10x-25=-40.$
Whence $x=6.$

23. $(x+2)^2-(x-4)^2+48=0.$
Expanding, $x^2+4x+4-x^2+8x-16+48=0.$
Whence $x=-3.$

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1. Area, $a \cdot b = ab$; perimeter, $2a+2b$.
2. Area, $3(x-4)=3x-12$; perimeter, $2(x-4)+2\cdot 3=2x-2$.
3. Area, $(2x-4)(x+2)=2x^2-8$;
   perimeter, $2(2x-4)+2(x+2)=6x-4$.
4. $4\cdot\$100=\$400$.
5. $n\cdot\$80=\$80n$.
6. $a\cdot b$ cents $= ab$ cents.
7. $(xa+yb)$ cents.
8. $x(b+10)$ dollars $=(bx+10x)$ dollars.
9. $x(b+2)$ dollars $+\,y(c-3)$ dollars $=(bx+2x+cy-3y)$ dollars.
10. 5% of $16=.8$; of $x=.05x$.

11. 3% of $(x + 120) = .03x + 3.6$; of $(12x - 300a) = .36x - 9a$.

12. $3(n + 4)$ years $= (3n + 12)$ years.

13. (*a*) $.04x = 180$.
(*b*) $.03x = .05(1000 - x)$.
(*c*) $.05x = .04(1000 - x) - 20$.

14. Width, $10 + 2 + 2 = 14$ inches; length, $12 + 2 + 2 = 16$ inches.

15. Width, $10 + x + x = (10 + 2x)$ inches;
length, $12 + x + x = (12 + 2x)$ inches;
area, $(10 + 2x)(12 + 2x) = (120 + 44x + 4x^2)$ square inches;
area of picture $= 12 \cdot 10 = 120$ square inches;
area of frame $= 120 + 44x + 4x^2 - 120$
$= (44x + 4x^2)$ square inches.

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2. Let $n =$ the less number.
Then $49 - n =$ the greater number.
Therefore $2(49 - n) - 13 = 5n$.
Whence $n = 12\frac{1}{7}$,
and $49 - n = 36\frac{6}{7}$.

3. Let $n =$ the less number.
Then $143 - n =$ the greater number.
Therefore $10n + 5(143 - n) = 950$.
Whence $n = 47$,
and $143 - n = 96$.

4. Let $n =$ the greater number.
Then $45 - n =$ the less number.
Therefore $5n + 4(45 - n) = 207$.
Whence $n = 27$,
and $45 - n = 18$.

5. Let $n =$ the less number.
Then $88 - n =$ the greater number.
Therefore $3(88 - n) = 5n + 29$.
Whence $n = 29\frac{3}{8}$,
and $88 - n = 58\frac{5}{8}$.

6. Let $n =$ the greater number.
Then $93 - n =$ the less number.
Therefore $7(93 - n) - 7 = 6n$.
Whence $n = 49\frac{7}{13}$,
and $93 - n = 43\frac{6}{13}$.

7. Let $n =$ the less number.
Then $48 - n =$ the greater number.
Therefore $2(48 - n) - 7 = 3n - 5.$
Whence $n = 18\frac{4}{5},$
and $48 - n = 29\frac{1}{5}.$

8. Let $n =$ one number.
Then $12\frac{1}{2} - n =$ the other number.
Therefore $7n - 10(12\frac{1}{2} - n) = 45.$
Whence $n = 10,$
and $12\frac{1}{2} - n = 2\frac{1}{2}.$

9. Let n and $121 - n =$ the two parts of 121.
Therefore $4n + 8 = 3(121 - n).$
Whence $n = 50\frac{5}{7},$ one part,
and $121 - n = 70\frac{2}{7},$ the other.

10. Let n and $15 - n =$ the numbers.
Therefore $2n - 5(15 - n) = 240.$
Whence $n = 45,$ one number,
and $15 - n = -30,$ the other number.

11. Let $n =$ one number.
Then $14 - n =$ the other.
Therefore $9n - 11(14 - n) = 0.$
Whence $n = 7\frac{7}{10},$
and $14 - n = 6\frac{3}{10}.$

12. Let $n =$ the first number.
Then $n + 1 =$ the next consecutive number.
Therefore $n^2 + n^2 + 2n + 1 = 17 + 2n^2.$
Whence $n = 8,$
and $n + 1 = 9.$

13. Let $n =$ one number.
Then $n + 1 =$ the next number.
Therefore $(n^2 + 2n + 1) - n^2 = 75.$
Whence $n = 37,$
and $n + 1 = 38.$

14. Let n and $n + 1 =$ the numbers.
Therefore $(n^2 + 2n + 1) - n^2 = 23.$
Whence $n = 11,$ the first number,
and $n + 1 = 12,$ the next number.

15. Let $n =$ one odd number.
Then $n + 2 =$ the next odd number.
Therefore $(n^2 + 4n + 4) - n^2 = 104.$
Whence $n = 25,$
and $n + 2 = 27.$

16. Let $n =$ one odd number.
Then $n + 2 =$ the next odd number.
Therefore $(n^2 + 4n + 4) - n^2 = 40.$
Whence $n = 9,$
and $n + 2 = 11.$

17. Let $n =$ one even number.
Then $n + 2 =$ the next even number.
Therefore $n(n + 2) = (n^2 + 4n + 4) - 56.$
Whence $n = 26,$
and $n + 2 = 28.$

18. Let n and $n + 2 =$ two consecutive odd numbers.
Therefore $n(n + 2) = n^2 + 46.$
Whence $n = 23,$
and $n + 2 = 25.$

20. Let $s =$ the side of the square field in rods.
Then $s + 30 =$ the length of the rectangular field in rods,
and $s - 20 =$ the breadth of the rectangular field in rods.
Therefore $s^2 = (s + 30)(s - 20).$
Whence $s = 60,$
$s + 30 = 90,$
and $s - 20 = 40.$

Area of square $= 60 \cdot 60 = 3600$ square rods, or $22\frac{1}{2}$ acres.
Area of rectangle $= 90 \cdot 40 = 3600$ square rods, or $22\frac{1}{2}$ acres.

21. Let $x =$ the width of the court in feet.
Then $2x + 24 =$ the length of the court in feet.
Therefore $2x + 2(2x + 24) = 210.$
Whence $x = 27,$
and $2x + 24 = 78.$

22. Let $x =$ the width of the court in feet.
Then $2x + 6 =$ the length of the court in feet.
Therefore $2x + 2(2x + 6) = 228.$
Whence $x = 36,$
and $2x + 6 = 78.$

23. Let $l =$ the length of the court in feet.
Then $l - 20 =$ the breadth of the court in feet.
Therefore $2l + 2(l - 20) = 80 \cdot 3.$
Whence $l = 70,$
and $l - 20 = 50.$

24. Let $n =$ the breadth of the field in yards.
Then $3n - 50 =$ the length of the field in yards.
Therefore $2n + 2(3n - 50) = \frac{780}{3}.$
Whence $n = 45,$
and $3n - 50 = 85.$

26. Let $d =$ the number of dimes.
Then $38 - d =$ the number of quarters,
and $10d =$ the value of the dimes in cents.
Also $(38 - d)25 =$ the value of the quarters in cents.
Therefore $10d + (38 - d)25 = 530.$
Solving, $d = 28,$
and $38 - d = 10.$

27. Let $d =$ the number of dimes.
Then $d + 5 =$ the number of nickels,
and $2d + 5 =$ the number of quarters.
Also $10d =$ the value of the dimes in cents,
$5(d + 5) =$ the value of the nickels in cents,
and $25(2d + 5) =$ the value of the quarters in cents.
Therefore

$$10d + 5(d + 5) + 25(2d + 5) = 605.$$

Solving, $d = 7,$
$d + 5 = 12,$
and $2d + 5 = 19.$

28. Let $n =$ the number of nickels.
Then $40 - n =$ the number of dimes.
Also $5n =$ the value of the nickels in cents,
and $10(40 - n) =$ the value of the dimes in cents.
Therefore $5n + 10(40 - n) = 290.$
Solving, $n = 22,$
and $40 - n = 18.$

29. Let $b =$ B's age now in years.
Then $b + 20 =$ A's age now in years.
Therefore $b + 20 + 10 = 2(b + 10).$
Whence $b = 10,$
and $b + 20 = 30.$

30. Let $n =$ B's age now in years.
Then $4n =$ A's age now in years.
Therefore $4n + 20 = 2(n + 20)$.
Whence $n = 10$,
and $4n = 40$.

31. Let $n =$ B's age now in years.
Then $2n + 8 =$ A's age now in years.
Therefore $2n + 8 - 16 = 4(n - 16)$.
Solving, $n = 28$,
and $2n + 8 = 64$.

33. Let $x =$ the number of dollars invested at 5%.
Then $1400 - x =$ the number of dollars invested at 6%.
Hence $.05x =$ the income from the 5% investment,
and $.06(1400 - x) =$ the income from the 6% investment.
Therefore $.05x + .06(1400 - x) = 76$.
Multiplying by 100,

$$5x + 6(1400 - x) = 7600.$$

Solving, $x = 800$,
and $1400 - x = 600$.
Hence \$800 was invested at 5%, and \$600 at 6%.

34. Let $x =$ the number of dollars invested at 6%.
Then $x - 125 =$ the number of dollars invested at 8%.
Hence $.06x =$ the yearly income from the 6% investment,
and $.08(x - 125) =$ the yearly income from the 8% investment.
Therefore $.06x + .08(x - 125) = 53$.
Solving, $x = 450$,
and $x - 125 = 325$.
Hence \$450 is invested at 6%, and \$325 at 8%.

35. Let $x =$ the number of dollars invested at 5%.
Then $1240 - x =$ the number of dollars invested at 6%.
Hence $.05x =$ the income from the 5% investment,
and $.06(1240 - x) =$ the income from the 6% investment.
Therefore $.06(1240 - x) = 15 + .05x$.
Solving, $x = 540$,
and $1240 - x = 700$.
Hence \$540 is invested at 5%, and \$700 at 6%.

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1. $-10 \div 2 = -5.$
2. $12 \div (-3) = -4.$
3. $-16 \div (-4) = 4.$
4. $8a^3 \div a^2 = 8a.$
5. $-4a^6 \div 2a^2 = -2a^4.$
6. $6x^2 \div (-3x) = -2x.$
7. $-18x^7 \div (-6x^4) = 3x^3.$
8. $-25ax^3 \div 5ax = -5x^2.$
9. $12ax^3 \div (-3bx^3) = -\dfrac{4a}{b}.$
10. $-28ay^4 \div (-7cy^3) = \dfrac{4ay}{c}.$
11. $70x^4y^9 \div (-10x^2y^8) = -7x^2y.$
12. $48ax^5 \div (-16bx^4) = -\dfrac{3ax}{b}.$
13. $-36x^6y^6 \div (-6x^2y^3) = 6x^4y^3.$
14. $63c^3d^5 \div (-9bcd^3) = \dfrac{-7c^2d^2}{b}.$
15. $64a^5b^7 \div (-16ab^7) = -4a^4.$
16. $-28a^6b^{12} \div (-7a^4b^8) = 4a^2b^4.$
17. $\dfrac{15x^2y^4z^5}{75xy^4z} = \dfrac{xz^4}{5}.$
18. $\dfrac{42x^{24}y^{56}}{-6x^8y^7} = -7x^{16}y^{49}.$
19. $\dfrac{x^a}{x^2} = x^{a-2}.$
20. $\dfrac{-x^{3a}}{x^a} = -x^{2a}.$
21. $\dfrac{-17a^9b^8c^7}{51a^7b^8c} = \dfrac{-a^2c^6}{3}.$
22. $\dfrac{39x^{13}y^{26}z^{39}}{-13x^{13}y^{13}z^{13}} = -3y^{13}z^{26}.$
23. $\dfrac{-11a^2b^{11}c^{13}}{66ac^{12}} = \dfrac{-ab^{11}c}{6}.$
24. $\dfrac{-121a^{11}b^{22}c^{33}}{-11a^{11}b^{11}c^{11}} = 11b^{11}c^{22}.$
25. $\dfrac{x^{a+b}}{x^b} = x^a.$
26. $\dfrac{x^c}{x} = x^{c-1}.$
27. $\dfrac{3a^3x^{3a}}{a^2x^a} = 3ax^{2a}.$
28. $\dfrac{6x^{2a+3}}{-2x^4} = -3x^{2a-1}.$

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1. $\dfrac{6x^2-4x}{2x} = 3x-2.$
2. $\dfrac{9x-18x^4}{-3x} = \dfrac{9x}{-3x} + \dfrac{-18x^4}{-3x} = -3+6x^3.$
3. $\dfrac{4xy-12x^2}{-4x} = \dfrac{4xy}{-4x} + \dfrac{-12x^2}{-4x} = -y+3x.$
4. $\dfrac{9ax^3-12x^5}{-3ax^2} = \dfrac{9ax^3}{-3ax^2} + \dfrac{-12x^5}{-3ax^2} = -3x+\dfrac{4x^3}{a}.$
5. $\dfrac{25x^2y+30xy^5}{-5xy} = -5x-6y^4.$
6. $\dfrac{16bx^4-36x^2}{4bx^2} = 4x^2-\dfrac{9}{b}.$
7. $\dfrac{14x^3y^4-28x^5y^6}{7x^2y^3} = 2xy-4x^3y^3.$

8. $\dfrac{4\,x^4y - 8\,x^6y^2 + 12\,x^8y^4}{4\,x^4y} = 1 - 2\,x^2y + 3\,x^4y^3.$

9. $\dfrac{a^3cd^2 - a^2c^3}{a^2cd} = ad - \dfrac{c^2}{d}.$

10. $\dfrac{ax^4 - bx^3 + cx^2}{-\,x^2} = -\,ax^2 + bx - c.$

11. $\dfrac{15\,a^2b^2 + 9\,a^4b^2 - 30\,a^6b^2}{-\,3\,a^2b^2} = -\,5 - 3\,a^2 + 10\,a^4.$

12. $\dfrac{16\,a^4b^5c^6 - 24\,a^5b^6c^7 - 48\,a^6b^7c^8}{8\,a^3b^2c} = 2\,ab^3c^5 - 3\,a^2b^4c^6 - 6\,a^3b^5c^7.$

13. $\dfrac{85\,xyz - 51\,x^2yz^2 + 102\,x^3yz^3 - 170\,x^5y^5z}{-\,17\,xyz} = -\,5 + 3\,xz - 6\,x^2z^2 + 10\,x^4y^4.$

14. $\dfrac{4\,(x-3) + a\,(x-3)}{x-3} = 4 + a.$

15. $\dfrac{3\,x\,(3\,x+4) - 4\,y\,(3\,x+4)}{3\,x+4} = 3\,x - 4\,y.$

16. $\dfrac{5\,a\,(2\,x^2 - y) - 3\,b\,(2\,x^2 - y)}{2\,x^2 - y} = 5\,a - 3\,b.$

17. $\dfrac{(a+b)^4 - 3\,(a+b)^3}{(a+b)^2} = (a+b)^2 - 3\,(a+b).$

18. $\dfrac{21\,(x-y)^7 - 35\,(x-y)^5}{-\,7\,(x-y)^5} = -\,3\,(x-y)^2 + 5.$

19. $\dfrac{16\,(3\,x-4)^4 - 24\,(3\,x-4)^5 - 48\,(3\,x-4)^7}{-\,8\,(3\,x-4)^4} = -2 + 3\,(3\,x-4) + 6\,(3\,x-4)^3.$

20. $\dfrac{-\,5\,(ac^2 - 2\,d)^3 + x\,(ac^2 - 2\,d)}{5\,(ac^2 - 2\,d)} = -\,(ac^2 - 2\,d)^2 + \dfrac{x}{5}.$

21. $\dfrac{4\,x^4 - 8\,x^{3a} - 6\,x^{2a-2}}{2\,x^3} = 2\,x - 4\,x^{3a-3} - 3\,x^{2a-5}.$

22. $\dfrac{3\,x^a - 2\,x^{a+1} - x^{a+2} + x^2}{x^2} = 3\,x^{a-2} - 2\,x^{a-1} - x^a + 1.$

23. $\dfrac{6\,x^{2a-3} - 12\,x^{4a+4} - 18\,x^{3a+5}}{-\,3\,x^{2a}} = -\,2\,x^{-3} + 4\,x^{2a+4} + 6\,x^{a+5}.$

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1. $$\begin{array}{l|l} x^2 + 7\,x + 12 & x + 3 \\ \cline{2-2} \underline{x^2 + 3\,x} & x + 4 \\ \cline{1-2} \qquad\;\; 4\,x + 12 & \\ \qquad\;\; \underline{4\,x + 12} & \end{array}$$

2. $$\begin{array}{l|l} x^2 - 2\,x - 15 & x - 5 \\ \cline{2-2} \underline{x^2 - 5\,x} & x + 3 \\ \cline{1-2} \qquad\;\; 3\,x - 15 & \\ \qquad\;\; \underline{3\,x - 15} & \end{array}$$

3.
$$\begin{array}{l|l} x^2+5x+6 & x+3 \\ \cline{2-2} x^2+3x & x+2 \\ \hline \multicolumn{1}{l}{\quad 2x+6} & \\ \multicolumn{1}{l}{\quad \underline{2x+6}} & \end{array}$$

4.
$$\begin{array}{l|l} x^2-7x+6 & x-1 \\ \cline{2-2} x^2-x & x-6 \\ \hline \multicolumn{1}{l}{\quad -6x+6} & \\ \multicolumn{1}{l}{\quad \underline{-6x+6}} & \end{array}$$

5.
$$\begin{array}{l|l} 6x^2-13x+6 & 2x-3 \\ \cline{2-2} 6x^2-9x & 3x-2 \\ \hline \multicolumn{1}{l}{\quad -4x+6} & \\ \multicolumn{1}{l}{\quad \underline{-4x+6}} & \end{array}$$

6.
$$\begin{array}{l|l} 25x^4+30x^2-7 & 5x^2+7 \\ \cline{2-2} 25x^4+35x^2 & 5x^2-1 \\ \hline \multicolumn{1}{l}{\quad -5x^2-7} & \\ \multicolumn{1}{l}{\quad \underline{-5x^2-7}} & \end{array}$$

7.
$$\begin{array}{l|l} 12a^2+19a-21 & 4a-3 \\ \cline{2-2} 12a^2-9a & 3a+7 \\ \hline \multicolumn{1}{r}{28a-21} & \\ \multicolumn{1}{r}{\underline{28a-21}} & \end{array}$$

8.
$$\begin{array}{l|l} x^3-2x^2+4x-8 & x-2 \\ \cline{2-2} x^3-2x^2 & x^2+4 \\ \hline \multicolumn{1}{r}{4x-8} & \\ \multicolumn{1}{r}{\underline{4x-8}} & \end{array}$$

9.
$$\begin{array}{l|l} a^3+3a^2b+3ab^2+b^3 & a+b \\ \cline{2-2} \underline{a^3+a^2b} & \underline{a^2+2ab+b^2} \\ \multicolumn{1}{l}{\quad 2a^2b+3ab^2} & \\ \multicolumn{1}{l}{\quad \underline{2a^2b+2ab^2}} & \\ \multicolumn{1}{r}{ab^2+b^3} & \\ \multicolumn{1}{r}{\underline{ab^2+b^3}} & \end{array}$$

10.
$$\begin{array}{l|l} x^3-15x^2+65x-63 & x-7 \\ \cline{2-2} \underline{x^3-7x^2} & \underline{x^2-8x+9} \\ \multicolumn{1}{l}{\quad -8x^2+65x} & \\ \multicolumn{1}{l}{\quad \underline{-8x^2+56x}} & \\ \multicolumn{1}{r}{9x-63} & \\ \multicolumn{1}{r}{\underline{9x-63}} & \end{array}$$

11.
$$\begin{array}{l|l} -25x^3+5x^2+5x-1 & 5x^2-1 \\ \cline{2-2} \underline{-25x^3 \qquad\quad +5x} & -5x+1 \\ \qquad\quad 5x^2 \qquad -1 & \\ \qquad\quad \underline{5x^2 \qquad -1} & \end{array}$$

12.
$$\begin{array}{l|l} 2x^3-14x^2+14x+12 & 2x-4 \\ \cline{2-2} \underline{2x^3-\ \ 4x^2} & x^2-5x-3 \\ \quad -10x^2+14x & \\ \quad \underline{-10x^2+20x} & \\ \qquad\qquad -\ \ 6x+12 & \\ \qquad\qquad \underline{-\ \ 6x+12} & \end{array}$$

13.
$$\begin{array}{l|l} 3a^3+28a^2+\ \ 89a-140 & 3a-5 \\ \cline{2-2} \underline{3a^3-\ \ 5a^2} & a^2+11a+48+\dfrac{100}{3a-5} \\ \qquad 33a^2+\ \ 89a & \\ \qquad \underline{33a^2-\ \ 55a} & \\ \qquad\qquad 144a-140 & \\ \qquad\qquad \underline{144a-240} & \\ \qquad\qquad\qquad 100 & \end{array}$$

14.
$$\begin{array}{l|l} 6x^3-23x^2+37x-24 & 2x-3 \\ \cline{2-2} \underline{6x^3-\ \ 9x^2} & 3x^2-7x+8 \\ \quad -14x^2+37x & \\ \quad \underline{-14x^2+21x} & \\ \qquad\qquad 16x-24 & \\ \qquad\qquad \underline{16x-24} & \end{array}$$

15.
$$\begin{array}{l|l} 12a^3-53a^2+53a+8 & 4a^2-7a-1 \\ \cline{2-2} \underline{12a^3-21a^2-\ \ 3a} & 3a-8 \\ \quad -32a^2+56a+8 & \\ \quad \underline{-32a^2+56a+8} & \end{array}$$

16.
$$\begin{array}{l|l} 15a^3-56a^2+99a-70 & 3a^2-7a+10 \\ \cline{2-2} \underline{15a^3-35a^2+50a} & 5a-7 \\ \quad -21a^2+49a-70 & \\ \quad \underline{-21a^2+49a-70} & \end{array}$$

17.
$$\begin{array}{l|l} a^4+11a^3+23a^2-55a-140 & a^2-5 \\ \cline{2-2} \underline{a^4 \qquad\qquad -\ \ 5a^2} & a^2+11a+28 \\ \quad 11a^3+28a^2 & \\ \quad \underline{11a^3 \qquad\quad -55a} & \\ \qquad\quad 28a^2 \qquad\quad -140 & \\ \qquad\quad \underline{28a^2 \qquad\quad -140} & \end{array}$$

18.
$$\begin{array}{l|l}
a^4+4a^3+6a^2+4a+1 & a^2+2a+1 \\ \cline{2-2}
\underline{a^4+2a^3+a^2} & \underline{a^2+2a+1} \\
\qquad 2a^3+5a^2+4a & \\
\qquad \underline{2a^3+4a^2+2a} & \\
\qquad\qquad\quad a^2+2a+1 & \\
\qquad\qquad\quad \underline{a^2+2a+1} &
\end{array}$$

19.
$$\begin{array}{l|l}
a^4-8a^3+24a^2-32a+16 & a^2-4a+4 \\ \cline{2-2}
\underline{a^4-4a^3+4a^2} & \underline{a^2-4a+4} \\
\quad -4a^3+20a^2-32a & \\
\quad \underline{-4a^3+16a^2-16a} & \\
\qquad\qquad\quad 4a^2-16a+16 & \\
\qquad\qquad\quad \underline{4a^2-16a+16} &
\end{array}$$

20.
$$\begin{array}{l|l}
x^4+4x^3-31x^2+40x+21 & x^2-7x-3 \\ \cline{2-2}
\underline{x^4-7x^3-3x^2} & x^2+11x+49+\dfrac{416x+168}{x^2-7x-3} \\ \cline{2-2}
\qquad 11x^3-28x^2+40x & \\
\qquad \underline{11x^3-77x^2-33x} & \\
\qquad\qquad\quad 49x^2+73x+21 & \\
\qquad\qquad\quad \underline{49x^2-343x-147} & \\
\qquad\qquad\qquad\qquad 416x+168 &
\end{array}$$

21.
$$\begin{array}{l|l}
10x^4-27x^3-42x^2+11x-36 & 2x^2-5x+9 \\ \cline{2-2}
\underline{10x^4-25x^3+45x^2} & 5x^2-x-46+\dfrac{378-210x}{2x^2-5x+9} \\ \cline{2-2}
\qquad -2x^3-87x^2+11x & \\
\qquad \underline{-2x^3+5x^2-9x} & \\
\qquad\qquad\quad -92x^2+20x-36 & \\
\qquad\qquad\quad \underline{-92x^2+230x-414} & \\
\qquad\qquad\qquad\qquad -210x+378 &
\end{array}$$

22.
$$\begin{array}{l|l}
x^4-3a^2x^2+a^4 & x^2-ax+a^2 \\ \cline{2-2}
\underline{x^4-ax^3+a^2x^2} & x^2+ax-3a^2+\dfrac{4a^4-4a^3x}{x^2-ax+a^2} \\ \cline{2-2}
\qquad ax^3-4a^2x^2+a^4 & \\
\qquad \underline{ax^3-a^2x^2+a^3x} & \\
\qquad\quad -3a^2x^2-a^3x+a^4 & \\
\qquad\quad \underline{-3a^2x^2+3a^3x-3a^4} & \\
\qquad\qquad\qquad\quad -4a^3x+4a^4 &
\end{array}$$

23.
$$\begin{array}{l|l}
a^4+3a^2b^2+4b^4 & a^2-ab+2b^2 \\ \cline{2-2}
\underline{a^4-a^3b+2a^2b^2} & \underline{a^2+ab+2b^2} \\
\qquad a^3b+a^2b^2+4b^4 & \\
\qquad \underline{a^3b-a^2b^2+2ab^3} & \\
\qquad\qquad 2a^2b^2-2ab^3+4b^4 & \\
\qquad\qquad \underline{2a^2b^2-2ab^3+4b^4} &
\end{array}$$

24.
$$\begin{array}{l|l}
16x^4-60x^2y^2+25y^4 & 4x^2-10xy-5y^2 \\ \hline
\underline{16x^4-40x^3y-20x^2y^2} & 4x^2+10xy+15y^2+\dfrac{200xy^3+100y^4}{4x^2-10xy-5y^2} \\
\qquad 40x^3y-40x^2y^2+25y^4 & \\
\qquad \underline{40x^3y-100x^2y^2-50xy^3} & \\
\qquad\qquad 60x^2y^2+50xy^3+25y^4 & \\
\qquad\qquad \underline{60x^2y^2-150xy^3-75y^4} & \\
\qquad\qquad\qquad 200xy^3+100y^4 &
\end{array}$$

25.
$$\begin{array}{l|l}
9a^4+26a^2b^2+49b^4 & 3a^2+4ab+7b^2 \\ \hline
\underline{9a^4+12a^3b+21a^2b^2} & 3a^2-4ab+7b^2 \\
\qquad -12a^3b+5a^2b^2+49b^4 & \\
\qquad \underline{-12a^3b-16a^2b^2-28ab^3} & \\
\qquad\qquad 21a^2b^2+28ab^3+49b^4 & \\
\qquad\qquad \underline{21a^2b^2+28ab^3+49b^4} &
\end{array}$$

26.
$$\begin{array}{l|l}
4a^8-44a^4b^4+100b^8 & 2a^4+2a^2b^2-10b^4 \\ \hline
\underline{4a^8+4a^6b^2-20a^4b^4} & 2a^4-2a^2b^2-10b^4 \\
\qquad -4a^6b^2-24a^4b^4+100b^8 & \\
\qquad \underline{-4a^6b^2-4a^4b^4+20a^2b^6} & \\
\qquad\qquad -20a^4b^4-20a^2b^6+100b^8 & \\
\qquad\qquad \underline{-20a^4b^4-20a^2b^6+100b^8} &
\end{array}$$

27.
$$\begin{array}{l|l}
25x^3-10x^2+40x-18 & 5x-6 \\ \hline
\underline{25x^3-30x^2} & 5x^2+4x+12\tfrac{4}{5}+\dfrac{58\tfrac{4}{5}}{5x-6} \\
\qquad 20x^2+40x & \\
\qquad \underline{20x^2-24x} & \\
\qquad\qquad 64x-18 & \\
\qquad\qquad \underline{64x-76\tfrac{4}{5}} & \\
\qquad\qquad\qquad 58\tfrac{4}{5} &
\end{array}$$

28.
$$\begin{array}{l|l}
x^3-y^3 & x-y \\ \hline
\underline{x^3-x^2y} & x^2+xy+y^2 \\
\qquad x^2y & \\
\qquad \underline{x^2y-xy^2} & \\
\qquad\qquad xy^2-y^3 & \\
\qquad\qquad \underline{xy^2-y^3} &
\end{array}$$

29.
$$\begin{array}{l|l}
a^3-125b^3 & a-5b \\ \hline
\underline{a^3-5a^2b} & a^2+5ab+25b^2 \\
\qquad 5a^2b & \\
\qquad \underline{5a^2b-25ab^2} & \\
\qquad\qquad 25ab^2-125b^3 & \\
\qquad\qquad \underline{25ab^2-125b^3} &
\end{array}$$

30.
$$\begin{array}{l|l}
a^6+343b^3 & a^2+7b \\ \hline
\underline{a^6+7a^4b} & a^4-7a^2b+49b^2 \\
\qquad -7a^4b & \\
\qquad \underline{-7a^4b-49a^2b^2} & \\
\qquad\qquad 49a^2b^2+343b^3 & \\
\qquad\qquad \underline{49a^2b^2+343b^3} &
\end{array}$$

31.
$$\begin{array}{l|l}
x^4 - 16 & x + 2 \\ \cline{2-2}
\underline{x^4 + 2x^3} & x^3 - 2x^2 + 4x - 8 \\
\quad -2x^3 & \\
\quad \underline{-2x^3 - 4x^2} & \\
\qquad\qquad 4x^2 & \\
\qquad\qquad \underline{4x^2 + 8x} & \\
\qquad\qquad\qquad -8x - 16 & \\
\qquad\qquad\qquad \underline{-8x - 16} &
\end{array}$$

32.
$$\begin{array}{l|l}
y^5 - 5y^2 - 3000 & y - 5 \\ \cline{2-2}
\underline{y^5 - 5y^4} & y^4 + 5y^3 + 25y^2 + 120y + 600 \\
\quad 5y^4 & \\
\quad \underline{5y^4 - 25y^3} & \\
\qquad\quad 25y^3 - 5y^2 & \\
\qquad\quad \underline{25y^3 - 125y^2} & \\
\qquad\qquad\quad 120y^2 & \\
\qquad\qquad\quad \underline{120y^2 - 600y} & \\
\qquad\qquad\qquad\quad 600y - 3000 & \\
\qquad\qquad\qquad\quad \underline{600y - 3000} &
\end{array}$$

33.
$$\begin{array}{l|l}
x^4 + y^4 & x - y \\ \cline{2-2}
\underline{x^4 - x^3y} & x^3 + x^2y + xy^2 + y^3 + \dfrac{2y^4}{x - y} \\
\quad x^3y & \\
\quad \underline{x^3y - x^2y^2} & \\
\qquad\quad x^2y^2 & \\
\qquad\quad \underline{x^2y^2 - xy^3} & \\
\qquad\qquad\quad xy^3 + y^4 & \\
\qquad\qquad\quad \underline{xy^3 - y^4} & \\
\qquad\qquad\qquad\quad 2y^4 &
\end{array}$$

34.
$$\begin{array}{l|l}
x^4 + y^4 & x + y \\ \cline{2-2}
\underline{x^4 + x^3y} & x^3 - x^2y + xy^2 - y^3 + \dfrac{2y^4}{x + y} \\
\quad -x^3y & \\
\quad \underline{-x^3y - x^2y^2} & \\
\qquad\quad x^2y^2 & \\
\qquad\quad \underline{x^2y^2 + xy^3} & \\
\qquad\qquad\quad -xy^3 + y^4 & \\
\qquad\qquad\quad \underline{-xy^3 - y^4} & \\
\qquad\qquad\qquad\quad 2y^4 &
\end{array}$$

35.
$$\begin{array}{l|l}
x^4 - y^4 & x + y \\ \cline{2-2}
\underline{x^4 + x^3y} & \underline{x^3 - x^2y + xy^2 - y^3} \\
\quad - x^3y & \\
\quad \underline{- x^3y - x^2y^2} & \\
\qquad\qquad x^2y^2 & \\
\qquad\qquad \underline{x^2y^2 + xy^3} & \\
\qquad\qquad\qquad - xy^3 - y^4 & \\
\qquad\qquad\qquad \underline{- xy^3 - y^4} &
\end{array}$$

36.
$$\begin{array}{l|l}
x^4 - y^4 & x - y \\ \cline{2-2}
\underline{x^4 - x^3y} & \underline{x^3 + x^2y + xy^2 + y^3} \\
\qquad x^3y & \\
\qquad \underline{x^3y - x^2y^2} & \\
\qquad\qquad x^2y^2 & \\
\qquad\qquad \underline{x^2y^2 - xy^3} & \\
\qquad\qquad\qquad xy^3 - y^4 & \\
\qquad\qquad\qquad \underline{xy^3 - y^4} &
\end{array}$$

37.
$$\begin{array}{l|l}
x^5 + y^5 & x + y \\ \cline{2-2}
\underline{x^5 + x^4y} & \underline{x^4 - x^3y + x^2y^2 - xy^3 + y^4} \\
\quad - x^4y & \\
\quad \underline{- x^4y - x^3y^2} & \\
\qquad\qquad x^3y^2 & \\
\qquad\qquad \underline{x^3y^2 + x^2y^3} & \\
\qquad\qquad\quad - x^2y^3 & \\
\qquad\qquad\quad \underline{- x^2y^3 - xy^4} & \\
\qquad\qquad\qquad\qquad xy^4 + y^5 & \\
\qquad\qquad\qquad\qquad \underline{xy^4 + y^5} &
\end{array}$$

38.
$$\begin{array}{l|l}
x^5 + y^5 & x - y \\ \cline{2-2}
\underline{x^5 - x^4y} & \\
\qquad\quad x^4y & \underline{x^4 + x^3y + x^2y^2 + xy^3 + y^4 + \dfrac{2\,y^5}{x - y}} \\
\qquad \underline{x^4y - x^3y^2} & \\
\qquad\qquad x^3y^2 & \\
\qquad\qquad \underline{x^3y^2 - x^2y^3} & \\
\qquad\qquad\qquad x^2y^3 & \\
\qquad\qquad\qquad \underline{x^2y^3 - xy^4} & \\
\qquad\qquad\qquad\qquad xy^4 + \quad y^5 & \\
\qquad\qquad\qquad\qquad \underline{xy^4 - \quad y^5} & \\
\qquad\qquad\qquad\qquad\qquad 2\,y^5 &
\end{array}$$

39.
$$\begin{array}{l|l}
x^5 - \quad y^5 & x + y \\ \cline{2-2}
\underline{x^5 + x^4y} & \\
\quad - x^4y & \underline{x^4 - x^3y + x^2y^2 - xy^3 + y^4 - \dfrac{2\,y^5}{x + y}} \\
\quad \underline{- x^4y - x^3y^2} & \\
\qquad\qquad x^3y^2 & \\
\qquad\qquad \underline{x^3y^2 + x^2y^3} & \\
\qquad\qquad\quad - x^2y^3 & \\
\qquad\qquad\quad \underline{- x^2y^3 - xy^4} & \\
\qquad\qquad\qquad\qquad xy^4 - \quad y^5 & \\
\qquad\qquad\qquad\qquad \underline{xy^4 + \quad y^5} & \\
\qquad\qquad\qquad\qquad\qquad - 2\,y^5 &
\end{array}$$

40.
$$\begin{array}{l|l}
x^5 - y^5 & x - y \\ \cline{2-2}
x^5 - x^4y & x^4 + x^3y + x^2y^2 + xy^3 + y^4 \\ \cline{1-1}
\multicolumn{1}{l}{\quad x^4y} & \\
\multicolumn{1}{l}{\quad \underline{x^4y - x^3y^2}} & \\
\multicolumn{1}{l}{\qquad x^3y^2} & \\
\multicolumn{1}{l}{\qquad \underline{x^3y^2 - x^2y^3}} & \\
\multicolumn{1}{l}{\qquad\quad x^2y^3} & \\
\multicolumn{1}{l}{\qquad\quad \underline{x^2y^3 - xy^4}} & \\
\multicolumn{1}{l}{\qquad\qquad xy^4 - y^5} & \\
\multicolumn{1}{l}{\qquad\qquad \underline{xy^4 - y^5}} &
\end{array}$$

41.
$$\begin{array}{r|l}
x^{2a} - 5x^a + 6 & x^a - 3 \\
x^{2a} - 3x^a \phantom{{}+6} & x^a - 2 \\ \hline
- 2x^a + 6 & \\
\underline{- 2x^a + 6} &
\end{array}$$

42.
$$\begin{array}{r|l}
x^{6a} - 7x^{3a} + 12 & x^{3a} - 4 \\
x^{6a} - 4x^{3a} \phantom{{}+12} & x^{3a} - 3 \\ \hline
- 3x^{3a} + 12 & \\
\underline{- 3x^{3a} + 12} &
\end{array}$$

43.
$$\begin{array}{r|l}
x^{2a} + 3x^a + 2x^{a+1} + 3x & x^a + 2x \\ \cline{2-2}
x^{2a} \phantom{{}+3x^a} + 2x^{a+1} \phantom{{}+3x} & x^a + 3 - \dfrac{3x}{x^a + 2x} \\ \hline
3x^a \phantom{{}+2x^{a+1}} + 3x & \\
\underline{3x^a \phantom{{}+2x^{a+1}} + 6x} & \\
- 3x &
\end{array}$$

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1. $x + 2a = 6a.$

Whence $x = 4a.$

2. $x + a = b.$

Whence $x = b - a.$

3. $bx + b = 4b.$

Dividing by b,

$x + 1 = 4.$

Whence $x = 3.$

4. $cx + c^2 = 6c^3.$

$x + c = 6c^2.$

Whence $x = 6c^2 - c.$

5. $5(b - x) = 10b.$

$b - x = 2b.$

Whence $x = - b.$

6. $bx - (b + c) = 5b - c.$

$bx - b - c = 5b - c.$

Whence $x = 6.$

7. $3ax - ab = 2ax - ac.$

Dividing by a,

$3x - b = 2x - c.$

Whence $x = b - c.$

8. $4bx - 7a^2b = 6ab^2 + 3bx.$

Dividing by b,

$x - 7a^2 = 6ab.$

Whence $x = 7a^2 + 6ab.$

9. $ax + bx = ac + bc.$

Grouping,

$x(a + b) = c(a + b).$

Whence $x = c.$

10. $a^2x + 1 - a^4 - x = 0.$

Grouping, $x(a^2 - 1) = a^4 - 1.$

Whence $x = a^2 + 1.$

11. $ax + 2ab = 2a^2 + bx.$

Grouping, $ax - bx = 2a^2 - 2ab.$

Whence $x = 2a.$

12. $ax - a^3 - 4 = 3a - x.$

Arranging, $ax + x = a^3 + 3a + 4.$

Dividing by $a + 1$, $x = a^2 - a + 4.$

13. $4b^2c^2 + (a + bx)c = (a - bx)c.$

$4b^2c + a + bx = a - bx.$

Whence $x = -2bc.$

14. $ax - ac + bc = 2ac - 5bc + 2bx.$

Arranging, $ax - 2bx = 3ac - 6bc.$

Whence $x = 3c.$

15. $(x + a)(x + b) = x^2 + 2a^2 + 3ab.$

$x^2 + ax + bx + ab = x^2 + 2a^2 + 3ab.$

Whence $x = 2a.$

16. $15(x - a) - 6(x + a) = 3(5a - 3x).$

Simplifying, $5x - 5a - 2x - 2a = 5a - 3x.$

Whence $x = 2a.$

17. $4x - cx - 8 + 2a + 6c = 6a - 3ac + 2cx.$

Arranging, $4x - 3cx = 4a - 3ac + 8 - 6c.$

Whence $x = a + 2.$

18. $9ab + (x - 3a)(x - 3b) = (x + 3a)(x - 3a) - 9a^2.$

Expanding,

$9ab + x^2 - 3ax - 3bx + 9ab = x^2 - 9a^2 - 9a^2.$

Whence $x = 6a.$

19. $(5a - 4b)x - 5(b^2 + 4a^2 + 6ab) = 10b^2 - 3(2a^2 + 3bx) - a(2x - b).$

Expanding,

$5ax - 4bx - 5b^2 - 20a^2 - 30ab = 10b^2 - 6a^2 - 9bx - 2ax + ab.$

$7ax + 5bx = 14a^2 + 31ab + 15b^2.$

Whence $x = 2a + 3b.$

20. $a^2x + 3ax + 10a = a^3 + x + 3.$

Arranging, $a^2x + 3ax - x = a^3 - 10a + 3.$

Whence $x = a - 3.$

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1. Let $x =$ the number of hours which must elapse before A and B are 120 miles apart.

Then $8x + 10x = 120.$

Whence $x = 6\frac{2}{3}.$

Let $y =$ the number of hours which must elapse before A and B are 180 miles apart.
Then $8y + 10y = 180.$
Whence $y = 10.$

2. Let $r =$ the rate of B in miles per hour.
Then $2r =$ the rate of A in miles per hour.
Therefore $5 \cdot 2r + 5r = 135.$
Whence $r = 9,$
and $2r = 18.$

3. Let $r =$ the rate of B in miles per hour.
Then $r + 2 =$ the rate of A in miles per hour.
Therefore $8(r + 2) + 8r = 96.$
Whence $r = 5,$
and $r + 2 = 7.$

4. Let $r =$ the rate of B in miles per hour.
Then $r + 4 =$ the rate of A in miles per hour.
Therefore $6(r + 4) + 6r = 168.$
Whence $r = 12,$
and $r + 4 = 16.$

5. Let $r =$ the rate of A in miles per hour.
Then $r - 3 =$ the rate of B in miles per hour.
Therefore $r - 3 = \frac{3r}{4}.$
Whence $r = 12,$
and $r - 3 = 9.$
Let $h =$ the required number of hours.
Therefore $12h + 9h = 168.$
Whence $h = 8.$

6. Let $r =$ the rate of B in miles per hour.
Then $2r =$ the rate of A in miles per hour.
Therefore $3 \cdot 2r + 5r = 77.$
Whence $r = 7,$
and $2r = 14.$
A travels $3 \cdot 14$, or 42 miles, and B travels $5 \cdot 7$, or 35 miles.

7. Let $r =$ the rate of A in miles per hour.
Then $r - 4 =$ the rate of B in miles per hour.
Therefore $6 \cdot r = 9(r - 4).$
Whence $r = 12,$
and $r - 4 = 8.$
$6 \cdot 12 + 9 \cdot 8 = 144$ miles, the total distance traveled.

8. Let $x =$ the rate of each in miles per hour.
Therefore $8x + 8x = 144.$
Whence $x = 9.$

9. Let $x =$ the rate of A in miles per hour.
Then $x + 2 =$ the rate of B in miles per hour.
Therefore $9x + 9(x + 2) = 144.$
Whence $x = 7,$
and $x + 2 = 9.$

10. Let $r =$ the rate of A in miles per hour.
Then $3r =$ the rate of B in miles per hour.
Therefore $12 \cdot r + 12 \cdot 3r = 144.$
Whence $r = 3,$
and $3r = 9.$

11. Let $r =$ the rate of A in miles per hour.
Then $r + 4 =$ the rate of B in miles per hour.
Therefore $6 \cdot r + 6(r + 4) = 144.$
Whence $r = 10,$
and $r + 4 = 14.$

12. Let $r =$ the rate of A in miles per hour.
Then $r + 2 =$ the rate of B in miles per hour.
Therefore $8r + 8(r + 2) = 144.$
Whence $r = 8,$
and $r + 2 = 10.$

13. Let $r =$ the rate of B in miles per hour.
Then $r + 4 =$ the rate of A in miles per hour.
Therefore $9(r + 4) + 9r = 144.$
Whence $r = 6,$
and $r + 4 = 10.$

14. Let $r =$ the rate of A in miles per hour.
Then $r + 6 =$ the rate of B in miles per hour.
Therefore $r + 6 = 2r.$
Whence $r = 6,$
and $r + 6 = 12.$
Let $t =$ the required time in hours.
Therefore $t \cdot 6 + t \cdot 12 = 144.$
Whence $t = 8.$

15. Let $h =$ the required number of hours.
Therefore $6 \cdot h + 9(h - 4) = 144.$
Whence $h = 12.$

16. Let $h =$ the required number of hours.
Therefore $7(h-3)+9(h-5)=144.$
Whence $h = 13\frac{1}{8}$ hours.

17. Let $t =$ the required number of hours.
Therefore $3t+4t=42.$
Whence $t=6.$

18. Let $t =$ the required number of hours.
Therefore $8t+10t=72.$
Whence $t=4.$

19. Let $t =$ the required number of hours.
Therefore $37t+38t=285.$
Whence $t=3\frac{4}{5}.$

20. Let $x =$ the number of hours that A travels.
Then $x-4 =$ the number of hours that B travels.
Therefore $8x=10(x-4).$
Whence $x=20.$

21. Let $h =$ the number of hours traveled by the first.
Then $h-2 =$ the number of hours traveled by the second.
Therefore $10h+12(h-2)=108.$
Whence $h=6.$
A has traveled $10 \cdot 6$, or 60 miles, and B $12 \cdot 4$, or 48 miles.

22. Let $x =$ the time in hours of the passenger train.
Then $x+2 =$ the time in hours of the freight train.
Therefore $42x+24(x+2)=246.$
Whence $x=3.$

23. Let $x =$ the rate of the second messenger in miles per hour.
Therefore $8 \cdot 8 = 6 \cdot x.$
Whence $x=10\frac{2}{3}.$

24. Let $h =$ the required number of hours.
Therefore $45h-18h=144.$
Whence $h=5\frac{1}{3}.$

25. $\frac{216}{9+15}=9$ hours. $9 \cdot 9 = 81$ miles, A's distance; $9 \cdot 15 = 135$ miles, B's distance.

26. $\frac{5}{6} \cdot 216 = 180$ miles, B's distance; $\frac{216}{6} = 36$ miles, A's distance.

27. $\frac{216-56}{2}=80$ miles, B's distance; $80+56=136$ miles, A's distance.

28. $108-24=84$ miles, A's distance; $108+24=132$ miles, B's distance.

29. $\frac{108 + 54}{6} = 27$ miles per hour, A's rate; $\frac{54}{9} = 6$ miles per hour, B's rate.

30. $\frac{108 + 36}{6} = 24$ miles per hour, A's rate; $\frac{72}{6} = 12$ miles per hour, B's rate.

31. $\frac{108 + 12}{6} = 20$ miles per hour, A's rate; $\frac{108 - 12}{6} = 16$ miles per hour, B's rate.

32. $\frac{216 - 96}{6} = 20$ miles per hour, A's rate; $\frac{96}{6} = 16$ miles per hour, B's rate.

33. Let $x =$ the rate of B in miles per hour.

Then $x + 4 =$ the rate of A in miles per hour.

Therefore $12 \cdot (x + 4) + 12 \cdot x = 216.$

Whence $x = 7,$

and $x + 4 = 11.$

34. Let $r =$ the rate of B in miles per hour.

Then $r + 2 =$ the rate of A in miles per hour.

Therefore $6(r + 2) + 6r = 216.$

Whence $r = 17,$

and $r + 2 = 19.$

Then $6 \cdot 17 = 102$ miles, B's distance,

and $6 \cdot 19 = 114$ miles, A's distance.

35. Let $r =$ the rate of B in miles per hour.

Then $r + 4 =$ the rate of A in miles per hour.

Therefore $9(r + 4) + 9r = 216.$

Whence $r = 10,$

and $r + 4 = 14.$

Then $9 \cdot 10 = 90$ miles, B's distance,

and $9 \cdot 14 = 126$ miles, A's distance.

36. Let $v =$ the velocity of the bullet in feet per second.

Therefore $\frac{1650}{v} + \frac{1650}{1100} = \frac{5}{2}.$

Whence $v = 1650.$

37. Let $v =$ the velocity of the bullet in feet per second.

Therefore $\frac{825}{v} + \frac{825}{1100} = \frac{7}{4}.$

Whence $v = 825.$

38. Let $x =$ the distance in feet from the boy to the target.

Therefore $\frac{3000 - x}{1100} = \frac{3000}{1650} + \frac{x}{1100}.$

Whence $x = 500.$

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1. $5a - 7x + 3b - 10c - 14a + 12y - 8x + 12a - 11x + 9b$
$= 3a - 26x + 12b - 10c + 12y.$

2. $2cd - a^2b + 7cd^2 - 12ab + 17ad - cd + 4c^2d$
$= cd + 7cd^2 + 4c^2d - a^2b - 12ab + 17ad.$

3.
$$\begin{array}{rrrr}
10x - & 9 + & 4ax - & 2cd \\
-15x & - & 6ax + & 12cd \\
 & 10 - & ax + & 5cd \\
11x - & 19 - & 7ax - & 6cd \\
\hline
6x - & 18 - & 10ax + & 9cd
\end{array}$$

4.
$$\begin{array}{rrrr}
a^3 + & 4a^2 - & a - & 3 \\
-5a^3 + & 3a^2 - & a & \\
6a^3 - & 3a^2 - & 2a + & 8 \\
-2a^3 - & 5a^2 - & a + & 5 \\
2a^3 - & 3a^2 + & 4a - & 5 \\
\hline
2a^3 - & 4a^2 - & a + & 5
\end{array}$$

5.
$$\begin{array}{rrrr}
Ax^3 + & Bx^2 + & Cx & \\
 & Ax^2 + & Bx & + C \\
\hline
Ax^3 + & (A+B)x^2 + & (B+C)x & + C
\end{array}$$

6.
$$\begin{array}{rrr}
7(a-b) - & 10(c-d) + & 8(x-y) \\
-4(a-b) - & 6(c-d) - & 7(x-y) \\
7(a-b) - & 15(c-d) + & 12(x-y) \\
\hline
10(a-b) - & 31(c-d) + & 13(x-y)
\end{array}$$

7.
$$\begin{array}{rrr}
-4(a+b) - & 2(x+y) + & 3(x-y) \\
4(a+b) - & 9(x+y) + & 8(x-y) \\
-\quad (a+b) - & 10(x+y) + & (x-y) \\
\hline
-\quad (a+b) - & 21(x+y) + & 12(x-y)
\end{array}$$

8.
$$\begin{array}{rrrr}
\frac{1}{3}ax + & \frac{2}{3}ax^2 - & \frac{5}{6}a^2x^2 + & a \\
-\quad ax + & \frac{1}{6}ax^2 + & 2\ a^2x^2 - & 9\ a \\
 & \frac{1}{2}ax^2 + & a^2x^2 - & \frac{1}{4}a \\
\hline
-\frac{2}{3}ax + & \frac{4}{3}ax^2 + & 2\frac{1}{6}a^2x^2 - & 8\frac{1}{4}a
\end{array}$$

9.
$$\begin{array}{rrrr}
x^2 + & 10xy - & 5y^2 - & 9 \\
2x^2 - & 3xy + & y^2 - & 1 \\
\hline
-x^2 + & 13xy - & 6y^2 - & 8
\end{array}$$

10.
$$\begin{array}{rrrrr}
-\quad a^2y + & 12ay^2 - & ay - & 15 & \\
3a^2y - & 7ay^2 - & 5ay & & + z \\
\hline
-4a^2y + & 19ay^2 + & 4ay - & 15 & - z
\end{array}$$

11.
$$\begin{array}{rrrrr}
3ab^2c^2d^3 & - 12 & & & \\
-4ab^2c^2d^3 & & -17a^2bcd^2 & + 3abcd & + ax \\
\hline
7ab^2c^2d^3 & - 12 & + 17a^2bcd^2 & - 3abcd & - ax
\end{array}$$

12.
$$\begin{array}{r} 0 \\ \underline{27\,xy + 13\,z - 12 + ab} \\ -27\,xy - 13\,z + 12 - ab \end{array}$$

13. $(a + b) - (a - b) + 4\,a - (a + 3\,b) + c$
$= a + b - a + b + 4\,a - a - 3\,b + c$
$= 3\,a - b + c.$

14. $15\,x - 2\,(12\,x - 16\,y) + xy + 3\,(3\,x - 4\,xy) - x$
$= 14\,x - 24\,x + 32\,y + xy + 9\,x - 12\,xy$
$= - x - 11\,xy + 32\,y.$

15. $ab - (7\,x + 4\,y - 2\,z) + 42 + 5\,(5\,ab - x - y)$
$= ab - 7\,x - 4\,y + 2\,z + 42 + 25\,ab - 5\,x - 5\,y$
$= 26\,ab - 12\,x - 9\,y + 2\,z + 42.$

16. $[x - (- 4\,x + 8\,y + 2\,z) + b - 4\,(3\,x - z)] - 5\,b$
$= x + 4\,x - 8\,y - 2\,z + b - 12\,x + 4\,z - 5\,b$
$= - 7\,x - 8\,y + 2\,z - 4\,b.$

17. $[ab - \{- 5\,ac + 7\,(d - a) + 4\,ca + 7\,d\} - 6\,ba] + 13\,a$
$= ab - \{- ac + 14\,d - 7\,a\} - 6\,ba + 13\,a$
$= - 5\,ab + ac - 14\,d + 7\,a + 13\,a$
$= 20\,a - 5\,ab + ac - 14\,d.$

18. $\{5\,x^2 - 2\,[- 4 - (2\,x^2 - 3\,x)] + 5\} - (x^2 + 4\,x - 2)$
$= 5\,x^2 - 2\,(- 4 - 2\,x^2 + 3\,x) + 5 - x^2 - 4\,x + 2$
$= 4\,x^2 + 8 + 4\,x^2 - 6\,x + 7 - 4\,x$
$= 8\,x^2 - 10\,x + 15.$

19.
$$\begin{array}{lllllll}
 & x^4 - 4\,x^2y^2 + 4\,y^4 & & & & & (1) \\
 & \underline{x^4 + 4\,x^2y^2 + 4\,y^4} & & & & & (2) \\
 & x^8 & - 4\,x^6y^2 & + 4\,x^4y^4 & & & \\
 & & + 4\,x^6y^2 & - 16\,x^4y^4 & + 16\,x^2y^6 & & \\
 & & & + 4\,x^4y^4 & - 16\,x^2y^6 & + 16\,y^8 & \\
\hline
(1) \cdot (2), & x^8 & & - 8\,x^4y^4 & & + 16\,y^8 &
\end{array}$$

20.
$$\begin{array}{lllllllll}
 & 4\,x^4 - x^2 + 5 & & & & & & & (1) \\
 & \underline{3\,x^3 - x + 7} & & & & & & & (2) \\
 & 12\,x^7 & - 3\,x^5 & & + 15\,x^3 & & & & \\
 & & - 4\,x^5 & & + \;\; x^3 & & - 5\,x & & \\
 & & & + 28\,x^4 & & - 7\,x^2 & & + 35 & \\
\hline
(1) \cdot (2), & 12\,x^7 & - 7\,x^5 & + 28\,x^4 & + 16\,x^3 & - 7\,x^2 & - 5\,x & + 35 &
\end{array}$$

$$\begin{array}{llllllllllll}
\textbf{21.} & \multicolumn{10}{l}{a^2+b^2+c^2-ab-ac-bc} & (1)\\
 & \multicolumn{10}{l}{a+b+c} & (2)\\ \cline{2-11}
 & a^3 & +ab^2 & +ac^2 & -a^2b & -a^2c & -abc & & & & & \\
 & & -ab^2 & & +a^2b & & -abc & +b^3 & +bc^2 & -b^2c & & \\
 & & & -ac^2 & & +a^2c & -abc & & -bc^2 & +b^2c & +c^3 & \\ \cline{2-11}
(1)\cdot(2), & a^3 & & & & & -3abc & +b^3 & & & +c^3 &
\end{array}$$

$$\begin{array}{llllllllll}
\textbf{22.} & \multicolumn{8}{l}{x^6-x^5y+x^4y^2-x^3y^3+x^2y^4-xy^5+y^6} & (1)\\
 & \multicolumn{8}{l}{x+y} & (2)\\ \cline{2-9}
 & x^7 & -x^6y & +x^5y^2 & -x^4y^3 & +x^3y^4 & -x^2y^5 & +xy^6 & & \\
 & & +x^6y & -x^5y^2 & +x^4y^3 & -x^3y^4 & +x^2y^5 & -xy^6 & +y^7 & \\ \cline{2-9}
(1)\cdot(2), & x^7 & & & & & & & +y^7 &
\end{array}$$

$$\begin{array}{lllllllllll}
\textbf{23.} & \multicolumn{9}{l}{-9x^5-4x^4-3x^3+x^2+5x+10} & (1)\\
 & \multicolumn{9}{l}{\quad 2x^4-x^3-x^2+3x} & (2)\\ \cline{2-10}
 & -18x^9 & -8x^8 & -6x^7 & +2x^6 & +10x^5 & +20x^4 & & & & \\
 & & +9x^8 & +4x^7 & +3x^6 & -x^5 & -5x^4 & -10x^3 & & & \\
 & & & +9x^7 & +4x^6 & +3x^5 & -x^4 & -5x^3 & -10x^2 & & \\
 & & & & -27x^6 & -12x^5 & -9x^4 & +3x^3 & +15x^2 & +30x & \\ \cline{2-10}
(1)\cdot(2), & -18x^9 & +x^8 & +7x^7 & -18x^6 & & +5x^4 & -12x^3 & +5x^2 & +30x &
\end{array}$$

$$\begin{array}{llllllll}
\textbf{24.} & 3x^2-4y & & (1) & & 9x^4-16y^2 & & (3)\\
 & 3x^2+4y & & (2) & & 9x^4+16y^2 & & (4)\\ \cline{2-3} \cline{6-7}
 & 9x^4 & -12x^2y & & & 81x^8 & -144x^4y^2 & \\
 & & +12x^2y-16y^2 & & & & +144x^4y^2-256y^4 & \\ \cline{2-3} \cline{6-7}
(1)\cdot(2), & 9x^4 & -16y^2 & (3) & (3)\cdot(4) & 81x^8 & -256y^4 &
\end{array}$$

$$\begin{array}{llllllll}
\textbf{25.} & \multicolumn{6}{l}{a-3b+2c} & (1)\\
 & \multicolumn{6}{l}{a-3b+2c} & (2)\\ \cline{2-7}
 & a^2 & -3ab & +2ac & & & & \\
 & & -3ab & & +9b^2 & -6bc & & \\
 & & & +2ac & & -6bc & +4c^2 & \\ \cline{2-7}
(1)\cdot(2), & a^2 & -6ab & +4ac & +9b^2 & -12bc & +4c^2 &
\end{array}$$

$$\begin{array}{llllllll}
\textbf{26.} & \multicolumn{6}{l}{x-y+z} & (1)\\
 & \multicolumn{6}{l}{x-y+z} & (2)\\ \cline{2-7}
 & x^2 & -xy & +xz & & & & \\
 & & -xy & & +y^2 & -yz & & \\
 & & & +xz & & -yz & +z^2 & \\ \cline{2-7}
(1)\cdot(2), & x^2 & -2xy & +2xz & +y^2 & -2yz & +z^2 & (3)
\end{array}$$

$$\begin{array}{llllllllllll}
 & \multicolumn{10}{l}{x-y+z} & (4)\\ \cline{2-11}
 & x^3 & -2x^2y & +2x^2z & +xy^2 & -2xyz & +xz^2 & & & & & \\
 & & -x^2y & & +2xy^2 & -2xyz & & -y^3 & +2y^2z & -yz^2 & & \\
 & & & +x^2z & & -2xyz & +2xz^2 & & +y^2z & -2yz^2 & +z^3 & \\ \cline{2-11}
(3)\cdot(4), & x^3 & -3x^2y & +3x^2z & +3xy^2 & -6xyz & +3xz^2 & -y^3 & +3y^2z & -3yz^2 & +z^3 &
\end{array}$$

27.

$$\begin{array}{lr}
 & x^a - y^b \quad (1)\\
 & \underline{x^a + y^b} \quad (2)\\
 & x^{2a} - x^a y^b \\
 & \underline{\qquad + x^a y^b - y^{2b}}\\
(1)\cdot(2), & x^{2a} \qquad\qquad - y^{2b}
\end{array}$$

28.

$$\begin{array}{lr}
 & x^a + 2y^b + 3z^c \quad (1)\\
 & \underline{x^a - 3y^b} \quad (2)\\
 & x^{2a} + 2x^a y^b + 3x^a z^c \\
 & \underline{\quad - 3x^a y^b \qquad\qquad - 6y^{2b} - 9y^b z^c}\\
(1)\cdot(2), & x^{2a} - x^a y^b + 3x^a z^c - 6y^{2b} - 9y^b z^c
\end{array}$$

29.

$$\begin{array}{l|l}
6a^5 - 13a^4 + 4a^3 + 3a^2 & -a^2 + a \\ \cline{2-2}
\underline{6a^5 - 6a^4} & \underline{-6a^3 + 7a^2 + 3a} \\
\quad -7a^4 + 4a^3 & \\
\quad \underline{-7a^4 + 7a^3} & \\
\qquad\quad -3a^3 + 3a^2 & \\
\qquad\quad \underline{-3a^3 + 3a^2} &
\end{array}$$

30.

$$\begin{array}{l|l}
-30a^4 - 11a^3 + 82a^2 + 12a - 48 & 3a^2 + 2a - 4 \\ \cline{2-2}
\underline{-30a^4 - 20a^3 + 40a^2} & \underline{-10a^2 + 3a + 12} \\
\qquad 9a^3 + 42a^2 + 12a & \\
\qquad \underline{9a^3 + 6a^2 - 12a} & \\
\qquad\qquad 36a^2 + 24a - 48 & \\
\qquad\qquad \underline{36a^2 + 24a - 48} &
\end{array}$$

31.

$$\begin{array}{l|l}
10x^6 - 11x^5 - 3x^4 + 20x^3 + 10x^2 + 2 & 5x^3 - 3x^2 + 2x - 2 \\ \cline{2-2}
\underline{10x^6 - 6x^5 + 4x^4 - 4x^3} & 2x^3 - x^2 - 2x + 4 \\
\quad -5x^5 - 7x^4 + 24x^3 + 10x^2 & \qquad + \dfrac{24x^2 - 12x + 10}{5x^3 - 3x^2 + 2x - 2} \\
\quad \underline{-5x^5 + 3x^4 - 2x^3 + 2x^2} & \\ \cline{2-2}
\qquad -10x^4 + 26x^3 + 8x^2 + 2 & \\
\qquad \underline{-10x^4 + 6x^3 - 4x^2 + 4x} & \\
\qquad\qquad 20x^3 + 12x^2 - 4x + 2 & \\
\qquad\qquad \underline{20x^3 - 12x^2 + 8x - 8} & \\
\qquad\qquad\quad 24x^2 - 12x + 10 &
\end{array}$$

32.

$$\begin{array}{l|l}
a^8 - 3a^6 + a^4 - 7a^2 + 3 & a^4 - 2a^2 + 1 \\ \cline{2-2}
\underline{a^8 - 2a^6 + a^4} & a^4 - a^2 - 2 + \dfrac{5 - 10a^2}{a^4 - 2a^2 + 1} \\ \cline{2-2}
\quad -a^6 \qquad - 7a^2 + 3 & \qquad a^2 - 1 \\
\quad \underline{-a^6 + 2a^4 - a^2} & \qquad \underline{a^2 - 1} \\
\qquad -2a^4 - 6a^2 + 3 & \qquad a^4 - a^2 \\
\qquad \underline{-2a^4 + 4a^2 - 2} & \qquad \underline{\quad - a^2 + 1} \\
\qquad\quad -10a^2 + 5 & \qquad a^4 - 2a^2 + 1
\end{array}$$

33.
$$\begin{array}{l|l}
a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 7ab^4 - b^5 & a^2 - 2ab + b^2 \\ \hline
\underline{a^5 - 2a^4b + a^3b^2} & a^3 - 3a^2b + 3ab^2 - b^3 \\
\qquad -3a^4b + 9a^3b^2 - 10a^2b^3 & \qquad + \dfrac{2ab^4}{a^2 - 2ab + b^2} \\
\qquad \underline{-3a^4b + 6a^3b^2 - 3a^2b^3} & \\ \hline
\qquad\qquad\qquad 3a^3b^2 - 7a^2b^3 + 7ab^4 & \\
\qquad\qquad\qquad \underline{3a^3b^2 - 6a^2b^3 + 3ab^4} & \\
\qquad\qquad\qquad\qquad - a^2b^3 + 4ab^4 - b^5 & \\
\qquad\qquad\qquad\qquad \underline{- a^2b^3 + 2ab^4 - b^5} & \\
\qquad\qquad\qquad\qquad\qquad\qquad 2ab^4 &
\end{array}$$

$$\begin{array}{l}
a - b \\
\underline{a - b} \\
a^2 - ab \\
\underline{ - ab + b^2} \\
a^2 - 2ab + b^2
\end{array}$$

34.
$$\begin{array}{l}
6a^2 + 5a - 6 \\
\underline{2a^2 - 13a + 20} \\
12a^4 + 10a^3 - 12a^3 \\
\qquad - 78a^3 - 65a^2 + 78a \\
\qquad\qquad\qquad \underline{+ 120a^2 + 100a - 120}
\end{array}
\qquad
\begin{array}{l}
3a - 2 \\
\underline{2a - 5} \\
6a^2 - 4a \\
\underline{ - 15a + 10} \\
6a^2 - 19a + 10
\end{array}$$

$$\begin{array}{l|l}
12a^4 - 68a^3 + 43a^2 + 178a - 120 & 6a^2 - 19a + 10 \\ \hline
\underline{12a^4 - 38a^3 + 20a^2} & 2a^2 - 5a - 12 \\
\qquad - 30a^3 + 23a^2 + 178a & \\
\qquad \underline{- 30a^3 + 95a^2 - 50a} & \\
\qquad\qquad\qquad - 72a^2 + 228a - 120 & \\
\qquad\qquad\qquad \underline{- 72a^2 + 228a - 120} &
\end{array}$$

35.
$$\begin{array}{l|l}
x^3 + 36xy - 27y^3 + 64 & x - 3y + 4 \\ \hline
x^2 - 3x^2y + 4x^2 & x^2 + 3xy - 4x + 9y^2 + 12y + 16 \\ \hline
\qquad 3x^2y - 4x^2 \qquad\qquad + 36xy & \\
\qquad \underline{3x^2y \qquad\qquad - 9xy^2 + 12xy} & \\
\qquad\qquad - 4x^2 + 9xy^2 + 24xy & \\
\qquad\qquad \underline{- 4x^2 \qquad\qquad + 12xy - 16x} & \\
\qquad\qquad\qquad\qquad 9xy^2 + 12xy + 16x - 27y^3 + 64 & \\
\qquad\qquad\qquad\qquad \underline{9xy^2 \qquad\qquad\qquad\qquad - 27y^3 \qquad + 36y^2} & \\
\qquad\qquad\qquad\qquad\qquad 12xy + 16x \qquad\qquad + 64 - 36y^2 & \\
\qquad\qquad\qquad\qquad\qquad \underline{12xy \qquad\qquad\qquad\qquad\qquad - 36y^2 + 48y} & \\
\qquad\qquad\qquad\qquad\qquad\qquad 16x \qquad\qquad + 64 \qquad\qquad - 48y & \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{16x \qquad\qquad + 64 \qquad\qquad - 48y} &
\end{array}$$

36. $6y - 20 + 5y - 18 = 36y - 4 - 40y + 9.$

Collecting, $11y - 38 = -4y + 5.$

Whence $y = 2\tfrac{13}{15}.$

37. $4y - 3(6 - y) + 2 = 6(y - 2) - 13y + 24.$

Simplifying, $4y - 18 + 3y = 6y - 12 - 13y + 22.$

Whence $y = 2.$

38. $6y - 4(1 - 2y) + 10 = 4(2y - 5) - 4y + 1.$
Simplifying, $6y - 4 + 8y + 9 = 8y - 20 - 4y.$
Whence $y = -2\frac{1}{2}.$

39. $(y + 7)(y - 11) = (y + 8)(y - 5) - 2.$
Expanding, $y^2 - 4y - 77 = y^2 + 3y - 40 - 2.$
Whence $y = -5.$

40. $(y - 3)^3 = (y^2 - 6y + 9)(5 + y) - 8y^2.$
Expanding, $y^3 - 9y^2 + 27y - 27 = y^3 - y^2 - 21y + 45 - 8y^2.$
Whence $y = \frac{3}{2}.$

41. $y^2 + a^2 - (y + 1)^2 - (a + 1)^2 = 0.$
Expanding,
$y^2 + a^2 - y^2 - 2y - 1 - a^2 - 2a - 1 = 0.$
Whence $y = -a - 1.$

42. $3ay + 6ac + 4b^2 = 3ab + 4by + 8bc.$
Arranging, $3ay - 4by = 3ab - 4b^2 - 6ac + 8bc.$
Whence $y = b - 2c.$

43. $x = \dfrac{-10 + \sqrt{100 + 44}}{2} = 1.$

44. $x = \dfrac{6 + \sqrt{36 + 160}}{10} = \dfrac{6 + 14}{10} = 2.$

45. $x = \dfrac{-5 - \sqrt{25 + 96}}{6} = \dfrac{-5 - 11}{6} = -2\frac{2}{3}.$

46. $x = \dfrac{-7 - \sqrt{49 + 176}}{4} = \dfrac{-7 - 15}{4} = -5\frac{1}{2}.$

47. $12s = \frac{1}{2} \cdot 18(20)^2.$
Whence $s = 300.$

48. $e = \frac{1}{2} \cdot 7(32.2 \times 40)^2 = \frac{1}{2} \cdot 7(1288)^2 = \frac{1}{2} \cdot 7(1{,}658{,}944) = 5{,}806{,}304.$

49. $s = \dfrac{3 + 4 + 5}{2} = 6.$

$$\sqrt{s(s - a)(s - b)(s - c)} = \sqrt{6 \cdot 3 \cdot 2 \cdot 1} = 6.$$

50. Let $n =$ the number.
Then $n + 1$ and $n + 2 =$ the next two numbers.
Therefore $n^2 + 38 = n^2 + 3n + 2.$
Whence $n = 12,$
$n + 1 = 13,$
and $n + 2 = 14.$

51. Let $n =$ a certain odd number.
Then $n + 2$ and $n + 4 =$ the next two odd numbers.
Therefore $n^2 + 98 = n^2 + 6n + 8.$
Whence $n = 15,$ the first odd number,
$n + 2 = 17,$ the second odd number,
and $n + 4 = 19,$ the third odd number.

52. Let $n =$ an odd number.
Then $n + 1$ and $n + 3 =$ the next two even numbers.
Therefore $n^2 + 47 = n^2 + 4n + 3.$
Whence $n = 11,$ the odd number,
$n + 1 = 12,$ the next even number,
and $n + 3 = 14,$ the second even number.

53. Let $n =$ the first number.
Then $n + 1$, $n + 2$, and $n + 3 =$ the next three numbers.
Therefore $n^2 + n + 42 = n^2 + 5n + 6.$
Whence $n = 9,$ the first number,
$n + 1 = 10,$ the second number,
$n + 2 = 11,$ the third number,
and $n + 3 = 12,$ the fourth number.

54. Let $x =$ the number of years sought.
Therefore $42 + x = 2(15 + x).$
Whence $x = 12.$

55. Let $x =$ the number of years sought.
Therefore $36 - x = 3(16 - x).$
Whence $x = 6.$

56. Let $x =$ the number of years sought.
Therefore $52 + x = 2(21 + x).$
Whence $x = 10.$

57. Let $x =$ the number of years sought.
Therefore $87 - x = 4(42 - x).$
Whence $x = 27.$

58. Let $q =$ the number of quarters.
Then $2q =$ the number of nickels,
and $32 - 3q =$ the number of dimes.
Then $25q =$ the value of quarters in cents,
$10q =$ the value of nickels in cents,
and $320 - 30q =$ the value of dimes in cents.
Therefore $25q + 10x + 320 - 30q = 360.$

Whence $q = 8$,
$2q = 16$,
and $32 - 3q = 8$.

59. Let $d =$ the distance to the target in feet.
Therefore $\frac{d}{1280} + \frac{d}{1120} = 6$.
Whence $d = 3584$.

60. Let $x =$ age of each.
$(x + 12)3 - 36 + x =$ the result.
$4x =$ the result.
Therefore he divided each result by 4.

61. Let $x =$ the per cent over 60 years.
Then $9x =$ the per cent between 20 and 60,
and $9x + 5 =$ the per cent under 20.
Therefore $x + 9x + 9x + 5 = 100$.
Whence $x = 5$,
$9x = 45$,
and $9x + 5 = 50$.

62. Let $x =$ the length in feet of the Hoosac tunnel.
Then $2x - 1320 =$ the length in feet of the St. Gothard tunnel,
and $2x - 10{,}320 =$ the length in feet of the Mount Cenis tunnel.
Therefore $x + 2x - 1320 + 2x - 10{,}320 = 113{,}760$.
Whence $x = 25{,}080$,
$2x - 1320 = 48{,}840$,
and $2x - 10{,}320 = 39{,}840$.

63. Let $x =$ the height in feet of Niagara Falls.
Then $2x + 32 =$ the height in feet of falls of Zambesi,
$8x + 188 =$ the height in feet of falls of Yosemite,
and $8x - 332 =$ the height in feet of Staubbach Falls.
Therefore $x + 2x + 32 + 8x + 188 + 8x - 332 = 3004$.
Whence $x = 164$,
$2x + 32 = 360$,
$8x + 188 = 1500$,
and $8x - 332 = 980$.

64. Let $x =$ the weight in pounds of cubic foot of water.
Then $x - 11.5 =$ the weight in pounds of cubic foot of alcohol,
and $16x - 151.3 =$ the weight in pounds of cubic foot of mercury.
Therefore $x + x - 11.5 + 16x - 151.3 = 962.2$.

Whence $x = 62.5,$
$x - 11.5 = 51,$
and $16x - 151.3 = 848.7.$

65. Let $x =$ the rainfall of San Diego in inches.
Then $5x - 1 =$ the rainfall of Boston in inches,
$5x - 2 =$ the rainfall of St. Louis in inches,
and $15x - 12 =$ the rainfall of Neahbay in inches.
Therefore $x + 5x - 1 + 5x - 2 + 15x - 12 = 219.$
Whence $x = 9,$
$5x - 1 = 44,$
$5x - 2 = 43,$
and $15x - 12 = 123.$

66. Let $x =$ the number of years in the expectation of one at 65.
Then $2x - .94 =$ the number of years in the expectation of one at 45,
$2x + 14.37 =$ the number of years in the expectation of one at 10,
and $2x + 8.98 =$ the number of years in the expectation of one at 21.
Therefore $x + 2x - .94 + 2x + 14.37 + 2x + 8.98 = 109.42.$
Whence $x = 12.43,$
$2x - .94 = 23.92,$
$2x + 14.37 = 39.23,$
and $2x + 8.98 = 33.84.$

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3. $15^2 = (10 + 5)^2 = 100 + 25 + 2 \cdot 10 \cdot 5 = 225.$
4. $17^2 = (20 - 3)^2 = 400 + 9 - 2 \cdot 20 \cdot 3 = 289.$
5. $18^2 = (20 - 2)^2 = 400 + 4 - 2 \cdot 20 \cdot 2 = 324.$
6. $19^2 = (20 - 1)^2 = 400 + 1 - 2 \cdot 20 \cdot 1 = 361.$
7. $(x + y)^2 = x^2 + y^2 + 2xy.$
8. $(x - c)^2 = x^2 + c^2 - 2xc.$
9. $(x + 1)^2 = x^2 + 1 + 2x.$
10. $(a + 2)^2 = a^2 + 4a + 4.$
11. $(b^2 + 3)^2 = b^4 + 6b^2 + 9.$
12. $(c - 4)^2 = c^2 - 8c + 16.$
13. $(2c^2 + d)^2 = 4c^4 + 4c^2d + d^2.$
14. $(3x - y)^2 = 9x^2 - 6xy + y^2.$
15. $(x + 2y)^2 = x^2 + 4xy + 4y^2.$
16. $(a - 4b)^2 = a^2 - 8ab + 16b^2.$
17. $(y^5 - 2)^2 = y^{10} - 4y^5 + 4.$
18. $(2a + 1)^2 = 4a^2 + 4a + 1.$
19. $(3a^2 + 2)^2 = 9a^4 + 12a^2 + 4.$
20. $(4a + 3)^2 = 16a^2 + 24a + 9.$
21. $(2a^2 - 3b)^2 = 4a^4 - 12a^2b + 9b^2.$
22. $(3c^2 - 4a^2)^2 = 9c^4 - 24c^2a^2 + 16a^4.$
23. $(2x^3 - 6)^2 = 4x^6 - 24x^3 + 36.$
24. $(3xy + y^2)^2 = 9x^2y^2 + 6xy^3 + y^4.$
25. $(7x^2 - 3y^4)^2 = 49x^4 - 42x^2y^4 + 9y^8.$
26. $(8y^3 + 2bx)^2 = 64y^6 + 32y^3bx + 4b^2x^2.$
27. $(6xy - 2a^2)^2 = 36x^2y^2 - 24xya^2 + 4a^4.$

28. $(4xy - 2x^2)^2 = 16x^2y^2 - 16x^3y + 4x^4.$

29. $(3x^4 - 5x^6)^2 = 9x^8 - 30x^{10} + 25x^{12}.$

30. $(2x^2 - 6x^3y)^2 = 4x^4 - 24x^5y + 36x^6y^2.$

31. $41^2 = (40 + 1)^2 = 1681.$

32. $92^2 = (90 + 2)^2 = 8464.$

33. $101^2 = (100 + 1)^2 = 10,201.$

34. $202^2 = (200 + 2)^2 = 40,804.$

35. $1001^2 = (1000 + 1)^2 = 1,002,001.$

36. $\dfrac{a^2 + 2ab + b^2}{a + b} = a + b.$

37. $\dfrac{a^2 - 2ac + c^2}{a - c} = a - c.$

38. $\dfrac{a^2 - 4a + 4}{a - 2} = a - 2.$

39. $\dfrac{9x^2 - 6x + 1}{3x - 1} = 3x - 1.$

40. $\dfrac{y^2 + 40y + 400}{20 + y} = 20 + y.$

41. $\dfrac{9x^2 - 12xy + 4y^2}{2y - 3x} = 2y - 3x.$

42. $\dfrac{4a^2 - 20a + 25}{-5 + 2a} = -5 + 2a.$

43. $\dfrac{4x^2 - 12xy + 9y^2}{3y - 2x} = 3y - 2x.$

44. $(x^2 - 2xy + y^2) \div (x - y) = x - y.$

45. $(a^2 + 2a + 1) \div (a + 1) = a + 1.$

46. $(a^2 + 4a + 4) \div (a + 2) = a + 2.$

47. $(4x^2 + 4x + 1) \div (2x + 1) = 2x + 1.$

48. $(16x^2 - 8x + 1) \div (4x - 1) = 4x - 1.$

49. $(16x^2 - 8xy + y^2) \div (4x - y) = 4x - y.$

50. $(9x^4 - 6x^2 + 1) \div (3x^2 - 1) = 3x^2 - 1.$

51. $(16y^6 - 40y^3 + 25) \div (4y^3 - 5) = 4y^3 - 5.$

52. $c^2 + 2cd + d^2 = (c + d)(c + d).$

53. $a^2 - 2a + 1 = (a - 1)(a - 1).$

54. $x^2 - 10x + 25 = (x - 5)(x - 5).$

55. $4a^2 - 4a + 1 = (2a - 1)(2a - 1).$

56. $4x^2 + 12x + 9 = (2x + 3)(2x + 3).$

57. $9a^2b^2 - 6ab + 1 = (3ab - 1)(3ab - 1).$

58. $16x^4y^2 - 24x^2ya + 9a^2 = (4x^2y - 3a)(4x^2y - 3a).$

59. $25a^4b^6 - 60a^2b^3c^2 + 36c^4 = (5a^2b^3 - 6c^2)(5a^2b^3 - 6c^2).$

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1. $$\begin{aligned}(a + b + c)^2 &= [(a + b) + c][(a + b) + c]\\ &= (a + b)^2 + 2c(a + b) + c^2\\ &= a^2 + 2ab + b^2 + 2ac + 2bc + c^2.\end{aligned}$$

2. $$\begin{aligned}(a + b - c)^2 &= [(a + b) - c]^2\\ &= (a + b)^2 - 2c(a + b) + c^2\\ &= a^2 + 2ab + b^2 - 2ac - 2bc + c^2.\end{aligned}$$

3. $$\begin{aligned}(a + b - 1)^2 &= [(a + b) - 1]^2\\ &= (a + b)^2 - 2(a + b) + 1\\ &= a^2 + 2ab + b^2 - 2a - 2b + 1.\end{aligned}$$

4. $(a+b+1)^2=[(a+b)+1]^2$
$=(a+b)^2+2(a+b)+1$
$=a^2+2ab+b^2+2a+2b+1.$

5. $(a-b+c)^2=[(a-b)+c]^2$
$=(a-b)^2+2c(a-b)+c^2$
$=a^2-2ab+b^2+2ac-2bc+c^2.$

6. $(a-b-c)^2=[(a-b)-c]^2$
$=(a-b)^2-2c(a-b)+c^2$
$=a^2-2ab+b^2-2ac+2bc+c^2.$

7. $(a+2b-3)^2=[(a+2b)-3]^2$
$=(a+2b)^2-2\cdot 3(a+2b)+9$
$=a^2+4ab+4b^2-6a-12b+9.$

8. $(a-2b+c)^2=[(a-2b)+c]^2$
$=(a-2b)^2+2c(a-2b)+c^2$
$=a^2-4ab+4b^2+2ac-4bc+c^2.$

9. $(2a+b-4c)^2=[(2a+b)-4c]^2$
$=(2a+b)^2-8c(2a+b)+16c^2$
$=4a^2+4ab+b^2-16ac-8bc+16c^2.$

10. $(3a-2b+5c)^2=[(3a-2b)+5c]^2$
$=(3a-2b)^2+10c(3a-2b)+25c^2$
$=9a^2-12ab+4b^2+30ac-20bc+25c^2.$

11. $(2a-3b-4c)^2=[(2a-3b)-4c]^2$
$=(2a-3b)^2-2\cdot 4c(2a-3b)+16c^2$
$=4a^2-12ab+9b^2-16ac+24bc+16c^2.$

12. $(4c-1+2ab)^2=[(4c-1)+2ab]^2$
$=(4c-1)^2+2\cdot 2ab(4c-1)+4a^2b^2$
$=16c^2-8c+1+16abc-4ab+4a^2b^2.$

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1. $(6+3)(6-3)=36-9=27.$
2. $(8+4)(8-4)=64-16=48.$
3. $(9-2)(9+2)=81-4=77.$
4. $(10-4)(10+4)=100-16=84.$
5. $(7+2)(7-2)=49-4=45.$
6. $(12-3)(12+3)=144-9=135.$
8. $35\cdot 25=(30+5)(30-5)=900-25=875.$
9. $33\cdot 27=(30+3)(30-3)=900-9=891.$
10. $36\cdot 44=(40-4)(40+4)=1600-16=1584.$
11. $35\cdot 45=(40-5)(40+5)=1600-25=1575.$
12. $52\cdot 48=(50+2)(50-2)=2500-4=2496.$
13. $65\cdot 75=(70-5)(70+5)=4900-25=4875.$
14. $72\cdot 68=(70+2)(70-2)=4900-4=4896.$
15. $75\cdot 85=(80-5)(80+5)=6400-25=6375.$
16. $97\cdot 103=(100-3)(100+3)=10{,}000-9=9991.$

17. $(x+3)(x-3)=x^2-9.$

18. $(a+5)(a-5)=a^2-25.$

19. $(2x+4)(2x-4)=4x^2-16.$

20. $(3n+5)(3n-5)=9n^2-25.$

21. $(x+y)(x-y)=x^2-y^2.$

22. $(x-a)(x+a)=x^2-a^2.$

23. $(x-1)(x+1)=x^2-1.$

24. $(a+2)(a-2)=a^2-4.$

25. $(a^2+3)(a^2-3)=a^4-9.$

26. $(4+y^2)(4-y^2)=16-y^4.$

27. $(4-x)(x+4)=16-x^2.$

28. $(2c+a)(2c-a)=4c^2-a^2.$

29. $(3a+b)(b-3a)=b^2-9a^2.$

30. $(4b+2c)(4b-2c)=16b^2-4c^2.$

31. $(3xy-2)(3xy+2)=9x^2y^2-4.$

32. $(4ab-3)(4ab+3)=16a^2b^2-9.$

33. $(a^4-b^3)(a^4+b^3)=a^8-b^6.$

34. $(a^6+a^3)(-a^3+a^6)=a^{12}-a^6.$

35. $(x^3-2y)(x^3+2y)=x^6-4y^2.$

36. $(4ab-a^2)(4ab+a^2)=16a^2b^2-a^4.$

37. $(3cd^2+2d)(-2d+3cd^2)=9c^2d^4-4d^2.$

38. $(6cd+3)(-3+6cd)=36c^2d^2-9.$

39. $(4xy+2y)(2y-4xy)=4y^2-16x^2y^2.$

40. $(3abc-2bc)(2bc+3abc)=9a^2b^2c^2-4b^2c^2.$

41. $(a^2-b^2)\div(a+b)=a-b.$

42. $(c^2-d^2)\div(c-d)=c+d.$

43. $(9-b^2)\div(3+b)=3-b.$

44. $(36-a^2)\div(6-a)=6+a.$

45. $(9x^2-16)\div(3x+4)=3x-4.$

46. $(x^4-1)\div(x^2-1)=x^2+1.$

47. $(x^2-y^2)\div(x-y)=x+y.$

48. $(x^2-1)\div(x+1)=x-1.$

49. $(4x^2-9)\div(2x-3)=2x+3.$

50. $(25-16x^2)\div(5+4x)=5-4x.$

51. $(16-x^8)\div(4-x^4)=4+x^4.$

52. $(y^6-4)\div(y^3-2)=y^3+2.$

53. $(c^2-d^2)=(c-d)(c+d).$

54. $(n^2-4)=(n+2)(n-2).$

55. $n^2-16=(n+4)(n-4).$

56. $9-4a^2=(3-2a)(3+2a).$

57. $36b^2-1=(6b+1)(6b-1).$

58. $a^2-9=(a+3)(a-3).$

59. $25-4n^2=(5-2n)(5+2n).$

60. $100-9x^2=(10-3x)(10+3x).$

61. $[(x+y)+1][(x+y)-1]=(x+y)^2-1=x^2+2xy+y^2-1.$

62.
$$\begin{aligned}(x+a+3)(x+a-3)&=[(x+a)+3][(x+a)-3]\\&=x^2+2xa+a^2-9.\end{aligned}$$

63.
$$\begin{aligned}[(x-a)+3][(x-a)-3]&=(x-a)^2-9\\&=x^2-2ax+a^2-9.\end{aligned}$$

64.
$$\begin{aligned}(x+4+c)(x+4-c)&=[(x+4)+c][(x+4)-c]\\&=(x+4)^2-c^2\\&=x^2+8x+16-c^2.\end{aligned}$$

65.
$$\begin{aligned}(2a-b+c)(2a-b-c)&=[(2a-b)+c][(2a-b)-c]\\&=(2a-b)^2-c^2\\&=4a^2-4ab+b^2-c^2.\end{aligned}$$

66. $[x+(b+c)][x-(b+c)]=x^2-(b+c)^2=x^2-b^2-2bc-c^2.$

67. $[x+(b-c)][x-(b-c)]=x^2-(b-c)^2=x^2-b^2+2bc-c^2.$

68. $[3+(x-y)][3-(x-y)]=9-(x-y)^2=9-x^2+2xy-y^2.$

69.
$$\begin{aligned}[4x+(2y-x)][4x-(2y-x)]&=16x^2-(2y-x)^2\\&=16x^2-4y^2+4xy-x^2\\&=15x^2-4y^2+4xy.\end{aligned}$$

70. $$\begin{aligned}[10-(a-5)][10+(a-5)] &= 100-(a-5)^2\\ &= 100-a^2+10a-25\\ &= 75+10a-a^2.\end{aligned}$$

71. $49x^2-1=(7x+1)(7x-1)$.

72. $64x^2-25=(8x-5)(8x+5)$.

73. $(a+b)^2-1=(a+b+1)(a+b-1)$.

74. $(x-y)^2-4=(x-y-2)(x-y+2)$.

75. $(2x-1)^2-a^2=(2x-1+a)(2x-1-a)$.

76. $(3y-z)^2-b^2=(3y-z+b)(3y-z-b)$.

77. $b^2-(x+y)^2=[b+(x+y)][b-(x+y)]=(b+x+y)(b-x-y)$.

78. $b^2-(x-y)^2=[b+(x-y)][b-(x-y)]$.

79. $c^2-(x+c)^2=[c+(x+c)][c-(x+c)]$.

80. $4-(x-2)^2=[2+(x-2)][2-(x-2)]$.

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1. $(x+1)(x+2)=x^2+3x+2$. **2.** $(x+2)(x+3)=x^2+5x+6$.

3. $(x+3)(x+4)=x^2+7x+12$.

4. $(x+4)(x+5)=x^2+9x+20$.

5. $(x+5)(x+6)=x^2+11x+30$.

6. $(n+1)(n-2)=n^2-n-2$.

7. $(n-2)(n+3)=n^2+n-6$.

8. $(n-3)(n+4)=n^2+n-12$.

9. $(n-4)(n+5)=n^2+n-20$.

10. $(n-5)(n+6)=n^2+n-30$.

11. $(a-1)(a-2)=a^2-3a+2$.

12. $(a-2)(a-3)=a^2-5a+6$.

13. $(a-3)(a-4)=a^2-7a+12$.

14. $(a-4)(a-5)=a^2-9a+20$.

15. $(a-4)(a-6)=a^2-10a+24$.

16. $(2y+3)(2y+4)=4y^2+14y+12$.

17. $(2y+2)(2y+3)=4y^2+10y+6$.

18. $(3a+1)(3a+4)=9a^2+15a+4$.

19. $(2n+3)(2n+5)=4n^2+16n+15$.

20. $(2n+3)(2n-5)=4n^2-4n-15$.

21. $(2n+3)(2n-4)=4n^2-2n-12$.

22. $(2a+1)(2a-5)=4a^2-8a-5$.

23. $(3a-5)(3a+1)=9a^2-12a-5$.

24. $(4a+3)(4a-5)=16a^2-8a-15$.

25. $(4a-5)(4a-6)=16a^2-44a+30$.

26. $(4ab+1)(4ab-6)=16a^2b^2-20ab-6$.

27. $(3x-2)(3x+5)=9x^2+9x-10$.

28. $(4a - 3b)(4a + 5b) = 16a^2 + 8ab - 15b^2.$
29. $(5a - 6b)(5a - 7b) = 25a^2 - 65ab + 42b^2.$
30. $\dfrac{x^2 + 3x + 2}{x + 1} = x + 2.$
31. $\dfrac{x^2 + 5x + 6}{x + 2} = x + 3.$
32. $\dfrac{x^2 + 4x + 3}{x + 3} = x + 1.$
33. $\dfrac{x^2 + 6x + 5}{x + 1} = x + 5.$
34. $\dfrac{x^2 + 7x + 12}{x + 3} = x + 4.$
35. $\dfrac{x^2 - 7x + 12}{x - 3} = x - 4.$
36. $\dfrac{x^2 - 5x + 6}{x - 3} = x - 2.$
37. $\dfrac{x^2 - 5x + 4}{x - 1} = x - 4.$
38. $\dfrac{x^2 - 6x + 5}{x - 5} = x - 1.$
39. $\dfrac{x^2 - 7x + 6}{x - 6} = x - 1.$
40. $\dfrac{x^2 - 7x + 12}{x - 4} = x - 3.$
41. $x^2 + 3x + 2 = (x + 1)(x + 2).$
42. $x^2 + 5x + 6 = (x + 3)(x + 2).$
43. $x^2 + 7x + 12 = (x + 4)(x + 3).$
44. $x^2 + 6x + 8 = (x + 4)(x + 2).$
45. $x^2 + 7x + 10 = (x + 2)(x + 5).$
46. $x^2 + 8x + 15 = (x + 3)(x + 5).$
47. $x^2 - 3x + 2 = (x - 2)(x - 1).$
48. $x^2 - 5x + 6 = (x - 3)(x - 2).$
49. $x^2 - 6x + 8 = (x - 4)(x - 2).$
50. $x^2 - 8x + 15 = (x - 5)(x - 3).$
51. $x^2 - 8x + 12 = (x - 6)(x - 2).$
52. $x^2 - 9x + 14 = (x - 7)(x - 2).$
53. $x^2 + 8x + 7 = (x + 1)(x + 7).$
54. $x^2 + 9x + 8 = (x + 1)(x + 8).$
55. $x^2 - 10x + 9 = (x - 9)(x - 1).$
56. $x^2 - 11x + 10 = (x - 10)(x - 1).$
57. $x^2 - 10x + 16 = (x - 2)(x - 8).$
58. $x^2 + 10x + 21 = (x + 3)(x + 7).$
59. $x^2 + 9x + 20 = (x + 4)(x + 5).$
60. $x^2 - 9x + 18 = (x - 6)(x - 3).$
61. $x^2 + 9x + 14 = (x + 2)(x + 7).$
62. $x^2 - 12x + 32 = (x - 4)(x - 8).$
63. $x^2 + 11x + 10 = (x + 10)(x + 1).$

Page 100 (First set)

1. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc.$
2. $(a + b + 1)^2 = a^2 + b^2 + 1 + 2ab + 2a + 2b.$
3. $(a + b + 2)^2 = a^2 + b^2 + 4 + 2ab + 4a + 4b.$
4. $(2a + b - c)^2 = 4a^2 + b^2 + c^2 + 4ab - 4ac - 2bc.$
5. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc.$
6. $(2x + y - 1)^2 = 4x^2 + y^2 + 1 + 4xy - 4x - 2y.$
7. $(4x + y - 2)^2 = 16x^2 + y^2 + 4 + 8xy - 16x - 4y.$
8. $(a - bc + d)^2 = a^2 + b^2c^2 + d^2 - 2abc + 2ad - 2bcd.$
9. $(3a - 3b + 1)^2 = 9a^2 + 9b^2 + 1 - 18ab + 6a - 6b.$
10. $(a - b + c - d)^2$
$= a^2 + b^2 + c^2 + d^2 - 2ab + 2ac - 2ad - 2bc + 2bd - 2cd.$
11. $(x + y - a + 1)^2$
$= x^2 + y^2 + a^2 + 1 + 2xy - 2ax + 2x - 2ay + 2y - 2a.$
12. $(a - y + b - 3)^2$
$= a^2 + y^2 + b^2 + 9 - 2ay + 2ab - 6a - 2by + 6y - 6b.$

Page 100 (Second set)

1. $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3.$
2. $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3.$
3. $(x+1)^3 = x^3 + 3x^2 + 3x + 1.$
4. $(x-1)^3 = x^3 - 3x^2 + 3x - 1.$
5. $(x+2)^3 = x^3 + 6x^2 + 12x + 8.$
6. $(x-2)^3 = x^3 - 6x^2 + 12x - 8.$
7. $(x-3)^3 = x^3 - 9x^2 + 27x - 27.$
8. $(x+3)^3 = x^3 + 9x^2 + 27x + 27.$
9. $(2x+y)^3 = 8x^3 + 12x^2y + 6xy^2 + y^3.$
10. $(x-2y)^3 = x^3 - 6x^2y + 12xy^2 - 8y^3.$
11. $(2x-1)^3 = 8x^3 - 12x^2 + 6x - 1.$
12. $(3x+2)^3 = 27x^3 + 54x^2 + 36x + 8.$
13. $(3x-2y)^3 = 27x^3 - 54x^2y + 36xy^2 - 8y^3.$
14. $(4x+3y)^3 = 64x^3 + 144x^2y + 108xy^2 + 27y^3.$
15. $(5x-2y)^3 = 125x^3 - 150x^2y + 60xy^2 - 8y^3.$

16. The cube of the binomial $a + b$ is equal to the cube of a plus three times the square of a multiplied by b, plus three times the product of a and the square of b, plus the cube of b.

17. The cube of the binomial $a - b$ is equal to the cube of a minus three times the square of a multiplied by b, plus three times the product of a and the square of b, minus the cube of b.

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1. $\sqrt{4a^2} = 2a.$
2. $\sqrt{9a^4} = 3a^2.$
3. $\sqrt{25a^6x^4} = 5a^3x^2.$
4. $\sqrt{4x^4} = 2x^2.$
5. $\sqrt{9x^6} = 3x^3.$
6. $\sqrt{16x^{16}} = 4x^8.$
7. $\sqrt{49x^{14}y^{10}} = 7x^7y^5.$
8. $\sqrt{81x^{12}d^{20}} = 9x^6d^{10}.$
9. $\sqrt{121c^{22}d^6} = 11c^{11}d^3.$
10. $\sqrt{169x^6} = 13x^3.$
11. $\sqrt{196y^{10}} = 14y^5.$
12. $\sqrt{225y^4z^6} = 15y^2z^3.$
13. $\sqrt{256c^{12}} = 16c^6.$
14. $\sqrt{400a^2b^2c^4} = 20abc^2.$
15. $\sqrt{324b^8} = 18b^4.$
16. $\sqrt{625b^2y^{18}z^6} = 25by^9z^3.$
17. $\sqrt{x^{2n}} = x^n.$
18. $\sqrt{x^{4n}} = x^{2n}.$
19. $\sqrt{x^{6n}} = x^{3n}.$
20. $\sqrt{a^{4n}b^{8n}} = a^{2n}b^{4n}.$
21. $\sqrt[3]{8} = 2.$
22. $\sqrt[3]{-8a^3} = -2a.$
23. $\sqrt[3]{27} = 3.$
24. $\sqrt[3]{64} = 4.$
25. $\sqrt[3]{-125} = -5.$
26. $\sqrt[3]{216} = 6.$
27. $\sqrt[3]{-a^3} = -a.$
28. $\sqrt[3]{-a^6} = -a^2.$
29. $\sqrt[3]{a^9b^3} = a^3b.$
30. $\sqrt[3]{343y^3} = 7y.$
31. $\sqrt[3]{512x^{15}} = 8x^5.$
32. $\sqrt[3]{-729a^3b^9} = -9ab^3.$
33. $\sqrt[3]{1000a^6b^{12}} = 10a^2b^4.$
34. $\sqrt[3]{-27a^3b^6c^{12}} = -3ab^2c^4.$
35. $\sqrt[3]{8a^{12}b^3c^9} = 2a^4bc^3.$

36. $\sqrt[3]{-125\,a^6b^6z^6} = -5\,a^2b^2z^2.$

37. $\sqrt[3]{x^{3n}} = x^n.$

38. $\sqrt[3]{x^{6n}} = x^{2n}.$

39. $\sqrt[3]{-x^{9n}} = -x^{3n}.$

40. $\sqrt[3]{x^{12n}y^{6n}} = x^{4n}y^{2n}.$

41. $\sqrt[5]{x^{15}} = x^3.$

42. $\sqrt[5]{32\,x^{10}} = 2\,x^2.$

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1. $3\,x + 6 = 3\,(x + 2).$
2. $x^3 - x^2 = x^2(x - 1).$
3. $8\,x - 2\,x^4 = 2\,x\,(4 - x^3).$
4. $xy + y^2 = y\,(x + y).$
5. $5\,b^3 - 15\,b^6 = 5\,b^3(1 - 3\,b^3).$
6. $c^3 - c^2y = c^2(c - y).$
7. $x^3 - x + x^2 = x\,(x^2 - 1 + x).$
8. $c^5 - c^3 + c^2 = c^2(c^3 - c + 1).$
9. $3\,y - 9\,y^4 + 12 = 3\,(y - 3\,y^4 + 4).$
10. $c^2 + 2\,bc - c = c\,(c + 2\,b - 1).$
11. $3\,y^2 - 15\,y + 6\,y^3 = 3\,y\,(y - 5 + 2\,y^2).$
12. $10\,ab - 14\,bc - 8\,b^2 = 2\,b\,(5\,a - 7\,c - 4\,b).$
13. $y^5 + 3\,y^4z + 6\,y^3z^2 = y^3(y^2 + 3\,yz + 6\,z^2).$
14. $4\,a^2x^2 - 6\,a^3y^3 + 12\,a^4x^2y^2 = 2\,a^2(2\,x^2 - 3\,ay^3 + 6\,a^2x^2y^2).$
15. $12\,z^3 + 30\,a^4z^7 - 18\,c^3z^5 = 6\,z^3(2 + 5\,a^4z^4 - 3\,c^3z^2).$
16. $6\,c^6 + 10\,c^{10} - 20\,c^2 = 2\,c^2(3\,c^4 + 5\,c^8 - 10).$
17. $8\,y^8 - 4\,y^4 - 12\,y^{12} = 4\,y^4(2\,y^4 - 1 - 3\,y^8).$
18. $x^4 + x^3 - x^2 + x = x\,(x^3 + x^2 - x + 1).$
19. $-8\,y^6 - 4\,y^6 + 12\,y^3 + 6\,y^5 = 6\,y^3(-2\,y^3 + 2 + y^2).$
20. $14\,b^5 - 49\,b^7 + 21\,b^2 - 7\,b^3 = 7\,b^2(2\,b^3 - 7\,b^5 + 3 - b).$
21. $-a^2x^2y^3 - 3\,a^4x^3y^2 - 2\,a^3x^5y^6z^3 + 5\,a^4x^4y^4z$
    $= a^2x^2y^2(-y - 3\,a^2x - 2\,ax^3y^4z^3 + 5\,a^2x^2y^2z).$
22. $-18\,a^3b^3c^4 - 45\,a^4cy^2 - 36\,a^3c^2xz - 63\,a^4cxy$
    $= -9\,a^3c\,(2\,b^3c^3 + 5\,ay^2 + 4\,cxz + 7\,axy).$
23. $32\,c^7x^2 + 80\,c^{10}bx - 112\,c^6b + 48\,c^{12}x$
    $= 16\,c^6(2\,cx^2 + 5\,c^4bx - 7\,b + 3\,c^6x).$
24. $56\,a^4b^3x^2 - 28\,a^3x + 112\,a^5x^2y^2 - 196\,a^6x^3$
    $= 28\,a^3x\,(2\,ab^3x - 1 + 4\,a^2xy^2 - 7\,a^3x^2).$

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1. $2\,(x + y) + a\,(x + y) = (x + y)\,(2 + a).$
2. $b\,(x + 2\,y) + (x + 2\,y) = (x + 2\,y)\,(b + 1).$
3. $c\,(c - a) - a\,(c - a) = (c - a)\,(c - a).$
4. $2\,c\,(3\,b - d) + 3\,a\,(3\,b - d) = (3\,b - d)\,(2\,c + 3\,a).$
5. $6\,d\,(c + x) - (c + x) + 4\,k\,(c + x) = (c + x)\,(6\,d - 1 + 4\,k).$
6. $-5\,(2\,a - 3\,b) + 7\,a\,(2\,a - 3\,b) - 3\,b\,(2\,a - 3\,b)$
    $= (2\,a - 3\,b)\,(7\,a - 5 - 3\,b).$
7. $k\,(a - b) + 3\,(b - a) = k\,(a - b) - 3\,(a - b) = (a - b)\,(k - 3).$

8. $h(x-y)+k(y-x)=h(x-y)-k(x-y)$

$=(x-y)(h-k).$

9. $5x(c-2d)-6(2d-c)=5x(c-2d)+6(c-2d)$

$=(c-2d)(5x+6).$

10. $3y(k-4h)+(4h-k)=3y(k-4h)-(k-4h)$

$=(k-4h)(3y-1).$

11. $2c(4k-3c)-5h(3c-4k)=2c(4k-3c)+5h(4k-3c)$

$=(4k-3c)(2c+5h).$

12. $3y(5x-b)-7(b-5x)+6z(5x-b)$

$=3y(5x-b)+7(5x-b)+6z(5x-b).$

$=(5x-b)(3y+7+6z).$

16. $ax+bx+ay+by=(ax+bx)+(ay+by)$

$=x(a+b)+y(a+b)$

$=(a+b)(x+y).$

17. $ch+h^2+cx+hx=(ch+h^2)+(cx+hx)$

$=h(c+h)+x(c+h)$

$=(c+h)(h+x).$

18. $bh-ch+bk-ck=(bh-ch)+(bk-ck)$

$=h(b-c)+k(b-c)$

$=(b-c)(h+k).$

19. $bx+xz-ab-az=(bx+xz)+(-ab-az)$

$=x(b+z)-a(b+z)$

$=(b+z)(x-a).$

20. $ax+3a-bx-3b=(ax+3a)+(-bx-3b)$

$=a(x+3)-b(x+3)$

$=(x+3)(a-b).$

21. $2x^2+10xy-4x-20y=2(x^2+5xy-2x-10y)$

$=2[x(x+5y)-2(x+5y)]$

$=2(x-2)(x+5y).$

22. $6ab-2ac+3b-c=(6ab-2ac)+(3b-c)$

$=2a(3b-c)+(3b-c)$

$=(3b-c)(2a+1).$

23. $36ax+45ac-4x-5c=9a(4x+5c)-(4x+5c)$

$=(4x+5c)(9a-1).$

24. $6a^3+14ab-15a^2y-35by=2a(3a^2+7b)-5y(3a^2+7b)$

$=(3a^2+7b)(2a-5y).$

25. $3ax-ay-6bx+2by=a(3x-y)-2b(3x-y)$

$=(3x-y)(a-2b).$

26. $12ax-6ay-50cx+25cy=6a(2x-y)-25c(2x-y)$

$=(2x-y)(6a-25c).$

27. $2h^3-3h^2k-10h+15k=h^2(2h-3k)-5(2h-3k)$

$=(2h-3k)(h^2-5).$

28. $10dh - 45hk - 22cd + 99ck = 5h(2d - 9k) - 11c(2d - 9k)$
$= (2d - 9k)(5h - 11c).$

29. $28hx + 9ky - 21hy - 12kx = 28hx - 21hy - 12kx + 9ky$
$= 7h(4x - 3y) - 3k(4x - 3y)$
$= (4x - 3y)(7h - 3k).$

30. $-2ax + 7a^2 + 16bx - 56ab = -a(2x - 7a) + 8b(2x - 7a)$
$= (2x - 7a)(8b - a).$

31. $6hk + 15xy - 10ky - 9hx = 6hk - 9hx - 10ky + 15xy$
$= 3h(2k - 3x) - 5y(2k - 3x)$
$= (2k - 3x)(3h - 5y).$

32. $-21ax + 12cx - 4cd + 7ad = 12cx - 21ax - 4cd + 7ad$
$= 3x(4c - 7a) - d(4c - 7a)$
$= (4c - 7a)(3x - d).$

33. $7gy - 77hy - cg + 11ch = 7y(g - 11h) - c(g - 11h)$
$= (g - 11h)(7y - c).$

34. $5a^3 + 10a - 5a^2 - 10 = 5[a(a^2 + 2) - (a^2 + 2)]$
$= 5(a^2 + 2)(a - 1).$

35. $3a - 5ax^3 - 6ax + 10ax^4 = a[(3 - 5x^3) - 2x(3 - 5x^3)]$
$= a(3 - 5x^3)(1 - 2x).$

36. $4abxy - 24dxy - 3abgh + 18dgh = 4xy(ab - 6d) - 3gh(ab - 6d)$
$= (ab - 6d)(4xy - 3gh).$

37. $8acxy - 20bx^2y - 6abc^2 + 15b^2cx$
$= 4xy(2ac - 5bx) - 3bc(2ac - 5bx)$
$= (2ac - 5bx)(4xy - 3bc).$

38. $ax + bx + cx + ay + by + cy = x(a + b + c) + y(a + b + c)$
$= (a + b + c)(x + y).$

39. $cg + 2ch + ck - gx - 2hx - kx = c(g + 2h + k) - x(g + 2h + k)$
$= (g + 2h + k)(c - x).$

40. $3ab - bx + 2by - 3ac + cx - 2cy$
$= b(3a - x + 2y) - c(3a - x + 2y)$
$= (3a - x + 2y)(b - c).$

41. $4ax - ac + 2ay + 12bx - 3bc + 6by$
$= a(4x - c + 2y) + 3b(4x - c + 2y)$
$= (4x - c + 2y)(a + 3b).$

42. $ax - 6cy - 2bx + 4by + 3cx - 2ay$
$= ax - 2bx + 3cx - 2ay + 4by - 6cy$
$= x(a - 2b + 3c) - 2y(a - 2b + 3c)$
$= (a - 2b + 3c)(x - 2y).$

43. $-2xz + 6ay + cz + 4xy - 3az - 2cy$
$= 6ay + 4xy - 2cy - 3az - 2xz + cz$
$= 2y(3a + 2x - c) - z(3a + 2x - c)$
$= (3a + 2x - c)(2y - z).$

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1. $c^2 + 2cd + d^2$.
2. $a^2 + 4a + 4$.
3. $4a^2 + 4a + 1$.
4. $x^6 + 6x^3 + 9$.
5. $4x^2 + 20x + 25$.
6. $9x^6 + 12x^3y + 4y^2$.
7. $100x^2 + 120xy + 36y^2$.
8. $49 + 112y^4 + 64y^8$.
9. $x^2 + 2x + 1$.
10. $y^2 - 6y + 9$.
11. $4y^2 - 40y + 100$.
12. $9y^2 + 36y + 36$.
13. $16z^4 - 16xz^2 + 4x^2$.
14. $121y^6 - 88y^3z + 16z^2$.
15. $100a^8 + 240a^4x^5 + 144x^{10}$.
16. $y^4 - 6y^2 + 9$.
17. $y^8 - 10y^4 + 25$.
18. $16k^4 + 16k^2 + 4$.
19. $25k^4 - 80k^2h^2 + 64h^4$.
20. $16a^{16} + 104a^8k + 169k^2$.

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1. $c^2 + 2cd + d^2 = (c + d)^2$.
2. $x^2 + 2xy + y^2 = (x + y)^2$.
3. $x^2 + 2x + 1 = (x + 1)^2$.
4. $a^2 - 10a + 25 = (a - 5)^2$.
5. $a^2 + 8a + 16 = (a + 4)^2$.
6. $4h^2 + 4h + 1 = (2h + 1)^2$.
7. $9 - 6x + x^2 = (3 - x)^2$.
8. $16k^2 - 8k + 1 = (4k - 1)^2$.
9. $4h^2 - 12h + 9 = (2h - 3)^2$.
10. $x^4 + 1 + 2x^2 = (x^2 + 1)^2$.
11. $x^6 - 6x^3 + 9 = (x^3 - 3)^2$.
12. $25h^2 + 4k^2 - 20hk = (5h - 2k)^2$.
13. $9h^2 - 24hk + 16k^2 = (3h - 4k)^2$.
14. $x^2 - 24xz^2 + 144z^4 = (x - 12z^2)^2$.
15. $12x^4z^3 + 9z^6 + 4x^8 = (3z^3 + 2x^4)^2$.
16. $c^6d^{10} + 2c^3d^5 + 1 = (c^3d^5 + 1)^2$.
17. $24x^2y^2 + 16y^4 + 9x^4 = (4y^2 + 3x^2)^2$.
18. $x^4 + 4y^4 + 4x^2y^2 = (x^2 + 2y^2)^2$.
19. $4x^2y^8 - 4xy^4 + 1 = (2xy^4 - 1)^2$.
20. $25h^6k^2 - 30h^3kz + 9z^2 = (5h^3k - 3z)^2$.
21. $121c^6d^4 - 220c^3d^2g^2 + 100g^4 = (11c^3d^2 - 10g^2)^2$.
22. $169a^4 - 156a^2x^4y^3 + 36x^8y^6 = (13a^2 - 6x^4y^3)^2$.

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1. $x^3 + 2x^2 + x = x(x^2 + 2x + 1) = x(x + 1)(x + 1)$.
2. $x^5 - 4x^4 + 4x^3 = x^3(x^2 - 4x + 4) = x^3(x - 2)(x - 2)$.
3. $2a^4 + 12a^2 + 18 = 2(a^4 + 6a^2 + 9) = 2(a^2 + 3)(a^2 + 3)$.
4. $3a^2 + 18a + 27 = 3(a^2 + 6a + 9) = 3(a + 3)(a + 3)$.
5. $x^6 - 20x^4 + 100x^2 = x^2(x^4 - 20x^2 + 100) = x^2(x^2 - 10)(x^2 - 10)$.
6. $5cx^6 - 70cx^3 + 245c = 5c(x^6 - 14x^3 + 49) = 5c(x^3 - 7)(x^3 - 7)$.
7. $16x^8 - 24x^6 + 9x^4 = x^4(16x^4 - 24x^2 + 9) = x^4(4x^2 - 3)(4x^2 - 3)$.
8. $28x^2 - 28x^3 + 7x^4 = 7x^2(4 - 4x + x^2) = 7x^2(2 - x)(2 - x)$.
9. $3ax + 3ay + 3bx + 3by = 3[a(x + y) + b(x + y)] = 3(x + y)(a + b)$.

10. $4cx - 4cy + 4dx - 4dy = 4[c(x-y) + d(x-y)] = 4(x-y)(c+d)$.

11. $ax - a + x - 1 = a(x-1) + (x-1) = (x-1)(a+1)$.

12. $63xy - 84y^2 + 98yz = 7y(9x - 12y + 14z)$.

13. $30ax - 34bx - 15a + 17b = 2x(15a - 17b) - (15a - 17b)$
$= (15a - 17b)(2x - 1)$.

14. $ax - bx + cx + ay - by + cy = x(a - b + c) + y(a - b + c)$
$= (a - b + c)(x + y)$.

15. $14anx - 21bny - 7n = 7n(2ax - 3by - 1)$.

16. $3ax - 5by - 5ay + 3bx = 3x(a+b) - 5y(a+b)$
$= (a+b)(3x - 5y)$.

17. $56a^2 - 40ab + 63ac - 45bc = 8a(7a - 5b) + 9c(7a - 5b)$
$= (7a - 5b)(8a + 9c)$.

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1. $x^2 + 5x + 6 = (x+2)(x+3)$.
2. $x^2 + 9x + 20 = (x+4)(x+5)$.
3. $x^2 - 7x + 12 = (x-3)(x-4)$.
4. $x^2 - 9x + 14 = (x-2)(x-7)$.
5. $x^2 + 19x + 18 = (x+1)(x+18)$.
6. $x^2 - 18x + 17 = (x-1)(x-17)$.
7. $x^2 - x - 6 = (x+2)(x-3)$.
8. $x^2 - 3x - 40 = (x-8)(x+5)$.
9. $x^2 + x - 30 = (x+6)(x-5)$.
10. $x^2 - 2x - 35 = (x-7)(x+5)$.
11. $x^2 + 11x - 42 = (x+14)(x-3)$.
12. $6 - 5x - x^2 = (6+x)(1-x)$.
13. $14 - 5x - x^2 = -1(x^2 + 5x - 14) = -(x+7)(x-2) = (x+7)(2-x)$.
14. $21 + 4x - x^2 = -1(x^2 - 4x - 21) = -1(x-7)(x+3) = (7-x)(x+3)$.
15. $24 + 10a - a^2 = -1(a^2 - 10a - 24) = -1(a-12)(a+2) = (12-a)(a+2)$.
16. $10 + 3x - x^2 = -1(x^2 - 3x - 10) = -1(x-5)(x+2) = (5-x)(x+2)$.
17. $72 + c - c^2 = -1(c^2 - c - 72) = -1(c-9)(c+8) = (9-c)(c+8)$.
18. $x^2 + x - 90 = (x+10)(x-9)$.
19. $28 + x^2 - 11x = (x-4)(x-7)$.
20. $27 - 12x + x^2 = (-9+x)(-3+x) = (x-9)(x-3)$.
21. $8x + x^2 - 48 = (x+12)(x-4)$.
22. $x^2 - x - 90 = (x-10)(x+9)$.
23. $x^2 - 5ax + 6a^2 = (x-2a)(x-3a)$.
24. $x^2 + 7ax + 12a^2 = (x+3a)(x+4a)$.
25. $a^2x^2 - 2ax - 15 = (ax-5)(ax+3)$.
26. $h^2k^2 - 9hk + 20 = (hk-4)(hk-5)$.
27. $x^4 - 5x^2 - 14 = (x^2-7)(x^2+2)$.
28. $x^2 + x^4 - 110 = (x^2+11)(x^2-10)$.
29. $x^6 - 11x^3 + 18 = (x^3-9)(x^3-2)$.
30. $b^2x^2 - 3bxy - 28y^2 = (bx-7y)(bx+4y)$.
31. $h^2k^2 - 5hkx - 36x^2 = (hk-9x)(hk+4x)$.
32. $a^6b^2 - 5a^3bx - 24x^2 = (a^3b-8x)(a^3b+3x)$.
33. $h^4d^2 + 10h^2dg^3 - 24g^6 = (h^2d+12g^3)(h^2d-2g^3)$.
34. $9k^2x^4y^6 + k^4x^8 - 22y^{12} = (k^2x^4+11y^6)(k^2x^4-2y^6)$.

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1. $x^4 + 6x^3 + 9x^2 = x^2(x^2 + 6x + 9) = x^2(x + 3)(x + 3).$
2. $x^3 - 5x^2 + 6x = x(x^2 - 5x + 6) = x(x - 3)(x - 2).$
3. $3b^2 - 9b - 30 = 3(b^2 - 3b - 10) = 3(b - 5)(b + 2).$
4. $3ax^3 - 3ax^2 - 6ax = 3ax(x^2 - x - 2) = 3ax(x - 2)(x + 1).$
5. $4x^5 - 4x^3 - 360x = 4x(x^4 - x^2 - 90) = 4x(x^2 - 10)(x^2 + 9).$
6. $3x^4 - 21x^3 - 54x^2 = 3x^2(x^2 - 7x - 18) = 3x^2(x - 9)(x + 2).$
7. $4a^6y^4 - 64a^4y^2 + 256a^2 = 4a^2(a^4y^4 - 16a^2y^2 + 64)$
   $= 4a^2(a^2y^2 - 8)(a^2y^2 - 8).$
8. $3x - 3ax - 18a^2x = -3x(6a^2 + a - 1)$
   $= -3x(2a + 1)(3a - 1)$
   $= 3x(1 + 2a)(1 - 3a).$
9. $56x + x^2 - x^3 = -x(x^2 - x - 56)$
   $= -x(x - 8)(x + 7)$
   $= x(8 - x)(x + 7).$
10. $140h^2 + 23h^3 - h^4 = -h^2(h^2 - 23h - 140)$
    $= -h^2(h - 28)(h + 5)$
    $= h^2(28 - h)(h + 5).$
11. $3x^3 - 3x^4 + 60x^2 = -3x^2(x^2 - x - 20)$
    $= -3x^2(x - 5)(x + 4)$
    $= 3x^2(5 - x)(x + 4).$
12. $32a^2x^2 + 2a^2 - 16a^2x = 2a^2(16x^2 - 8x + 1) = 2a^2(4x - 1)^2.$
13. $4ay^2x^2 - 40ay^2x + 100ay^2 = 4ay^2(x^2 - 10x + 25)$
    $= 4ay^2(x - 5)(x - 5).$
14. $3x^4 - 3x^3 + 3x^2 - 3x = 3x(x^3 - x^2 + x - 1)$
    $= 3x[x^2(x - 1) + (x - 1)]$
    $= 3x(x - 1)(x^2 + 1).$
15. $2ax^3 - 4ax^2 - 12ax + 24a = 2a(x^3 - 2x^2 - 6x + 12)$
    $= 2a[x^2(x - 2) - 6(x - 2)]$
    $= 2a(x - 2)(x^2 - 6).$
16. $5xyz - 15yz + 10xz - 30z = 5z(xy - 3y + 2x - 6)$
    $= 5z[y(x - 3) + 2(x - 3)]$
    $= 5z(x - 3)(y + 2).$

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1. $2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$
   $= 2x(x + 1) + 3(x + 1)$
   $= (2x + 3)(x + 1).$
2. $3x^2 + 5x + 2 = 3x^2 + 3x + 2x + 2$
   $= 3x(x + 1) + 2(x + 1)$
   $= (3x + 2)(x + 1).$

3. $2x^2 - 7x + 3 = 2x^2 - 6x - x + 3$
$= 2x(x - 3) - (x - 3)$
$= (2x - 1)(x - 3).$

4. $x^2 + 6x + 9 = x^2 + 3x + 3x + 9$
$= x(x + 3) + 3(x + 3)$
$= (x + 3)(x + 3).$

5. $3x^2 + 8x + 4 = 3x^2 + 6x + 2x + 4$
$= 3x(x + 2) + 2(x + 2)$
$= (3x + 2)(x + 2).$

6. $5x^2 - 8x + 3 = 5x^2 - 5x - 3x + 3$
$= 5x(x - 1) - 3(x - 1)$
$= (5x - 3)(x - 1).$

7. $9x^2 + 18x + 8 = 9x^2 + 12x + 6x + 8$
$= 3x(3x + 4) + 2(3x + 4)$
$= (3x + 2)(3x + 4).$

8. $7x^2 + 9ax + 2a^2 = 7x^2 + 7ax + 2ax + 2a^2$
$= 7x(x + a) + 2a(x + a)$
$= (7x + 2a)(x + a).$

9. $4x^2 + 16x + 15 = 4x^2 + 6x + 10x + 15$
$= 2x(2x + 3) + 5(2x + 3)$
$= (2x + 5)(2x + 3).$

10. $2x^2 - 3x - 2 = 2x^2 + x - 4x - 2$
$= x(2x + 1) - 2(2x + 1)$
$= (x - 2)(2x + 1).$

11. $2x^2 - x - 3 = 2x^2 + 2x - 3x - 3$
$= 2x(x + 1) - 3(x + 1)$
$= (2x - 3)(x + 1).$

12. $4x^2 - 7x - 15 = 4x^2 - 12x + 5x - 15$
$= 4x(x - 3) + 5(x - 3)$
$= (x - 3)(4x + 5).$

13. $3x^4 + 7x^2 - 6 = 3x^4 - 2x^2 + 9x^2 - 6$
$= x^2(3x^2 - 2) + 3(3x^2 - 2)$
$= (x^2 + 3)(3x^2 - 2).$

14. $3x^2 + 5xy - 2y^2 = 3x^2 + 6xy - xy - 2y^2$
$= 3x(x + 2y) - y(x + 2y)$
$= (3x - y)(x + 2y).$

15. $3x^2 - x - 2 = 3x^2 - 3x + 2x - 2$
$= 3x(x - 1) + 2(x - 1)$
$= (3x + 2)(x - 1).$

16. $2x^2 - 5x - 3 = 2x^2 - 6x + x - 3$
$= 2x(x - 3) + (x - 3)$
$= (2x + 1)(x - 3).$

17. $2x^2 + xz - 3z^2 = 2x^2 + 3xz - 2xz - 3z^2$
$= x(2x + 3z) - z(2x + 3z)$
$= (x - z)(2x + 3z).$

18. $6x^2 - x - 2 = 6x^2 - 4x + 3x - 2$
$= 2x(3x - 2) + (3x - 2)$
$= (2x + 1)(3x - 2).$

19. $25x^2 - 20x + 3 = 25x^2 - 15x - 5x + 3$
$= 5x(5x - 3) - (5x - 3)$
$= (5x - 3)(5x - 1).$

20. $49x^2 - 21x + 2 = 49x^2 - 7x - 14x + 2$
$= 7x(7x - 1) - 2(7x - 1)$
$= (7x - 2)(7x - 1).$

21. $36x^2 - 36x + 5 = 36x^2 - 30x - 6x + 5$
$= 6x(6x - 5) - (6x - 5)$
$= (6x - 5)(6x - 1).$

22. $6x^4 + 13x^2 + 6 = 6x^4 + 9x^2 + 4x^2 + 6$
$= 3x^2(2x^2 + 3) + 2(2x^2 + 3)$
$= (3x^2 + 2)(2x^2 + 3).$

23. $9x^6 - 6x^3 - 8 = 9x^6 - 12x^3 + 6x^3 - 8$
$= 3x^3(3x^3 - 4) + 2(3x^3 - 4)$
$= (3x^3 + 2)(3x^3 - 4).$

24. $5x^2 + 2x - 16 = 5x^2 + 10x - 8x - 16$
$= 5x(x + 2) - 8(x + 2)$
$= (5x - 8)(x + 2).$

25. $3x^2 - 11x - 20 = 3x^2 - 15x + 4x - 20$
$= 3x(x - 5) + 4(x - 5)$
$= (3x + 4)(x - 5).$

26. $5x^4 + 18x^2y + 16y^2 = 5x^4 + 10x^2y + 8x^2y + 16y^2$
$= 5x^2(x^2 + 2y) + 8y(x^2 + 2y)$
$= (5x^2 + 8y)(x^2 + 2y).$

27. $6x^2 + 23x - 55 = 6x^2 - 10x + 33x - 55$
$= 2x(3x - 5) + 11(3x - 5)$
$= (2x + 11)(3x - 5).$

28. $6x^6 + 23x^3 + 21 = 6x^6 + 9x^3 + 14x^3 + 21$
$= 3x^3(2x^3 + 3) + 7(2x^3 + 3)$
$= (3x^3 + 7)(2x^3 + 3).$

29. $10x^2 - 7x - 12 = 10x^2 - 15x + 8x - 12$
$= 5x(2x - 3) + 4(2x - 3)$
$= (5x + 4)(2x - 3).$

30. $4c^2 + 11cg - 3g^2 = 4c^2 + 12cg - cg - 3g^2$
$= 4c(c + 3g) - g(c + 3g)$
$= (4c - g)(c + 3g).$

31. $8x^2 + 26xy + 15y^2 = 8x^2 + 6xy + 20xy + 15y^2$
$= 2x(4x + 3y) + 5y(4x + 3y)$
$= (2x + 5y)(4x + 3y).$

32. $35x^2 + 22x + 3 = 35x^2 + 15x + 7x + 3$
$= 5x(7x + 3) + (7x + 3)$
$= (7x + 3)(5x + 1).$

33. $14x^2 - x - 3 = 14x^2 - 7x + 6x - 3$
$= 7x(2x - 1) + 3(2x - 1)$
$= (7x + 3)(2x - 1).$

34. $21x^4 - x^2 - 2 = 21x^4 - 7x^2 + 6x^2 - 2$
$= 7x^2(3x^2 - 1) + 2(3x^2 - 1)$
$= (7x^2 + 2)(3x^2 - 1).$

35. $22x^2 - 3x - 7 = 22x^2 + 11x - 14x - 7$
$= 11x(2x + 1) - 7(2x + 1)$
$= (11x - 7)(2x + 1).$

36. $18x^2 + 65x + 7 = 18x^2 + 2x + 63x + 7$
$= 2x(9x + 1) + 7(9x + 1)$
$= (2x + 7)(9x + 1).$

37. $26x^2 + 9x - 2 = 26x^2 + 13x - 4x - 2$
$= 13x(2x + 1) - 2(2x + 1)$
$= (13x - 2)(2x + 1).$

38. $14x^2 - 39x + 10 = 14x^2 - 35x - 4x + 10$
$= 7x(2x - 5) - 2(2x - 5)$
$= (7x - 2)(2x - 5).$

39. $35x^2 - 39x - 36 = 35x^2 - 60x + 21x - 36$
$= 5x(7x - 12) + 3(7x - 12)$
$= (5x + 3)(7x - 12).$

40. $42x^2 - 9x - 6 = 3(14x^2 - 3x - 2)$
$= 3(14x^2 - 7x + 4x - 2)$
$= 3[7x(2x - 1) + 2(2x - 1)]$
$= 3(2x - 1)(7x + 2).$

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1. $4ax^2 + 8ax + 3a = a(4x^2 + 8x + 3)$
$= a(4x^2 + 6x + 2x + 3)$
$= a[2x(2x + 3) + (2x + 3)]$
$= a(2x + 1)(2x + 3).$

2. $2c^2x^2 + c^2x - 3c^2 = c^2(2x^2 + x - 3)$
$= c^2(2x^2 + 3x - 2x - 3)$
$= c^2[x(2x + 3) - (2x + 3)]$
$= c^2(x - 1)(2x + 3).$

3. $48\,ax^4 - 120\,ax^2 + 75\,a = 3\,a\,(16\,x^4 - 40\,x^2 + 25)$
$= 3\,a\,(4\,x^2 - 5)\,(4\,x^2 - 5).$
4. $18\,c^2d - 30\,cd - 28\,d = 2\,d(9\,c^2 - 15\,c - 14)$
$= 2\,d\,(9\,c^2 - 21\,c + 6\,c - 14)$
$= 2\,d\,[3\,c\,(3\,c - 7) + 2\,(3\,c - 7)]$
$= 2\,d\,(3\,c + 2)\,(3\,c - 7).$
5. $x^4 + x^3 - 110\,x^2 = x^2\,(x^2 + x - 110) = x^2\,(x + 11)\,(x - 10).$
6. $75\,x^3 + 60\,x^2 + 12\,x = 3\,x\,(25\,x^2 + 20\,x + 4) = 3\,x\,(5\,x + 2)\,(5\,x + 2).$
7. $4\,x^2y^2 - 4\,xy^2 - 120\,y^2 = 4\,y^2\,(x^2 - x - 30) = 4\,y^2\,(x - 6)\,(x + 5).$
8. $70\,x^3 - 85\,x^2y - 30\,xy^2 = 5\,x\,(14\,x^2 - 17\,xy - 6\,y^2)$
$= 5\,x\,(14\,x^2 - 21\,xy + 4\,xy - 6\,y^2)$
$= 5\,x\,[7\,x\,(2\,x - 3\,y) + 2\,y\,(2\,x - 3\,y)]$
$= 5\,x\,(7\,x + 2\,y)\,(2\,x - 3\,y).$
9. $18\,x^3 - 39\,x^2 + 18\,x = 3\,x\,(6\,x^2 - 13\,x + 6)$
$= 3\,x\,(6\,x^2 - 9\,x - 4\,x + 6)$
$= 3\,x\,[3\,x\,(2\,x - 3) - 2\,(2\,x - 3)]$
$= 3\,x\,(3\,x - 2)\,(2\,x - 3).$
10. $36\,x^4 - 6\,x^3 - 12\,x^2 = 6\,x^2\,(6\,x^2 - x - 2)$
$= 6\,x^2\,(6\,x^2 - 4\,x + 3\,x - 2)$
$= 6\,x^2\,[2\,x\,(3\,x - 2) + (3\,x - 2)]$
$= 6\,x^2\,(3\,x - 2)\,(2\,x + 1).$
11. $4\,x^4 - 20\,x^3 - 24\,x^2 = 4\,x^2(x^2 - 5\,x - 6) = 4\,x^2\,(x - 6)\,(x + 1).$
12. $5\,ax^2 + 5\,ax + 5\,a = 5\,a\,(x^2 + x + 1).$

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1. $k^2 - h^2 = (k + h)\,(k - h).$
2. $h^2 - 1 = (h + 1)\,(h - 1).$
3. $k^2 - 4 = (k + 2)\,(k - 2).$
4. $a^4 - x^2 = (a^2 + x)\,(a^2 - x).$
5. $1 - a^2 = (1 + a)\,(1 - a).$
6. $1 - 4\,a^2 = (1 + 2\,a)\,(1 - 2\,a).$
7. $4\,a^2 - 9 = (2\,a + 3)\,(2\,a - 3).$
8. $25 - x^2 = (5 - x)\,(5 + x).$
9. $h^4 - 81 = (h^2 + 9)\,(h^2 - 9)$
$= (h^2 + 9)(h + 3)(h - 3).$
10. $16\,x^4 - 25 = (4\,x^2 + 5)\,(4\,x^2 - 5).$
11. $36\,c^8 - 49\,d^8 = (6\,c^4 + 7\,d^4)\,(6\,c^4 - 7\,d^4).$
12. $a^2b^4 - 121 = (ab^2 + 11)\,(ab^2 - 11).$
13. $100 - z^8 = (10 + z^4)\,(10 - z^4).$
14. $x^4 - y^4 = (x^2 + y^2)\,(x^2 - y^2) = (x^2 + y^2)\,(x + y)\,(x - y).$
15. $64\,h^6 - 169 = (8\,h^3 + 13)\,(8\,h^3 - 13).$
16. $a^8 - b^8 = (a^4 + b^4)\,(a^4 - b^4)$
$= (a^4 + b^4)\,(a^2 + b^2)\,(a^2 - b^2)$
$= (a^4 + b^4)\,(a^2 + b^2)\,(a + b)\,(a - b).$
17. $1 - 16\,x^4 = (1 + 4\,x^2)\,(1 - 4\,x^2) = (1 + 4\,x^2)\,(1 + 2\,x)\,(1 - 2\,x).$
18. $16\,x^4 - y^8 = (4\,x^2 + y^4)\,(4\,x^2 - y^4) = (4\,x^2 + y^4)\,(2\,x + y^2)\,(2\,x - y^2).$

19. $$\begin{aligned} x^{16} - y^{16} &= (x^8 + y^8)(x^8 - y^8) \\ &= (x^8 + y^8)(x^4 + y^4)(x^4 - y^4) \\ &= (x^8 + y^8)(x^4 + y^4)(x^2 + y^2)(x^2 - y^2) \\ &= (x^8 + y^8)(x^4 + y^4)(x^2 + y^2)(x + y)(x - y). \end{aligned}$$

20. $x^{16} - 4y^{16} = (x^8 + 2y^8)(x^8 - 2y^8).$

21. $144a^2b^4 - 9 = 9(4ab^2 + 1)(4ab^2 - 1).$

22. $196 - x^4y^6 = (14 + x^2y^3)(14 - x^2y^3).$

23. $225a^4 - 16b^8c^{12} = (15a^2 + 4b^4c^6)(15a^2 - 4b^4c^6).$

24. $x^{2n} - y^2 = (x^n + y)(x^n - y).$

25. $x^2 - y^{4n} = (x + y^{2n})(x - y^{2n}).$

26. $a^{4n} - b^2 = (a^{2n} + b)(a^{2n} - b).$

27. $a^{6n} - b^{2n} = (a^{3n} + b^n)(a^{3n} - b^n).$

28. $a^{4n} - b^{6n} = (a^{2n} + b^{3n})(a^{2n} - b^{3n}).$

29. $4x^{2n} - y^{4n} = (2x^n + y^{2n})(2x^n - y^{2n}).$

30. $9x^{2n} - 4y^{6n} = (3x^n + 2y^{3n})(3x^n - 2y^{3n}).$

31. $(a + b)^2 - 4 = (a + b + 2)(a + b - 2).$

32. $(x - y)^2 - 4c^2 = (x - y + 2c)(x - y - 2c).$

33. $(a - 2b)^2 - 9x^2 = (a - 2b + 3x)(a - 2b - 3x).$

34. $4(x + y)^2 - 1 = (2x + 2y + 1)(2x + 2y - 1).$

35. $25(3b - c)^2 - 64 = (15b - 5c + 8)(15b - 5c - 8).$

37. $$\begin{aligned} 81 - (x - y)^2 &= [9 + (x - y)][9 - (x - y)] \\ &= (9 + x - y)(9 - x + y). \end{aligned}$$

38. $$\begin{aligned} 4a^2 - (2x + y)^2 &= [2a + (2x + y)][2a - (2x + y)] \\ &= (2a + 2x + y)(2a - 2x - y). \end{aligned}$$

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1. $(a - b)^2 - 9x^2 = (a - b + 3x)(a - b - 3x).$

2. $$\begin{aligned} 16y^2 - (h + 2k)^2 &= [4y + (h + 2k)][4y - (h + 2k)] \\ &= (4y + h + 2k)(4y - h - 2k). \end{aligned}$$

3. $$\begin{aligned} 25a^4 - (2b - 3c)^2 &= [5a^2 + (2b - 3c)][5a^2 - (2b - 3c)] \\ &= (5a^2 + 2b - 3c)(5a^2 - 2b + 3c). \end{aligned}$$

4. $$\begin{aligned} 49x^6 - (7c - 2d)^2 &= [7x^3 + (7c - 2d)][7x^3 - (7c - 2d)] \\ &= (7x^3 + 7c - 2d)(7x^3 - 7c + 2d). \end{aligned}$$

5. $$\begin{aligned} 1 - (5h - 3k)^2 &= [1 + (5h - 3k)][1 - (5h - 3k)] \\ &= (1 + 5h - 3k)(1 - 5h + 3k). \end{aligned}$$

7. $$\begin{aligned} (a - b)^2 - (a + b)^2 &= [(a - b) + (a + b)][(a - b) - (a + b)] \\ &= (a - b + a + b)(a - b - a - b) \\ &= 2a(-2b) \\ &= -4ab. \end{aligned}$$

8. $$\begin{aligned} (a - b)^2 - (x + y)^2 &= [(a - b) + (x + y)][(a - b) - (x + y)] \\ &= (a - b + x + y)(a - b - x - y). \end{aligned}$$

9. $(a+2b)^2-(a-3b)^2=[(a+2b)+(a-3b)][(a+2b)-(a-3b)]$
$=(a+2b+a-3b)(a+2b-a+3b)$
$=(2a-b)5b.$

10. $(2a-3b)^2-(3a-2b)^2$
$=[(2a-3b)+(3a-2b)][(2a-3b)-(3a-2b)]$
$=(2a-3b+3a-2b)(2a-3b-3a+2b)$
$=(5a-5b)(-a-b)$
$=5(b-a)(a+b).$

11. $(5h+3k)^2-(2c-9d)^2$
$=[(5h+3k)+(2c-9d)][(5h+3k)-(2c-9d)]$
$=(5h+3k+2c-9d)(5h+3k-2c+9d).$

12. $(4b-a)^2-(7a-6b)^2$
$=[(4b-a)+(7a-6b)][(4b-a)-(7a-6b)]$
$=(4b-a+7a-6b)(4b-a-7a+6b)$
$=(6a-2b)(10b-8a)$
$=4(3a-b)(5b-4a).$

13. $4(a-2b)^2-(2x-y)^2$
$=[2(a-2b)+(2x-y)][2(a-2b)-(2x-y)]$
$=(2a-4b+2x-y)(2a-4b-2x+y).$

14. $9(a+b)^2-(2a-5b)^2$
$=[3(a+b)+(2a-5b)][3(a+b)-(2a-5b)]$
$=(3a+3b+2a-5b)(3a+3b-2a+5b)$
$=(5a-2b)(a+8b).$

15. $(a-2b)^2-4(a+b)^2$
$=[(a-2b)+2(a+b)][(a-2b)-2(a+b)]$
$=(a-2b+2a+2b)(a-2b-2a-2b)$
$=3a(-a-4b)$
$=-3a(a+4b).$

17. $x^2+2ax+a^2-y^2=(x+a)^2-y^2=(x+a-y)(x+a+y).$

18. $x^2+2x+1-4z^2=(x+1)^2-4z^2=(x+1+2z)(x+1-2z).$

19. $c^2-2cd+d^2-16a^2=(c-d)^2-16a^2=(c-d+4a)(c-d-4a).$

20. $4x^2-12cx+9c^2-25y^2=(2x-3c)^2-25y^2$
$=(2x-3c+5y)(2x-3c-5y).$

21. $x^2-y^2-2ax+a^2=x^2-2ax+a^2-y^2$
$=(x-a)^2-y^2$
$=(x-a+y)(x-a-y).$

22. $k^2-g^4-4kh+4h^2=k^2-4kh+4h^2-g^4$
$=(k-2h)^2-g^4$
$=(k-2h+g^2)(k-2h-g^2).$

23. $6xy-c^2+9x^2+y^2=9x^2+6xy+y^2-c^2$
$=(3x+y)^2-c^2$
$=(3x+y-c)(3x+y+c).$

24. $12ab - 4h^2 + 4b^2 + 9a^2 = 9a^2 + 12ab + 4b^2 - 4h^2$
$= (3a + 2b)^2 - 4h^2$
$= (3a + 2b - 2h)(3a + 2b + 2h).$

25. $1 - 4ax + 4a^2x^2 - x^2 = (1 - 2ax)^2 - x^2$
$= (1 - 2ax + x)(1 - 2ax - x).$

26. $4c^2 - 20cd + 25d^2 - 9d^4 = (2c - 5d)^2 - (3d^2)^2$
$= (2c - 5d + 3d^2)(2c - 5d - 3d^2).$

27. $9d^2 - 25a^2 - 6cd + c^2 = 9d^2 - 6cd + c^2 - 25a^2$
$= (3d - c)^2 - 25a^2$
$= (3d - c + 5a)(3d - c - 5a).$

29. $a^2 - b^2 - 2bc - c^2 = a^2 - (b^2 + 2bc + c^2)$
$= a^2 - (b + c)^2$
$= (a + b + c)(a - b - c).$

30. $x^2 - a^2 - 2ac - c^2 = x^2 - (a^2 + 2ac + c^2)$
$= x^2 - (a + c)^2$
$= (x + a + c)(x - a - c).$

31. $y^2 - b^2 + 4bc - 4c^2 = y^2 - (b^2 - 4bc + 4c^2)$
$= y^2 - (b - 2c)^2$
$= (y - b + 2c)(y + b - 2c).$

32. $2bc - c^2 - b^2 + a^4 = a^4 - (b^2 - 2bc + c^2)$
$= a^4 - (b - c)^2$
$= (a^2 - b + c)(a^2 + b - c).$

33. $9x^2 - 4y^2 - a^2 - 4ay = 9x^2 - (4y^2 + 4ay + a^2)$
$= 9x^2 - (2y + a)^2$
$= (3x - 2y - a)(3x + 2y + a).$

34. $6x + 9y^2 - 9 - x^2 = 9y^2 - (x^2 - 6x + 9)$
$= 9y^2 - (x - 3)^2$
$= (3y - x + 3)(3y + x - 3).$

35. $4bc + 1 - 4c^2 - b^2 = 1 - (b^2 - 4bc + 4c^2)$
$= 1 - (b - 2c)^2$
$= (1 - b + 2c)(1 + b - 2c).$

36. $4bc - 4b^2 + 4x^6 - c^2 = 4x^6 - (4b^2 - 4bc + c^2)$
$= 4x^6 - (2b - c)^2$
$= (2x^3 - 2b + c)(2x^3 + 2b - c).$

38. $a^2 + 2ab + b^2 - c^2 - 2cd - d^2$
$= (a^2 + 2ab + b^2) - (c^2 + 2cd + d^2)$
$= (a + b)^2 - (c + d)^2$
$= (a + b + c + d)(a + b - c - d).$

39. $9k^2 - 6hk + h^2 - 4c^2 - 4cd - d^2$
$= (9k^2 - 6hk + h^2) - (4c^2 + 4cd + d^2)$
$= (3k - h)^2 - (2c + d)^2$
$= (3k - h - 2c - d)(3k - h + 2c + d).$

40. $x^2 - 1 + y^2 - a^2 + 2xy + 2a = (x^2 + 2xy + y^2) - (a^2 - 2a + 1)$
$= (x + y)^2 - (a - 1)^2$
$= (x + y - a + 1)(x + y + a - 1).$

41. $1 + 2bc + 2a - c^2 - b^2 + a^2 = (a^2 + 2a + 1) - (b^2 - 2bc + c^2)$
$= (a + 1)^2 - (b - c)^2$
$= (a + 1 + b - c)(a + 1 - b + c).$

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1. $x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1).$

2. $x^4 - 2x^2 + 1 = (x^2 - 1)^2 = (x + 1)(x + 1)(x - 1)(x - 1).$

3. $x^8 - 2x^4 + 1 = (x^4 - 1)^2$
$= (x^2 + 1)(x^2 - 1)(x^2 + 1)(x^2 - 1)$
$= (x^2 + 1)(x^2 + 1)(x + 1)(x + 1)(x - 1)(x - 1).$

4. $x^5 - 8x^3 + 16x = x(x^4 - 8x^2 + 16)$
$= x(x^2 - 4)(x^2 - 4)$
$= x(x + 2)(x - 2)(x + 2)(x - 2).$

5. $x^5 - x = x(x^4 - 1)$
$= x(x^2 + 1)(x^2 - 1)$
$= x(x^2 + 1)(x + 1)(x - 1).$

6. $x^{10} - x^2 = x^2(x^8 - 1)$
$= x^2(x^4 + 1)(x^4 - 1)$
$= x^2(x^4 + 1)(x^2 + 1)(x^2 - 1)$
$= x^2(x^4 + 1)(x^2 + 1)(x + 1)(x - 1).$

7. $x^4 - 10x^2 + 9 = (x^2 - 1)(x^2 - 9) = (x + 1)(x - 1)(x + 3)(x - 3).$

8. $x^4 - 13x^2 + 36 = (x^2 - 9)(x^2 - 4) = (x + 3)(x - 3)(x + 2)(x - 2).$

9. $3a^2x^4 - 12a^4x^2 + 12a^6 = 3a^2(x^4 - 4a^2x^2 + 4a^4)$
$= 3a^2(x^2 - 2a^2)(x^2 - 2a^2).$

10. $18a^2x^2 - 24a^2x - 10a^2 = 2a^2(9x^2 - 12x - 5)$
$= 2a^2(9x^2 - 15x + 3x - 5)$
$= 2a^2[3x(3x - 5) + (3x - 5)]$
$= 2a^2(3x + 1)(3x - 5).$

11. $3x^4 - 15x^2 + 12 = 3(x^4 - 5x^2 + 4)$
$= 3(x^2 - 1)(x^2 - 4)$
$= 3(x + 1)(x - 1)(x + 2)(x - 2).$

12. $12a - 39ay - 51ay^2 = 3a(4 - 13y - 17y^2)$
$= 3a(4 - 17y + 4y - 17y^2)$
$= 3a[(4 - 17y) + y(4 - 17y)]$
$= 3a(4 - 17y)(1 + y).$

13. $4n^6 + 48n^2 - 28n^4 = 4n^2(n^4 - 7n^2 + 12)$
$= 4n^2(n^2 - 3)(n^2 - 4)$
$= 4n^2(n^2 - 3)(n + 2)(n - 2).$

14. $16x^4 + 8x^2 - 3 = 16x^4 + 12x^2 - 4x^2 - 3$
$= 4x^2(4x^2 + 3) - (4x^2 + 3)$
$= (4x^2 - 1)(4x^2 + 3)$
$= (2x + 1)(2x - 1)(4x^2 + 3).$

15. $a^3 - a + a^2b - b = a(a^2 - 1) + b(a^2 - 1)$
$= (a + b)(a^2 - 1)$
$= (a + b)(a + 1)(a - 1).$

16. $x^3 - x^2 - 4x + 4 = x^2(x - 1) - 4(x - 1)$
$= (x - 1)(x^2 - 4)$
$= (x - 1)(x + 2)(x - 2).$

17. $3a^3 + 3a^2 - 27a - 27 = 3(a^3 + a^2 - 9a - 9)$
$= 3[a^2(a + 1) - 9(a + 1)]$
$= 3(a + 1)(a^2 - 9)$
$= 3(a + 1)(a + 3)(a - 3).$

18. $2a^3b + 3a^2b - 8ab - 12b = b(2a^3 + 3a^2 - 8a - 12)$
$= b[a^2(2a + 3) - 4(2a + 3)]$
$= b(2a + 3)(a^2 - 4)$
$= b(2a + 3)(a + 2)(a - 2).$

19. $4a^2 - a^4 + 81 + 10a^2x - 36a - 25x^2$
$= (4a^2 - 36a + 81) - (a^4 - 10a^2x + 25x^2)$
$= (2a - 9)^2 - (a^2 - 5x^2)^2$
$= (2a - 9 - a^2 + 5x)(2a - 9 + a^2 - 5x).$

20. $12cd^3 - 6a^3x - a^6 + 4c^2 + 9d^6 - 9x^2$
$= (4c^2 + 12cd^3 + 9d^6) - (a^6 + 6a^3x + 9x^2)$
$= (2c + 3d^3)^2 - (a^3 + 3x)^2$
$= (2c + 3d^3 - a^3 - 3x)(2c + 3d^3 + a^3 + 3x).$

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1. $x^3 + 1 = (x + 1)(x^2 - x + 1).$
2. $x^3 - 1 = (x - 1)(x^2 + x + 1).$
3. $a^3 - 8 = (a - 2)(a^2 + 2a + 4).$
4. $27 + b^3 = (3 + b)(9 - 3b + b^2).$
5. $64x^3 - 1 = (4x - 1)(16x^2 + 4x + 1).$
6. $x^3 - y^6 = (x - y^2)(x^2 + xy^2 + y^4).$
7. $x^3 + 8y^3 = (x + 2y)(x^2 - 2xy + 4y^2).$
8. $x^3 + y^9 = (x + y^3)(x^2 - xy^3 + y^6).$
9. $8x^3 - 27y^3 = (2x - 3y)[(2x)^2 + (2x)(3y) + (3y)^2]$
$= (2x - 3y)(4x^2 + 6xy + 9y^2).$
10. $27a^3 + 64b^3 = (3a + 4b)(9a^2 - 12ab + 16b^2).$
11. $125y^3 + 8x^3 = (5y + 2x)(25y^2 - 10xy + 4x^2).$
12. $216 - 27x^3 = 27(8 - x^3) = 27(2 - x)(4 + 2x + x^2).$
13. $x^6 + y^9 = (x^2 + y^3)(x^4 - x^2y^3 + y^6).$
14. $a^6 + b^6 = (a^2 + b^2)(a^4 - a^2b^2 + b^4).$
15. $a^6 - b^6 = (a^3 - b^3)(a^3 + b^3)$
$= (a - b)(a^2 + ab + b^2)(a + b)(a^2 - ab + b^2).$

16. $x^6 - 64 = (x^3 + 8)(x^3 - 8)$
$= (x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4).$

17. $64x^6 + 1 = (4x^2 + 1)(16x^4 - 4x^2 + 1).$

18. $1 - 64x^6 = (1 - 8x^3)(1 + 8x^3)$
$= (1 - 2x)(1 + 2x + 4x^2)(1 + 2x)(1 - 2x + 4x^2).$

19. $a^6b^6 - 64 = (a^3b^3 - 8)(a^3b^3 + 8)$
$= (ab - 2)(a^2b^2 + 2ab + 4)(ab + 2)(a^2b^2 - 2ab + 4).$

20. $x^{12} + y^{12} = (x^4)^3 + (y^4)^3 = (x^4 + y^4)(x^8 - x^4y^4 + y^8).$

21. $27x^{27} - 1 = (3x^9)^3 - 1^3 = (3x^9 - 1)(9x^{18} + 3x^9 + 1).$

22. $k^{12} - y^6 = (k^6 + y^3)(k^6 - y^3)$
$= [(k^2)^3 + y^3][(k^2)^3 - y^3]$
$= (k^2 + y)(k^4 - k^2y + y^2)(k^2 - y)(k^4 + k^2y + y^2).$

23. $x^{3a} + 1 = (x^{2a} - x^a + 1)(x^a + 1).$

24. $x^{3a} - y^{3a} = (x^a - y^a)(x^{2a} + x^ay^a + y^{2a}).$

25. $x^{3a} + y^{6a} = (x^a + y^{2a})(x^{2a} - x^ay^{2a} + y^{4a}).$

26. $8x^{6a} - y^{9a} = (2x^{2a})^3 - (y^{3a})^3 = (2x^{2a} - y^{3a})(4x^{4a} + 2x^{2a}y^{3a} + y^{6a}).$

27. $27x^{3m} - 64 = (3x^m - 4)(9x^{2m} + 12x^m + 16).$

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1. $x^6 + 2x^3 + 1 = (x^3 + 1)(x^3 + 1)$
$= (x + 1)(x^2 - x + 1)(x + 1)(x^2 - x + 1).$

2. $x^6 - 2x^3y^3 + y^6 = (x^3 - y^3)(x^3 - y^3)$
$= (x - y)(x^2 + xy + y^2)(x - y)(x^2 + xy + y^2).$

3. $x^4 + x = x(x^3 + 1) = x(x + 1)(x^2 - x + 1).$

4. $x^5 - x^2 = x^2(x^3 - 1) = x^2(x - 1)(x^2 + x + 1).$

5. $8x^3 - 64 = 8(x^3 - 8) = 8(x - 2)(x^2 + 2x + 4).$

6. $8x^6 + 216x^3 = 8x^3(x^3 + 27) = 8x^3(x + 3)(x^2 - 3x + 9).$

7. $x^5 - x^3y^2 - x^2y^3 + y^5 = x^3(x^2 - y^2) - y^3(x^2 - y^2)$
$= (x^3 - y^3)(x^2 - y^2)$
$= (x - y)(x^2 + xy + y^2)(x + y)(x - y).$

8. $x^5 + x^3 + 8x^2 + 8 = x^3(x^2 + 1) + 8(x^2 + 1)$
$= (x^3 + 8)(x^2 + 1)$
$= (x + 2)(x^2 - 2x + 4)(x^2 + 1).$

9. $x^7 - x^4 - 16x^3 + 16 = x^4(x^3 - 1) - 16(x^3 - 1)$
$= (x^4 - 16)(x^3 - 1)$
$= (x^2 + 4)(x^2 - 4)(x^3 - 1)$
$= (x^2 + 4)(x + 2)(x - 2)(x - 1)(x^2 + x + 1).$

10. $x^6 - 7x^3 - 8 = (x^3 + 1)(x^3 - 8)$
$= (x + 1)(x^2 - x + 1)(x - 2)(x^2 + 2x + 4).$

11. $8x^6 - 3 - 23x^3 = 8x^6 - 24x^3 + x^3 - 3$
$= (x^3 - 3)(2x + 1)(4x^2 - 2x + 1).$

12. $x^4 + x^3 - x - 1 = x^3(x+1) - (x+1) = (x+1)(x-1)(x^2+x+1).$

13. $x^6 - x^2y^2 = x^2(x^2+y)(x^2-y).$

14.
$$\begin{aligned} x^8 + 64x^2 &= x^2(x^6+64) \\ &= x^2[(x^2)^3+4^3] \\ &= x^2(x^2+4)(x^4-4x^2+16). \end{aligned}$$

16.
$$\begin{aligned} 1 - 3x + 3x^2 - x^3 &= (1-x^3) - (3x-3x^2) \\ &= (1-x)(1+x+x^2) - 3x(1-x) \\ &= (1-x)(1+x+x^2-3x) \\ &= (1-x)(1-2x+x^2) \\ &= (1-x)(1-x)(1-x). \end{aligned}$$

17.
$$\begin{aligned} x^3 + 3x^2y + 3xy^2 + y^3 &= (x^3+y^3) + (3x^2y+3xy^2) \\ &= (x+y)(x^2-xy+y^2) + 3xy(x+y) \\ &= (x+y)(x^2-xy+y^2+3xy) \\ &= (x+y)(x^2+2xy+y^2) \\ &= (x+y)(x+y)(x+y). \end{aligned}$$

18.
$$\begin{aligned} x^3 - 3x^2y + 3xy^2 - y^3 &= (x^3-y^3) - (3x^2y-3xy^2) \\ &= (x-y)(x^2+xy+y^2) - 3xy(x-y) \\ &= (x-y)(x^2+xy+y^2-3xy) \\ &= (x-y)(x^2-2xy+y^2) \\ &= (x-y)(x-y)(x-y). \end{aligned}$$

19.
$$\begin{aligned} x^3 + 2x^2 + 4x + 8 &= x^2(x+2) + 4(x+2) \\ &= (x+2)(x^2+4). \end{aligned}$$

20.
$$\begin{aligned} 8 - 4x + 2x^2 - x^3 &= 4(2-x) + x^2(2-x) \\ &= (2-x)(4+x^2). \end{aligned}$$

21.
$$\begin{aligned} x^3 + 9x^2 + 27x + 27 &= x^3 + 27 + 9x^2 + 27x \\ &= (x+3)(x^2-3x+9) + 9x(x+3) \\ &= (x+3)(x^2-3x+9+9x) \\ &= (x+3)(x+3)(x+3). \end{aligned}$$

22.
$$\begin{aligned} x^3 - 9x^2 + 27x - 27 &= x^3 - 27 - 9x^2 + 27x \\ &= (x-3)(x^2+3x+9) - 9x(x-3) \\ &= (x-3)(x^2+3x+9-9x) \\ &= (x-3)(x-3)(x-3). \end{aligned}$$

23.
$$\begin{aligned} x^3 + 12x^2 + 48x + 64 &= (x^3+64) + (12x^2+48x) \\ &= (x+4)(x^2-4x+16) + 12x(x+4) \\ &= (x+4)(x^2-4x+16+12x) \\ &= (x+4)(x^2+8x+16) \\ &= (x+4)(x+4)(x+4). \end{aligned}$$

24.
$$\begin{aligned} 64 - 48x + 12x^2 - x^3 &= (64-x^3) - (48x-12x^2) \\ &= (4-x)(16+4x+x^2) - 12x(4-x) \\ &= (4-x)(16+4x+x^2-12x) \\ &= (4-x)(16-8x+x^2) \\ &= (4-x)(4-x)(4-x). \end{aligned}$$

25. $8x^3 - 12x^2 + 6x - 1 = (8x^3 - 1) - (12x^2 - 6x)$
$= (2x - 1)(4x^2 + 2x + 1) - 6x(2x - 1)$
$= (2x - 1)^3.$
26. $x^4 + x^3 + x^2 - 1 = x^3(x + 1) + (x - 1)(x + 1)$
$= (x + 1)(x^3 + x - 1).$
27. $x^3 + a^3 + x^2 - a^2 = (x + a)(x^2 - ax + a^2) + (x + a)(x - a)$
$= (x + a)(x^2 - ax + a^2 + x - a).$

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1. $x^4 + 16x^2y^2 + 64y^4 = (x^2 + 8y^2)(x^2 + 8y^2).$
2. $4a^2x^4 + 4a^2x^2 - 120a^2 = 4a^2(x^4 + x^2 - 30) = 4a^2(x^2 + 6)(x^2 - 5).$
3. $9x^2 + 27x + 14 = 9x^2 + 6x + 21x + 14$
$= 3x(3x + 2) + 7(3x + 2)$
$= (3x + 7)(3x + 2).$
4. $3x^4 + x^2 - 2 = 3x^4 + 3x^2 - 2x^2 - 2$
$= 3x^2(x^2 + 1) - 2(x^2 + 1)$
$= (3x^2 - 2)(x^2 + 1).$
5. $3x^3 - 75x = 3x(x^2 - 25) = 3x(x + 5)(x - 5).$
6. $x^2 + 2a^2x^2 + x^2a^4 = x^2(1 + 2a^2 + a^4) = x^2(1 + a^2)(1 + a^2).$
7. $x^3 - 27 = (x - 3)(x^2 + 3x + 9).$
8. $x^2 - xz + 2xy - 2yz = x(x - z) + 2y(x - z)$
$= (x + 2y)(x - z).$
9. $2ac - bc + 6ad - 3bd = c(2a - b) + 3d(2a - b)$
$= (c + 3d)(2a - b).$
10. $25x^2y^2 + 16x^4 - 40x^3y = x^2(25y^2 - 40xy + 16x^2)$
$= x^2(5y - 4x)(5y - 4x).$
11. $x^4 - 15x^3 + 56x^2 = x^2(x^2 - 15x + 56) = x^2(x - 8)(x - 7).$
12. $9x^6 - 6x^3 - 35 = 9x^6 - 21x^3 + 15x^3 - 35$
$= 3x^3(3x^3 - 7) + 5(3x^3 - 7)$
$= (3x^3 + 5)(3x^3 - 7).$
13. $a^6 - 4096 = (a^3 + 64)(a^3 - 64)$
$= (a + 4)(a^2 - 4a + 16)(a - 4)(a^2 + 4a + 16)$
14. $a^{2x+2} - 1 = (a^{x+1} + 1)(a^{x+1} - 1).$
15. $4x^4 - 13x^2y^2 + 9y^4 = 4x^4 - 4x^2y^2 - 9x^2y^2 + 9y^4$
$= 4x^2(x^2 - y^2) - 9y^2(x^2 - y^2)$
$= (4x^2 - 9y^2)(x^2 - y^2)$
$= (2x + 3y)(2x - 3y)(x + y)(x - y).$
16. $216 + x^3 = 6^3 + x^3 = (6 + x)(36 - 6x + x^2).$
17. $x^9 - 512 = (x^3)^3 - 8^3$
$= (x^3 - 8)[(x^3)^2 + (x^3)8 + 8^2]$
$= (x - 2)(x^2 + 2x + 4)(x^6 + 8x^3 + 64).$

18. $$\begin{aligned} x^2 - 4y^2 + 9 - 6x &= x^2 - 6x + 9 - 4y^2 \\ &= (x-3)^2 - 4y^2 \\ &= (x - 3 + 2y)(x - 3 - 2y). \end{aligned}$$

19. $$\begin{aligned} ah - 2ak + 2bh - 4bk &= a(h - 2k) + 2b(h - 2k) \\ &= (h - 2k)(a + 2b). \end{aligned}$$

20. $4x^4 - 25y^8 = (2x^2 + 5y^4)(2x^2 - 5y^4).$

21. $$\begin{aligned} a^2 - 25c^2 + 10ab + 25b^2 &= (a^2 + 10ab + 25b^2) - 25c^2 \\ &= (a + 5b)^2 - 25c^2 \\ &= (a + 5b + 5c)(a + 5b - 5c). \end{aligned}$$

22. $y^6 - 343 = (y^2)^3 - 7^3 = (y^2 - 7)(y^4 + 7y^2 + 49).$

23. $$\begin{aligned} 9x^4 - 9x^2 - 28 &= 9x^4 - 21x^2 + 12x^2 - 28 \\ &= 3x^2(3x^2 - 7) + 4(3x^2 - 7) \\ &= (3x^2 + 4)(3x^2 - 7). \end{aligned}$$

24. $$\begin{aligned} 9x^2y^2 + 39xy^2 - 30y^2 &= 3y^2(3x^2 + 13x - 10) \\ &= 3y^2(3x^2 + 15x - 2x - 10) \\ &= 3y^2[3x(x + 5) - 2(x + 5)] \\ &= 3y^2(3x - 2)(x + 5). \end{aligned}$$

25. $$\begin{aligned} 3x^6y^2 + 63x^3y^2 - 300y^2 &= 3y^2(x^6 + 21x^3 - 100) \\ &= 3y^2(x^3 + 25)(x^3 - 4). \end{aligned}$$

26. $$\begin{aligned} 12 - 15a + 16x - 20ax &= 3(4 - 5a) + 4x(4 - 5a) \\ &= (4 - 5a)(3 + 4x). \end{aligned}$$

27. $$\begin{aligned} 2cx + 3dx - 2cy - 3dy &= x(2c + 3d) - y(2c + 3d) \\ &= (x - y)(2c + 3d). \end{aligned}$$

28. $y^4 - 2y^2z^4 + z^8 = (y^2 - z^4)(y^2 - z^4) = (y + z^2)(y - z^2)(y + z^2)(y - z^2).$

29. $81c^{10} - 64d^{10} = (9c^5 - 8d^5)(9c^5 + 8d^5).$

30. $100x^8 - 220x^4y^2 + 121y^4 = (10x^4 - 11y^2)(10x^4 - 11y^2).$

31. $27y^3 - 512 = (3y)^3 - 8^3 = (3y - 8)(9y^2 + 24y + 64).$

32. $81 - 16x^8 = (9 + 4x^4)(9 - 4x^4) = (9 + 4x^4)(3 + 2x^2)(3 - 2x^2).$

33. $25x^4 - 3025x^8 = 25x^4(1 - 121x^4) = 25x^4(1 + 11x^2)(1 - 11x^2).$

34. $$\begin{aligned} 16c^7 - 4c^5 - 72c^3 &= 4c^3(4c^4 - c^2 - 18) \\ &= 4c^3(4c^4 + 8c^2 - 9c^2 - 18) \\ &= 4c^3[4c^2(c^2 + 2) - 9(c^2 + 2)] \\ &= 4c^3(c^2 + 2)(4c^2 - 9) \\ &= 4c^3(c^2 + 2)(2c + 3)(2c - 3). \end{aligned}$$

35. $c^3 + c^2d + 3cd^2 + 3d^3 = c^2(c + d) + 3d^2(c + d) = (c^2 + 3d^2)(c + d).$

36. $$\begin{aligned} 1 - 18x^4 + 81x^8 &= (1 - 9x^4)(1 - 9x^4) \\ &= (1 + 3x^2)(1 - 3x^2)(1 + 3x^2)(1 - 3x^2). \end{aligned}$$

37. $x^4 + 12x^2 - 64 = (x^2 + 16)(x^2 - 4) = (x^2 + 16)(x + 2)(x - 2).$

38. $$\begin{aligned} c^9 - 512c^{12} &= c^9(1 - 512c^3) \\ &= c^9[1 - (8c)^3] \\ &= c^9(1 - 8c)(1 + 8c + 64c^2). \end{aligned}$$

39. $h^{6a} - k^{8a} = (h^{3a} + k^{4a})(h^{3a} - k^{4a}).$

40. $a^2 - b^4 - 12ab + 36b^2 = a^2 - 12ab + 36b^2 - b^4$
$= (a - 6b)^2 - b^4$
$= (a - 6b - b^2)(a - 6b + b^2).$

41. $5h^2 + 21hk - 20k^2 = 5h^2 - 4hk + 25hk - 20k^2$
$= h(5h - 4k) + 5k(5h - 4k)$
$= (5h - 4k)(h + 5k).$

42. $x^6 - x^3 - 2 = (x^3 - 2)(x^3 + 1) = (x^3 - 2)(x + 1)(x^2 - x + 1).$

43. $2x^2y^2 - y^4 - x^4 = -1(x^4 - 2x^2y^2 + y^4)$
$= -1(x^2 - y^2)(x^2 - y^2)$
$= (y^2 - x^2)(x^2 - y^2)$
$= (y + x)(y - x)(x + y)(x - y).$

44. $5h^3 - 9h^2k - 9k^3 + 5hk^2 = (5h^3 + 5hk^2) - (9h^2k + 9k^3)$
$= 5h(h^2 + k^2) - 9k(h^2 + k^2)$
$= (h^2 + k^2)(5h - 9k).$

45. $3x^2 - 17x + 10 = 3x^2 - 15x - 2x + 10$
$= 3x(x - 5) - 2(x - 5)$
$= (x - 5)(3x - 2).$

46. $9x^{5n} - x^{3n} = x^{3n}(9x^{2n} - 1) = x^{3n}(3x^n + 1)(3x^n - 1).$

47. $27a - 18ab^2 - 3a^9 + 3ab^4 = 3a(9 - 6b^2 - a^8 + b^4)$
$= 3a[(9 - 6b^2 + b^4) - a^8]$
$= 3a[(3 - b^2)^2 - a^8]$
$= 3a(3 - b^2 - a^4)(3 - b^2 + a^4).$

48. $1 + 2x^2 - 3x^4 = -1(3x^4 - 2x^2 - 1)$
$= -1(3x^4 - 3x^2 + x^2 - 1)$
$= -1[3x^2(x^2 - 1) + (x^2 - 1)]$
$= -1(x^2 - 1)(3x^2 + 1)$
$= -1(x + 1)(x - 1)(3x^2 + 1)$
$= (x + 1)(1 - x)(3x^2 + 1).$

49. $x^4 - x^2 - 12 = (x^2 - 4)(x^2 + 3) = (x + 2)(x - 2)(x^2 + 3).$

50. $16x^4 + 8x^2 - 3 = 16x^4 - 4x^2 + 12x^2 - 3$
$= 4x^2(4x^2 - 1) + 3(4x^2 - 1)$
$= (4x^2 - 1)(4x^2 + 3)$
$= (2x + 1)(2x - 1)(4x^2 + 3).$

51. $1024 - 64h^3 + h^6 = (32 - h^3)(32 - h^3).$

52. $h^2k^2 - h^2 - k^2 + 1 = h^2(k^2 - 1) - (k^2 - 1)$
$= (k^2 - 1)(h^2 - 1)$
$= (k + 1)(k - 1)(h + 1)(h - 1).$

53. $12 - 2h - 4h^2 = -2(2h^2 + h - 6)$
$= -2(2h^2 - 3h + 4h - 6)$
$= -2[h(2h - 3) + 2(2h - 3)]$
$= -2(2h - 3)(h + 2)$
$= 2(3 - 2h)(h + 2).$

54. $3 - y^2 + 3y^3 - y^5 = (3 - y^2) + y^3(3 - y^2)$
$= (3 - y^2)(1 + y^3)$
$= (3 - y^2)(1 + y)(1 - y + y^2).$

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1. $x^2 - 9 = 0.$
Factoring the left member,
$(x + 3)(x - 3) = 0.$
Whence $x = -3$ or $+3.$

2. $x^2 = 16.$
Then
$(x + 4)(x - 4) = 0.$
Whence $x = -4$ or $+4.$

3. $x^2 - 3x = 0.$
Then $x(x - 3) = 0.$
Whence $x = 0$ or $3.$

4. $x^2 = 7x.$
Then $x(x - 7) = 0.$
Whence $x = 0$ or $7.$

5. $x^2 - 7x = -12.$
Then
$(x - 3)(x - 4) = 0.$
Whence $x = 3$ or $4.$

6. $x^2 - x = 20.$
Then
$(x - 5)(x + 4) = 0.$
Whence $x = 5$ or $-4.$

7. $3x^2 - 15x = 0.$
Then $3x(x - 5) = 0.$
Whence $x = 0$ or $5.$

8. $5x^2 + 35x = 0.$
Then $5x(x + 7) = 0.$
Whence $x = 0$ or $-7.$

9. $8 - 9x = -x^2.$
Then
$(x - 8)(x - 1) = 0.$
Whence $x = 8$ or $1.$

10. $12x - 28 = -x^2.$
Then
$(x + 14)(x - 2) = 0.$
Whence $x = -14$ or $2.$

11. $4x^2 = 16x.$
Then $4x(x - 4) = 0.$
Whence $x = 0$ or $4.$

12. $x^2 - a^2 = 0.$
Then
$(x + a)(x - a) = 0.$
Whence $x = -a$ or $a.$

13. $x^2 = 9b^2.$
Then
$(x + 3b)(x - 3b) = 0.$
Whence $x = -3b$ or $3b.$

14. $x^2 - 2ax + a^2 = 0.$
Then
$(x - a)(x - a) = 0.$
Whence $x = a.$

15. $x^2 + 4b^2 = 4bx.$
Then
$(x - 2b)(x - 2b) = 0.$
Whence $x = 2b.$

16.
$x^2 + ax + 3x + 3a = 0.$
Then
$(x + a)(x + 3) = 0.$
Whence $x = -a$ or $-3.$

17. $x^2 + bx = 4x + 4b.$
Then
$(x - 4)(x + b) = 0.$
Whence $x = 4$ or $-b.$

18. $x^2 + ax + bx + ab = 0.$

Then $(x + a)(x + b) = 0.$

Whence $x = -a$ or $-b.$

19. $4x^2 + 8x + 3 = 0.$

Then $(2x + 1)(2x + 3) = 0.$

Whence $x = -\frac{1}{2}$ or $-\frac{3}{2}.$

20. $9x^2 = 3x + 2.$

Then $(3x - 2)(3x + 1) = 0.$

Whence $x = \frac{2}{3}$ or $-\frac{1}{3}.$

21. $16x^2 - 12x = 10.$

Then $(4x - 5)(2x + 1) = 0.$

Whence $x = \frac{5}{4}$ or $-\frac{1}{2}.$

22. $50x + 24 = 25x^2.$

Then $(5x - 12)(5x + 2) = 0.$

Whence $x = \frac{12}{5}$ or $-\frac{2}{5}.$

23. $x^2 + bx = 0.$

Then $x(x + b) = 0.$

Whence $x = 0$ or $-b.$

24. $x^2 - ax - bx = 0.$

Then $x(x - a - b) = 0.$

Whence $x = 0$ or $a + b.$

25. $x^2 + 3x = ax.$

Then $x(x + 3 - a) = 0.$

Whence $x = 0$ or $a - 3.$

26. $4x = x^2 + 4.$

Then $(x - 2)(x - 2) = 0.$

Whence $x = 2.$

27. $x^3 - 9x = 0.$

Then $x(x + 3)(x - 3) = 0.$

Whence $x = 0, -3,$ or $3.$

28. $x^3 + x^2 = 4x + 4.$

Then $(x + 2)(x - 2)(x + 1) = 0.$

Whence $x = -2, 2,$ or $-1.$

29. $x^3 - 5x^2 + 6x = 0.$

Then $x(x - 3)(x - 2) = 0.$

Whence $x = 0, 3,$ or $2.$

30. $2x^3 - x^2 = 32x - 16.$

Then $(x - 4)(x + 4)(2x - 1) = 0.$

Whence $x = 4, -4,$ or $\frac{1}{2}.$

31. $5x^2 + x^3 = 45 + 9x.$

Then $(x + 3)(x - 3)(x + 5) = 0.$

Whence $x = -3, 3,$ or $-5.$

32. $x^4 - 5x^2 + 4 = 0.$

Then $(x + 2)(x - 2)(x + 1)(x - 1) = 0.$

Whence $x = -2, 2, -1,$ or $1.$

33. $9 + x^4 = 10x^2.$

Then $(x + 3)(x - 3)(x + 1)(x - 1) = 0.$

Whence $x = -3, 3, -1,$ or $1.$

34. $x^4 - 36x^2 = 0.$

Then

$x^2(x + 6)(x - 6) = 0.$

Whence $x = 0, -6, \text{ or } 6.$

35. $x^4 + 15x^2 = 8x^3.$

Then

$x^2(x - 5)(x - 3) = 0.$

Whence $x = 0, 5, \text{ or } 3.$

36. $x^4 - 2x^2 + 1 = 0.$

Then $(x + 1)^2(x - 1)^2 = 0.$

Whence $x = -1 \text{ or } 1.$

37. It does not follow from $(x + 1)(x - 1) = x - 1$ that $x + 1 = 1$, for by the condition of the problem $x - 1 = 0$, and it is not permissible to divide each member of an equation by zero.

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1. Let $n =$ the number.

Then $n^2 + n = 90,$

or $(n - 9)(n + 10) = 0.$

Whence $n = 9 \text{ or } -10.$

2. Let $n =$ the number.

Then $n^2 - 2n = 35,$

or $(n - 7)(n + 5) = 0.$

Whence $n = 7 \text{ or } -5.$

3. Let $n =$ the number.

Then $n^2 + 2n + 5 = 148,$

or $(n + 13)(n - 11) = 0.$

Whence $n = -13 \text{ or } 11.$

4. Let $n =$ the number.

Then $4n^2 = 7n,$

or $n(n - \frac{7}{4}) = 0.$

Whence $n = 0 \text{ or } \frac{7}{4}.$

5. Let $n =$ the number.

Then

$(n + 20)(n + 21) = 930,$

or $(n - 10)(n + 51) = 0.$

Whence $n = 10 \text{ or } -51.$

6. Let $n =$ the number.

Then

$(17 - n)(23 - n) = 216,$

or $(n - 35)(n - 5) = 0.$

Whence $n = 35 \text{ or } 5.$

7. Let $n =$ the number.

Then

$(27 - n)(n + 21) = 540,$

or $(n - 9)(n + 3) = 0.$

Whence $n = 9 \text{ or } -3.$

8. Let $n =$ the number.

Then $(n + 15)(22 - n) = 70 + 23n,$

or $(n - 10)(n + 26) = 0.$

Whence $n = 10 \text{ or } -26.$

9. Let $n =$ the one number.

Then $n + 6 =$ the other.

Therefore $(n + 6)^2 - n^2 = 120.$

Whence $n = 7,$

and $n + 6 = 13.$

Also $n^2 - (n + 6)^2 = 120.$

Whence $n = -13,$

and $n + 6 = -7.$

The negative numbers are rejected.

10. Let $n =$ the number.

Then $9n^2 - 5n = 8n^2,$

or $n(n - 5) = 0.$

Whence $n = 0$ or $5.$

11. Let $f =$ the frontage in feet.

Then $4f =$ the depth in feet.

Therefore $4f^2 = 2500,$

or $(f - 25)(f + 25) = 0.$

Whence $f = 25$ or $-25,$

and $4f = 100$ or $-100.$

The negative lengths, being meaningless, are rejected.

12. Let $b =$ the breadth in feet.

Then $b + 6 =$ the length in feet.

Therefore $b(b + 6) = 216,$

or $(b + 18)(b - 12) = 0.$

Whence $b = -18$ or $12,$

and $b + 6 = -12$ or $18.$

The negative values are rejected.

13. Let $w =$ the width in rods.

Then $w + 6 =$ the length in rods.

Therefore $w(w + 6) = 24 \cdot 9,$

or $(w + 18)(w - 12) = 0.$

Whence $w = -18$ or $12,$

and $w + 6 = -12$ or $18.$

The negative values are rejected.

14. Let $n =$ the first number.

Then $n + 1 =$ the second.

Hence $n^2 + (n + 1)^2 = 145,$

or $(n + 9)(n - 8) = 0.$

Whence $n = -9$ or $8,$

and $n + 1 = -8$ or $9.$

Therefore the numbers are -9 and -8, or 8 and 9.

15. Let $n =$ the first odd number.

Then $n + 2 =$ the second.

Hence $n^2 + (n + 2)^2 = 290,$

or $(n + 13)(n - 11) = 0.$

Whence $n = -13$ or $11,$

and $n + 2 = -11$ or $13.$

Therefore the numbers are -13 and -11, or 11 and 13.

16. Let n, $n + 2$, and $n + 4$ represent the odd numbers.
Then $n^2 + (n+2)^2 + (n+4)^2 = 251,$
or $(n+11)(n-7) = 0.$
Whence $n = -11$ or 7.
Therefore the numbers are -11, -9, and -7; or 7, 9, and 11.

17. Let $n =$ the side of the square in inches.
Then $n^2 + 4 \cdot 8 \cdot n = 185,$
or $(n+37)(n-5) = 0.$
Whence $n = -37$ or 5.
Therefore n is 5.

18. Let $x =$ the edge of the cube in inches.
Then $6x^2 = 294,$
or $(x-7)(x+7) = 0.$
Whence $x = 7$ or -7.
Therefore the edge of the cube is 7 inches.

19. Let $d =$ the depth in feet,
$4d =$ the length in feet,
and $3d =$ the width in feet.
Then $2 \cdot d \cdot 4d + 2 \cdot d \cdot 3d + 2 \cdot 4d \cdot 3d = 608,$
or $(d-4)(d+4) = 0.$
Whence $d = 4$ or -4.
Therefore the depth is 4 feet, the length is 16 feet, and the width is 12 feet.

20. Let $x =$ the depth in inches.
Then $x + 3 =$ the length in inches,
and $x + 1 =$ the width in inches.
$2x(x+3) + 2x(x+1) + 2(x+3)(x+1) = 62,$
or $(3x+14)(x-2) = 0.$
Whence $x = -4\frac{2}{3}$ or 2.
Therefore the depth is 2 inches, the length is 5 inches, and the width is 3 inches.

21. Let $b =$ the base of the triangle in feet.
Then $\dfrac{6 \cdot b}{2} = 30.$
Whence $b = 10.$

22. Let $b =$ the base of the triangle in feet.
Then $3b =$ the altitude in feet.
Therefore $\dfrac{3b \cdot b}{2} = 54,$

10. Let $n =$ the number.
Then $9n^2 - 5n = 8n^2,$
or $n(n-5) = 0.$
Whence $n = 0$ or $5.$

11. Let $f =$ the frontage in feet.
Then $4f =$ the depth in feet.
Therefore $4f^2 = 2500,$
or $(f-25)(f+25) = 0.$
Whence $f = 25$ or $-25,$
and $4f = 100$ or $-100.$
The negative lengths, being meaningless, are rejected.

12. Let $b =$ the breadth in feet.
Then $b + 6 =$ the length in feet.
Therefore $b(b+6) = 216,$
or $(b+18)(b-12) = 0.$
Whence $b = -18$ or $12,$
and $b + 6 = -12$ or $18.$
The negative values are rejected.

13. Let $w =$ the width in rods.
Then $w + 6 =$ the length in rods.
Therefore $w(w+6) = 24 \cdot 9,$
or $(w+18)(w-12) = 0.$
Whence $w = -18$ or $12,$
and $w + 6 = -12$ or $18.$
The negative values are rejected.

14. Let $n =$ the first number.
Then $n + 1 =$ the second.
Hence $n^2 + (n+1)^2 = 145,$
or $(n+9)(n-8) = 0.$
Whence $n = -9$ or $8,$
and $n + 1 = -8$ or $9.$
Therefore the numbers are -9 and -8, or 8 and 9.

15. Let $n =$ the first odd number.
Then $n + 2 =$ the second.
Hence $n^2 + (n+2)^2 = 290,$
or $(n+13)(n-11) = 0.$
Whence $n = -13$ or $11,$
and $n + 2 = -11$ or $13.$
Therefore the numbers are -13 and -11, or 11 and 13.

16. Let n, $n + 2$, and $n + 4$ represent the odd numbers.

Then $n^2 + (n + 2)^2 + (n + 4)^2 = 251,$

or $(n + 11)(n - 7) = 0.$

Whence $n = -11$ or 7.

Therefore the numbers are -11, -9, and -7; or 7, 9, and 11.

17. Let $n =$ the side of the square in inches.

Then $n^2 + 4 \cdot 8 \cdot n = 185,$

or $(n + 37)(n - 5) = 0.$

Whence $n = -37$ or 5.

Therefore n is 5.

18. Let $x =$ the edge of the cube in inches.

Then $6x^2 = 294,$

or $(x - 7)(x + 7) = 0.$

Whence $x = 7$ or -7.

Therefore the edge of the cube is 7 inches.

19. Let $d =$ the depth in feet,

$4d =$ the length in feet,

and $3d =$ the width in feet.

Then $2 \cdot d \cdot 4d + 2 \cdot d \cdot 3d + 2 \cdot 4d \cdot 3d = 608,$

or $(d - 4)(d + 4) = 0.$

Whence $d = 4$ or -4.

Therefore the depth is 4 feet, the length is 16 feet, and the width is 12 feet.

20. Let $x =$ the depth in inches.

Then $x + 3 =$ the length in inches,

and $x + 1 =$ the width in inches.

$2x(x + 3) + 2x(x + 1) + 2(x + 3)(x + 1) = 62,$

or $(3x + 14)(x - 2) = 0.$

Whence $x = -4\frac{2}{3}$ or 2.

Therefore the depth is 2 inches, the length is 5 inches, and the width is 3 inches.

21. Let $b =$ the base of the triangle in feet.

Then $\dfrac{6 \cdot b}{2} = 30.$

Whence $b = 10.$

22. Let $b =$ the base of the triangle in feet.

Then $3b =$ the altitude in feet.

Therefore $\dfrac{3b \cdot b}{2} = 54,$

or $(b - 6)(b + 6) = 0.$

Whence $b = 6 \text{ or } - 6.$

Hence the base is 6 feet and the altitude is 18 feet.

23. Let $a =$ the altitude in feet.

Then $5a =$ the base in feet.

Therefore $\frac{5a \cdot a}{2} = 40,$

or $(a - 4)(a + 4) = 0.$

Whence $a = 4 \text{ or } - 4.$

Hence the altitude is 4 feet and the base is 20 feet.

24. Let $a =$ the altitude in meters.

Then $6a =$ the base in meters.

Therefore $\frac{6a \cdot a}{2} = 75,$

or $(a - 5)(a + 5) = 0.$

Whence $a = 5 \text{ or } - 5.$

Hence the altitude is 5 meters and the base is 30 meters.

25. If $x =$ the base in feet,

then $x + 2 =$ the altitude in feet.

Therefore $\frac{x(x + 2)}{2} = 24,$

or $(x - 6)(x + 8) = 0.$

Whence $x = 6 \text{ or } - 8.$

Therefore the base is 6 feet and the altitude is 8 feet.

26. Let $b =$ the base in feet.

Then $b + 3 =$ the altitude in feet.

Therefore $\frac{b(b + 3)}{2} = 54,$

or $(b - 9)(b + 12) = 0.$

Whence $b = 9 \text{ or } - 12.$

Therefore the base is 9 feet and the altitude is 12 feet.

27. If x and $x + 2$ represent the legs in feet,

then $\frac{x(x + 2)}{2} = 24,$

or $(x - 6)(x + 8) = 0.$

Whence $x = 6 \text{ or } - 8.$

Therefore the legs are 6 feet and 8 feet.

28. If x and $x+7$ represent the legs in feet,

then $\frac{x(x+7)}{2} = 30,$

or $(x-5)(x+12) = 0.$

Whence $x = 5$ or $-12.$

Therefore the legs are 5 feet and 12 feet.

29. Let $a =$ the altitude in inches.

Then $6 + 2a =$ the base in inches.

Therefore $\frac{a(6+2a)}{2} = 2\frac{5}{8} \cdot 144,$

or $(a-18)(a+21) = 0.$

Whence $a = 18$ or $-21.$

Therefore the altitude is 18 inches and the base is 42 inches.

30. Let $b =$ the base in feet.

Then $6 + 3b =$ the altitude in feet.

Therefore $\frac{b(6+3b)}{2} = 4 \cdot 9,$

or $(b-4)(b+6) = 0.$

Whence $b = 4$ or $-6.$

Therefore the base is 4 feet and the altitude is 18 feet.

31. Let $b =$ the base in centimeters.

Then $b - 5 =$ the altitude in centimeters.

Therefore $\frac{b(b-5)}{2} = 150,$

or $(b+15)(b-20) = 0.$

Whence $b = -15$ or $20.$

Therefore the base is 20 centimeters and the altitude is 15 centimeters.

32. $\frac{12(10+18)}{2} = 168,$ area.

33. If x and $x+4$ represent the bases in inches,

then $8x + 16 = 96.$

Whence $x = 10,$

and $x + 4 = 14.$

34. If $12 =$ one base in feet,

and $a =$ the altitude in feet,

then $2a =$ the other base in feet.

Therefore $\frac{a(2a+12)}{2} = 112,$

or $(a-8)(a+14)=0.$

Whence $a=8$ or $-14.$

Therefore the altitude is 8 feet and the other base is 16 feet.

35. If $b=$ the longer base in feet,

then $\frac{2b}{3}=$ the other base in feet,

and $\frac{b}{3}=$ the altitude in feet.

Hence $\frac{b}{3}\cdot\frac{1}{2}\left(b+\frac{2b}{3}\right)=360,$

or $(b-36)(b+36)=0.$

Whence $b=36$ or $-36.$

Therefore the longer base is 36 feet, the other base is 24 feet, and the altitude is 12 feet.

36. If $a=$ the altitude in feet,

then $a+4=$ the shorter base in feet,

and $a+6=$ the longer base in feet.

Therefore $\frac{a(a+4+a+6)}{2}=66,$

or $(a-6)(a+11)=0.$

Whence $a=6$ or $-11.$

Therefore the altitude is 6 feet, the shorter base is 10 feet, and the longer base is 12 feet.

37. If $a=$ the altitude in feet,

then $a+8=$ the shorter base in feet,

and $a+12=$ the longer base in feet.

Therefore $\frac{a(a+8+a+12)}{2}=16\cdot 9,$

or $(a-8)(a+18)=0.$

Whence $a=8$ or $-18.$

Therefore the altitude is 8 feet, the shorter base is 16 feet, and the longer base is 20 feet.

38. If $b=$ the shorter base in feet,

then $b+4=$ the longer base in feet,

and $b+2=$ the altitude in feet.

Therefore $\frac{(b+2)(b+b+4)}{2}=4\cdot 9,$

or $(b-4)(b+8)=0.$

Whence $b = 4 \text{ or } -8.$

Therefore the shorter base is 4 feet, the longer base is 8 feet, and the altitude is 6 feet.

39. If $b =$ one base in feet,
then $b =$ the altitude in feet,
and $b + 2 =$ the other base in feet.

Therefore $$\frac{b(b + b + 2)}{2} = 10 \cdot 9,$$

or $$(b - 9)(b + 10) = 0.$$

Whence $b = 9 \text{ or } -10.$

Therefore one base is 9 feet, the altitude is 9 feet, and the other base is 11 feet.

40. If $b =$ the shorter base in feet,
then $b + 10 =$ the longer base in feet,
and $5b + 2 =$ the altitude in feet.

Therefore $$\frac{(5b + 2)(b + b + 10)}{2} = 22 \cdot 9,$$

or $$(b - 4)(5b + 47) = 0.$$

Whence $b = 4 \text{ or } -9\tfrac{2}{5}.$

Therefore the shorter base is 4 feet, the longer base is 14 feet, and the altitude is 22 feet.

41. If $s =$ the shorter base in centimeters,
then $2s =$ the longer base in centimeters,
and $2s - 10 =$ the altitude in centimeters.

Therefore $$(2s - 10)\left(\frac{s + 2s}{2}\right) = 900,$$

or $$(s - 20)(s + 15) = 0.$$

Whence $s = 20 \text{ or } -15.$

Therefore the shorter base is 20 centimeters, the longer base is 40 centimeters, and the altitude is 30 centimeters.

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1. $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$. $\therefore$ H.C.F. $= 2 \cdot 3 = 6$.
2. $24 = 2^3 \cdot 3$ and $56 = 2^3 \cdot 7$. $\therefore$ H.C.F. $= 2^3 = 8$.
3. $96 = 2^5 \cdot 3$ and $144 = 2^4 \cdot 3^2$. $\therefore$ H.C.F. $= 2^4 \cdot 3 = 48$.
4. $84 = 2^2 \cdot 3 \cdot 7$ and $196 = 2^2 \cdot 7^2$. $\therefore$ H.C.F. $= 2^2 \cdot 7 = 28$.
5. $125 = 5^3$ and $225 = 5^2 \cdot 3^2$. $\therefore$ H.C.F. $= 5^2 = 25$.
6. $64 = 2^6$, $96 = 2^5 \cdot 3$, and $256 = 2^8$. $\therefore$ H.C.F. $= 2^5 = 32$.
7. $90 = 3^2 \cdot 2 \cdot 5$, $108 = 3^3 \cdot 2^2$, and $324 = 3^4 \cdot 2^2$. $\therefore$ H.C.F. $= 3^2 \cdot 2 = 18$.
8. $12x^4 = 2^2 \cdot 3 \cdot x^4$ and $18x^3 = 2 \cdot 3^2 \cdot x^3$. $\therefore$ H.C.F. $= 2 \cdot 3 \cdot x^3 = 6x^3$.

9. $16x^2y^4 = 2^4 \cdot x^2 \cdot y^4$ and $24x^3y^5 = 2^3 \cdot 3 \cdot x^3 \cdot y^5$. $\therefore$ H.C.F. $= 2^3x^2y^4 = 8x^2y^4$.

10. $27c^2d = 3^3 \cdot c^2 \cdot d$ and $21cd^4 = 3 \cdot 7 \cdot c \cdot d^4$. $\therefore$ H.C.F. $= 3cd$.

11. $32a^3bc^4 = 2^5 \cdot a^3 \cdot b \cdot c^4$ and $48a^2b^2d = 2^4 \cdot 3 \cdot a^2 \cdot b^2 \cdot d$.
$\therefore$ H.C.F. $= 2^4a^2b = 16a^2b$.

12. $125m^7n^5p = 5^3 \cdot m^7 \cdot n^5 \cdot p$ and $100ng^4 = 5^2 \cdot 2^2 \cdot n \cdot g^4$.
$\therefore$ H.C.F. $= 5^2 \cdot n = 25n$.

13. $18h^2k^2 = 2 \cdot 3^2 \cdot h^2 \cdot k^2$, $36h^4k = 2^2 \cdot 3^2 \cdot h^4 \cdot k$,
and $24h^3k^3 = 2^3 \cdot 3 \cdot h^3 \cdot k^3$. $\therefore$ H.C.F. $= 2 \cdot 3 \cdot h^2 \cdot k = 6h^2k$.

14. $9xy^4 = 3^2 \cdot x \cdot y^4$, $54x^5y = 2 \cdot 3^3 \cdot x^5 \cdot y$, and $15x^4y^5 = 3 \cdot 5 \cdot x^4 \cdot y^5$.
$\therefore$ H.C.F. $= 3xy$.

15. $27a^4b^5c^2 = 3^3 \cdot a^4 \cdot b^5 \cdot c^2$, $54a^3b^2d = 2 \cdot 3^3 \cdot a^3 \cdot b^2 \cdot d$,
and $81a^2b^2e^3 = 3^4 \cdot a^2 \cdot b^2 \cdot e^3$. $\therefore$ H.C.F. $= 3^3 \cdot a^2 \cdot b^2 = 27a^2b^2$.

16. $x^2 - 9 = (x + 3)(x - 3)$ and $x^2 - 5x + 6 = (x - 2)(x - 3)$.
$\therefore$ H.C.F. $= x - 3$.

17. $x^2 + 3x - 10 = (x + 5)(x - 2)$ and $x^2 + 6x + 5 = (x + 5)(x + 1)$.
$\therefore$ H.C.F. $= x + 5$.

18. $x^3 - 4x = x(x + 2)(x - 2)$ and $x^3 - 8x^2 + 12x = x(x - 2)(x - 6)$.
$\therefore$ H.C.F. $= x^2 - 2x$.

19. $2c^3 + 12c^2 + 18c = 2c(c + 3)^2$ and $c^3 - 2c^2 - 15c = c(c + 3)(c - 5)$.
$\therefore$ H.C.F. $= c^2 + 3c$.

20. $8 + y^3 = (2 + y)(4 - 2y + y^2)$ and $y^2 + 4y + 4 = (2 + y)^2$.
$\therefore$ H.C.F. $= 2 + y$.

21. $x^4 - 2x^2 + 1 = (x + 1)^2(x - 1)^2$ and $x^2 - 2x + 1 = (x - 1)^2$.
$\therefore$ H.C.F. $= x^2 - 2x + 1$.

22. $ab + 3b + ac + 3c = (a + 3)(b + c)$
and $2ab + 6b - 2ac - 6c = 2(a + 3)(b - c)$. $\therefore$ H.C.F. $= a + 3$.

23. $c^2 + 3cd + 2d^2 = (c + d)(c + 2d)$, $c^2 + 5cd + 6d^2 = (c + 2d)(c + 3d)$,
and $c^2 + cd - 2d^2 = (c - d)(c + 2d)$. $\therefore$ H.C.F. $= c + 2d$.

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1. $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$. $\therefore$ L.C.M. $= 2^2 \cdot 3^2 = 36$.

2. $32 = 2^5$ and $48 = 2^4 \cdot 3$. $\therefore$ L.C.M. $= 2^5 \cdot 3 = 96$.

3. $20 = 2^2 \cdot 5$ and $28 = 2^2 \cdot 7$. $\therefore$ L.C.M. $= 2^2 \cdot 5 \cdot 7 = 140$.

4. $96 = 2^5 \cdot 3$ and $144 = 2^4 \cdot 3^2$. $\therefore$ L.C.M. $= 2^5 \cdot 3^2 = 288$.

5. $64 = 2^6$, $120 = 2^3 \cdot 3 \cdot 5$, and $216 = 2^3 \cdot 3^3$. $\therefore$ L.C.M. $= 2^6 \cdot 3^3 \cdot 5 = 8640$.

6. $128 = 2^7$, $160 = 2^5 \cdot 5$, and $200 = 2^3 \cdot 5^2$. $\therefore$ L.C.M. $= 2^7 \cdot 5^2 = 3200$.

7. $x^2y = x^2 \cdot y$, $xy^2 = x \cdot y^2$, and $xy^3 = x \cdot y^3$. $\therefore$ L.C.M. $= x^2y^3$.

8. $6cd^3 = 2 \cdot 3 \cdot c \cdot d^3$, $4c^5de = 2^2 \cdot c^5 \cdot d \cdot e$, and $10c^2d^2e^3 = 2 \cdot 5 \cdot c^2 \cdot d^2 \cdot e^3$.
$\therefore$ L.C.M. $= 60c^5d^3e^3$.

9. $8abc = 2^3 \cdot a \cdot b \cdot c$, $3b^2c = 3 \cdot b^2 \cdot c$, and $12c^2 = 2^2 \cdot 3 \cdot c^2$.
$\therefore$ L.C.M. $= 24ab^2c^2$.

10. $18m^2 = 2 \cdot 3^2 \cdot m^2$, $15mn^2p = 3 \cdot 5 \cdot m \cdot n^2 \cdot p$, and $20m^3p^2 = 2^2 \cdot 5m^3 \cdot p^2$.
$\therefore$ L.C.M. $= 180\,m^3n^2p^2$.

11. $20 = 2^2 \cdot 5$, $18xy^4 = 2 \cdot 3^2 \cdot x \cdot y^4$, and $27\,ax^3y = 3^3 \cdot a \cdot x^3 \cdot y$.
$\therefore$ L.C.M. $= 540\,ax^3y^4$.

12. $36\,ab^2 = 2^2 \cdot 3^2 \cdot a \cdot b^2$, $42\,abc = 2 \cdot 3 \cdot 7 \cdot a \cdot b \cdot c$, and $63\,b^2c = 3^2 \cdot 7 \cdot b^2 \cdot c$.
$\therefore$ L.C.M. $= 252\,ab^2c$.

13. $4\,a = 2^2 \cdot a$, $a^2 - ab = a(a - b)$. $\therefore$ L.C.M. $= 4\,a^2 - 4\,ab$.

14. $12\,ax = 2^2 \cdot 3 \cdot ax$, $3\,a^3x^2 - 3\,ax^3 = 3\,ax^2(a^2 - x)$.
$\therefore$ L.C.M. $= 12\,a^3x^2 - 12\,ax^3$.

15. $cx + cy = c(x + y)$, $dy + dx = d(x + y)$. $\therefore$ L.C.M. $= cdx + cdy$.

16. $3\,x + 3\,z = 3(x + z)$, $6\,a^2x + 6\,a^2z = 2 \cdot 3\,a^2(x + z)$.
$\therefore$ L.C.M. $= 6\,a^2x + 6\,a^2z$.

17. $x^2 - xy = x(x - y)$, $ax + ay = a(x + y)$. $\therefore$ L.C.M. $= ax^3 - axy^2$.

18. $x^2 - 9 = (x - 3)(x + 3)$, $x^2 - 5\,x + 6 = (x - 2)(x - 3)$.
$\therefore$ L.C.M. $= x^3 - 2\,x^2 - 9\,x + 18$.

19. $c^2 - 4 = (c + 2)(c - 2)$, $c^2 - 8\,c - 20 = (c - 10)(c + 2)$.
$\therefore$ L.C.M. $= c^3 - 10\,c^2 - 4\,c + 40$.

20. $4\,ax = 2^2 \cdot a \cdot x$, $4\,x^2 - 1 = (2\,x + 1)(2\,x - 1)$,
$4\,x^2 + 4\,x + 1 = (2\,x + 1)^2$.
$\therefore$ L.C.M. $= 32\,ax^4 - 8\,ax^2 + 16\,ax^3 - 4\,ax$.

21. $x^2 + 1 = x^2 + 1$, $x^4 - 1 = (x^2 + 1)(x + 1)(x - 1)$,
$x^4 - 2\,x^2 + 1 = (x + 1)^2(x - 1)^2$. $\therefore$ L.C.M. $= x^6 - x^4 - x^2 + 1$.

22. $4 - c^2 = (2 + c)(2 - c)$, $c^3 + 8 = (c + 2)(c^2 - 2\,c + 4)$,
$c^2 + 6\,c + 8 = (c + 2)(c + 4)$.
$\therefore$ L.C.M. $= 64 - 16\,c - 8\,c^2 + 8\,c^3 - 2\,c^4 - c^5$.

23. $ac - 2\,bd + 2\,ad - bc = (a - b)(c + 2\,d)$,
$a^2 + ab - 2\,b^2 = (a + 2\,b)(a - b)$.
$\therefore$ L.C.M. $= a^2c + abc - 2\,b^2c + 2\,a^2d + 2\,abd - 4\,b^2d$.

24. $x^2 - y^2 = (x + y)(x - y)$, $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$,
$x^2 + 2\,xy + y^2 = (x + y)^2$.
$\therefore$ L.C.M. $= x^5 + 2\,x^4y + x^3y^2 - x^2y^3 - 2\,xy^4 - y^5$.

25. $ax^2 + bxy = x(ax + by)$, $2\,ax + 2\,by = 2(ax + by)$,
$a^2x^2 - b^2y^2 = (ax + by)(ax - by)$. $\therefore$ L.C.M. $= 2\,a^2x^3 - 2\,b^2xy^2$.

26. $2\,x^3 - 2\,x = 2\,x(x + 1)(x - 1)$, $3\,x^4 + 15\,x^3 - 18\,x^2 = 3\,x^2(x + 6)(x - 1)$,
$x^2 - 36 = (x + 6)(x - 6)$.
$\therefore$ L.C.M. $= 6\,x^6 - 222\,x^4 + 216\,x^2$.

27. $8 - y^3 = (2 - y)(4 + 2\,y + y^2)$, $y^2 - 4 = (y + 2)(y - 2)$,
$4\,y^2 + 2\,y^3 + y^4 = y^2(4 + 2\,y + y^2)$.
$\therefore$ L.C.M. $= - y^6 - 2\,y^5 + 8\,y^3 + 16\,y^2$.

28. $2\,x^3 - 6\,x^2 + 4\,x^2y = 2\,x^2(x - 3 + 2\,y)$,
$x^2 - 4\,y^2 - 6\,x + 9 = (x - 3 + 2\,y)(x - 3 - 2\,y)$.
$\therefore$ L.C.M. $= 2\,x^4 - 8\,x^2y^2 - 12\,x^3 + 18\,x^2$.

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1. $\frac{a^3b}{a^2b^4} = \frac{a}{b^3}.$

2. $\frac{12\,x^2y^6}{18\,xy^7} = \frac{2\,x}{3\,y}.$

3. $\frac{32\,a^4b^4c}{48\,a^3b^6c^2} = \frac{2\,a}{3\,b^2c}.$

4. $\frac{45\,cd^5e^2}{20\,c^5d^3} = \frac{9\,d^2e^2}{4\,c^4}.$

5. $\frac{36\,a^2b^2}{54\,a^5bc^3} = \frac{2\,b}{3\,a^3c^3}.$

6. $\frac{2\,a}{2\,a+2} = \frac{a}{a+1}.$

7. $\frac{3\,xy+3\,y^2}{3\,y^4} = \frac{x+y}{y^3}.$

8. $\frac{21\,d^2+14\,c}{14\,c} = \frac{3\,d^2+2\,c}{2\,c}.$

9. $\frac{a^2-1}{a^2-2\,a+1} = \frac{a+1}{a-1}.$

10. $\frac{4\,x^2+4\,x+1}{4\,x^2-1} = \frac{2\,x+1}{2\,x-1}.$

11. $\frac{2\,a^2-2\,b^2}{4\,a^2-8\,ab+4\,b^2} = \frac{a+b}{2\,a-2\,b}.$

12. $\frac{18\,ax^3+9\,ay}{12\,bx^3+6\,by} = \frac{3\,a}{2\,b}.$

13. $\frac{x^2-25}{x^2-x-30} = \frac{x-5}{x-6}.$

14. $\frac{c^2-5\,cd+4\,d^2}{c^2-16\,d^2} = \frac{c-d}{c+4\,d}.$

15. $\frac{2\,x^2-2\,x-180}{2\,x^2-162} = \frac{x-10}{x-9}.$

16. $\frac{21+10\,x+x^2}{x^2-9} = \frac{x+7}{x-3}.$

17. $\frac{4\,c^3-5\,c^2-4\,c+5}{8\,c^4-10\,c^3+12\,c-15} = \frac{(4\,c-5)(c^2-1)}{(4\,c-5)(2\,c^3+3)} = \frac{c^2-1}{2\,c^3+3}.$

18. $\frac{x^2-y^2}{(x-y)^2} = \frac{x+y}{x-y}.$

19. $\frac{a^2-b^2}{(a-b)^3} = \frac{a+b}{a^2-2\,ab+b^2}.$

20. $\frac{c^2-d^2}{c^3-d^3} = \frac{(c+d)(c-d)}{(c^2+cd+d^2)(c-d)} = \frac{c+d}{c^2+cd+d^2}.$

21. $\frac{y^3-z^3}{(y-z)^3} = \frac{(y-z)(y^2+yz+z^2)}{(y-z)(y^2-2\,yz+z^2)} = \frac{y^2+yz+z^2}{y^2-2\,yz+z^2}.$

22. $\frac{x^3-8}{x^2-4} = \frac{(x^2+2\,x+4)(x-2)}{(x+2)(x-2)} = \frac{x^2+2\,x+4}{x+2}.$

23. $\frac{x^6-1}{x^2-1} = \frac{(x+1)(x-1)(x^2+x+1)(x^2-x+1)}{(x+1)(x-1)} = x^4+x^2+1.$

24. $\frac{x^4-y^4}{x^6-y^6} = \frac{(x+y)(x-y)(x^2+y^2)}{(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)} = \frac{x^2+y^2}{x^4+x^2y^2+y^4}.$

25. $\frac{64\,x^3+1}{1+8\,x+16\,x^2} = \frac{(16\,x^2-4\,x+1)(4\,x+1)}{(1+4\,x)(4\,x+1)} = \frac{16\,x^2-4\,x+1}{1+4\,x}.$

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1. $\frac{-x}{y} = -\frac{x}{y} = \frac{x}{-y} = -\frac{-x}{-y}.$

2. $\frac{x}{-y} = -\frac{x}{y} = \frac{-x}{y} = -\frac{-x}{-y}.$

3. $\frac{a}{a-b} = \frac{-a}{-a+b} = -\frac{-a}{a-b} = -\frac{a}{-a+b}.$

4. $\frac{x-3}{2x+4}=\frac{-x+3}{-2x-4}=-\frac{-x+3}{2x+4}=-\frac{x-3}{-2x-4}.$

5. $-\frac{c^2-d}{-2c+d}=-\frac{-c^2+d}{2c-d}=+\frac{-c^2+d}{-2c+d}=+\frac{c^2-d}{2c-d}.$

6. $-\frac{x-y}{x+y-3}=-\frac{-x+y}{-x-y+3}=+\frac{-x+y}{x+y-3}=+\frac{x-y}{-x-y+3}.$

7. $\frac{x^2-5x+6}{x^2-7x+12}=\frac{-x^2+5x-6}{-x^2+7x-12}=-\frac{-x^2+5x-6}{x^2-7x+12}=-\frac{x^2-5x+6}{-x^2+7x-12}.$

8. $-\frac{-x+x^2-2}{x^3-x^2y-1}=-\frac{x-x^2+2}{-x^3+x^2y+1}=+\frac{x-x^2+2}{x^3-x^2y-1}=+\frac{-x+x^2-2}{-x^3+x^2y+1}.$

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1. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ equal $\frac{3}{6}$, $\frac{2}{6}$, and $\frac{1}{6}$ respectively.
2. $\frac{5}{24}, \frac{7}{32}$ equal $\frac{20}{96}$ and $\frac{21}{96}$ respectively.
3. $\frac{4}{3d}, \frac{2c}{5d}$ equal $\frac{20}{15d}$ and $\frac{6c}{15d}$ respectively.
4. $\frac{3a+b}{12}, \frac{3a}{16}$ equal $\frac{12a+4b}{48}$ and $\frac{9a}{48}$ respectively.
5. $\frac{5x+y}{21}, \frac{4x+3y}{35}$ equal $\frac{25x+5y}{105}$ and $\frac{12x+9y}{105}$ respectively.
6. $\frac{4}{5a^2b}, \frac{3}{abc}$ equal $\frac{4c}{5a^2bc}$ and $\frac{15a}{5a^2bc}$ respectively.
7. $\frac{5c}{3mn^3}, \frac{12d}{11m^2n^2}$ equal $\frac{55cm}{33m^2n^3}$ and $\frac{36nd}{33m^2n^3}$ respectively.
8. $\frac{3a}{2b^2c}, \frac{2b}{3a^2c}, \frac{c}{6ab}$ equal $\frac{9a^3}{6a^2b^2c}$, $\frac{4b^3}{6a^2b^2c}$ and $\frac{abc^2}{6a^2b^2c}$ respectively.
9. $\frac{3b}{2cd^3}, \frac{9a}{4cde}, \frac{7ab}{5cde^2}$ equal $\frac{30be^2}{20cd^3d^2}$, $\frac{45ad^2e}{20cd^3e^2}$ and $\frac{28abd^2}{20cd^3e^2}$ respectively.
10. $\frac{x+y}{3xy^2}, \frac{x-2y}{2x^2y}$ equal $\frac{2x^2+2xy}{6x^2y^2}$ and $\frac{3xy-6y^2}{6x^2y^2}$ respectively.
11. $\frac{2+m^2}{2m}, \frac{4+m^2}{4n}$ equal $\frac{4n+2m^2n}{4mn}$ and $\frac{4m+m^3}{4mn}$ respectively.
12. $\frac{2}{x+2}, \frac{4}{3x+6}$ equal $\frac{6}{3x+6}$ and $\frac{4}{3x+6}$ respectively.
13. $\frac{4}{x+1}, \frac{5}{x-1}$ equal $\frac{4x-4}{x^2-1}$ and $\frac{5x+5}{x^2-1}$ respectively.
14. $\frac{3c}{c^2-d^2}, \frac{4}{c+d}$ equal $\frac{3c}{c^2-d^2}$ and $\frac{4c-4d}{c^2-d^2}$ respectively.
15. $\frac{3x}{x+3}, \frac{5}{x^2+5x+6}$ equal $\frac{3x^2+6x}{x^2+5x+6}$ and $\frac{5}{x^2+5x+6}$ respectively.

16. $\frac{2x}{x^2 - xy}$, $\frac{3y}{x^2 - 2xy + y^2}$ equal $\frac{2x^2 - 2xy}{x^3 - 2x^2y + xy^2}$ and $\frac{3xy}{x^3 - 2x^2y + xy^2}$ respectively.

17. $\frac{3c+2}{c^2 - d^2}$, $\frac{4-c}{c^2 - 7cd + 6d^2}$ equal $\frac{3c^2 + 2c - 18cd - 12d}{c^3 - cd^2 - 6c^2d + 6d^3}$ and $\frac{4c - c^2 + 4d - cd}{c^3 - cd^2 - 6c^2d + 6d^3}$ respectively.

18. $\frac{2x+5}{x^2 - 1}$, $\frac{2x-4}{x^3 - 3x^2 + 2x}$ equal $\frac{2x^3 + x^2 - 10x}{x^4 - 2x^3 - x^2 + 2x}$ and $\frac{2x^2 - 2x - 4}{x^4 - 2x^3 - x^2 + 2x}$ respectively.

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1. $\frac{2x}{3} + \frac{3x}{5} = \frac{10x}{15} + \frac{9x}{15} = \frac{19x}{15}.$
2. $\frac{2a}{3} - \frac{5a}{6} + \frac{a}{4} = \frac{8a}{12} - \frac{10a}{12} + \frac{3a}{12} = \frac{a}{12}.$
3. $\frac{3c}{7} + \frac{c}{21} - \frac{1}{3} = \frac{9c}{21} + \frac{c}{21} - \frac{7}{21} = \frac{10c-7}{21}.$
4. $\frac{9a}{16} - \frac{12m}{8} - \frac{5m}{24} = \frac{27a}{48} - \frac{72m}{48} - \frac{10m}{48} = \frac{27a - 82m}{48}.$
5. $\frac{x-3}{4} + \frac{2x-5}{6} = \frac{3x-9}{12} + \frac{4x-10}{12} = \frac{7x-19}{12}.$
6. $\frac{5a+7}{14} - \frac{9a+x}{21} = \frac{15a+21}{42} - \frac{18a+2x}{42} = \frac{21-3a-2x}{42}.$
7. $\frac{5c}{3} - \frac{3c-x}{5} + \frac{x-3c}{10} = \frac{50c}{30} - \frac{18c-6x}{30} + \frac{3x-9c}{30} = \frac{23c+9x}{30}.$
8. $\frac{4m-3}{6} - \frac{7-9m}{9} - \frac{3a+5m-4}{27}$

$$= \frac{36m-27}{54} - \frac{42-54m}{54} - \frac{6a+10m-8}{54}$$

$$= \frac{80m-6a-61}{54}.$$

9. $\frac{2}{a} + \frac{3}{4a^2} = \frac{8a}{4a^2} + \frac{3}{4a^2} = \frac{8a+3}{4a^2}.$
10. $\frac{3}{x^2} + \frac{4}{x} - \frac{8}{3x^3} = \frac{9x}{3x^3} + \frac{12x^2}{3x^3} - \frac{8}{3x^3} = \frac{12x^2+9x-8}{3x^3}.$
11. $\frac{x}{y} - \frac{5}{2y^3} + \frac{a}{4} = \frac{4xy^2}{4y^3} - \frac{10}{4y^3} + \frac{ay^3}{4y^3} = \frac{4xy^2+ay^3-10}{4y^3}.$

12. $\frac{a}{b}+\frac{b}{c}-\frac{c}{a}=\frac{a^2c}{abc}+\frac{ab^2}{abc}-\frac{bc^2}{abc}=\frac{a^2c+ab^2-bc^2}{abc}.$

13. $\frac{7}{m^2}-\frac{5}{mn}+\frac{6}{n^2}=\frac{7n^2}{m^2n^2}-\frac{5mn}{m^2n^2}+\frac{6m^2}{m^2n^2}=\frac{7n^2-5mn+6m^2}{m^2n^2}.$

14. $\frac{3}{xy}-\frac{5}{2x^2}-\frac{6}{5xy^3}=\frac{30xy^2}{10x^2y^3}-\frac{25y^3}{10x^2y^3}-\frac{12x}{10x^2y^3}=\frac{30xy^2-25y^3-12x}{10x^2y^3}.$

15. $\frac{3x-1}{6x^2}-\frac{7-2x^2}{4x^3}+\frac{5x^3-8}{9x^4}=\frac{18x^3-6x^2}{36x^4}-\frac{63x-18x^3}{36x^4}+\frac{20x^3-32}{36x^4}$

$=\frac{56x^3-6x^2-63x-32}{36x^4}.$

16. $\frac{4c^2-9}{2cd}-\frac{6-c}{5c^2}-\frac{c^2-4}{cd^2}=\frac{20c^3d-45cd}{10c^2d^2}-\frac{12d^2-2cd^2}{10c^2d^2}-\frac{10c^3-40c}{10c^2d^2}$

$=\frac{20c^3d-45cd-12d^2+2cd^2-10c^3+40c}{10c^2d^2}.$

17. $\frac{3+x}{x-3}+\frac{3}{4}=\frac{(3+x)4}{(x-3)4}+\frac{3(x-3)}{4(x-3)}=\frac{7x+3}{4x-12}.$

18. $\frac{3}{x-5}-\frac{2}{x+5}=\frac{3(x+5)}{(x-5)(x+5)}-\frac{2(x-5)}{(x+5)(x-5)}=\frac{x+25}{x^2-25}.$

19. $\frac{5x}{x^2+xy}-\frac{7}{x}=\frac{5x}{x(x+y)}-\frac{7(x+y)}{x(x+y)}=\frac{-2x-7y}{x^2+xy}.$

20. $\frac{3a-b}{a^2-b^2}+\frac{2}{a-b}=\frac{3a-b}{a^2-b^2}+\frac{2(a+b)}{(a-b)(a+b)}=\frac{5a+b}{a^2-b^2}.$

21. $\frac{10}{25-m^2}-\frac{1}{m^2+16m+55}$

$=\frac{10(m+11)}{(5+m)(5-m)(m+11)}-\frac{1(5-m)}{(m+5)(m+11)(5-m)}$

$=\frac{105+11m}{275+25m-11m^2-m^3}.$

22. $\frac{3}{a^2-16}+\frac{5}{a^2-6a+8}=\frac{3(a-2)}{(a+4)(a-4)(a-2)}+\frac{5(a+4)}{(a-2)(a-4)(a+4)}$

$=\frac{8a+14}{a^3-2a^2-16a+32}.$

23. $\frac{2x+1}{x^2-1}+\frac{4}{x^2-3x+2}=\frac{(2x+1)(x-2)}{(x+1)(x-1)(x-2)}+\frac{4(x+1)}{(x-1)(x-2)(x+1)}$

$=\frac{2x^2+x+2}{x^3-2x^2-x+2}.$

24. $\frac{c-5}{c^2-6c}+\frac{2c-3}{c^2-8c+12}=\frac{(c-5)(c-2)}{c(c-6)(c-2)}+\frac{(2c-3)c}{(c-6)(c-2)c}$

$=\frac{3c^2-10c+10}{c^3-8c^2+12c}.$

25. $$\frac{x^2-3xy+y^2}{9-6x+x^2}-\frac{5x-3y}{9-3x}=\frac{(x^2-3xy+y^2)3}{(3-x)^2\cdot 3}-\frac{(5x-3y)(3-x)}{3(3-x)(3-x)}$$
$$=\frac{8x^2-12xy-15x+9y+3y^2}{3x^2-18x+27}.$$

26. $$\frac{x+2}{x^2+x}+\frac{1}{x}+\frac{3-x}{x^2+2x+1}=\frac{(x+2)(x+1)}{x(x+1)(x+1)}+\frac{1(x+1)^2}{x(x+1)^2}+\frac{(3-x)x}{(x+1)^2\cdot x}$$
$$=\frac{x^2+8x+3}{x^3+2x^2+x}.$$

27. $$\frac{a+b}{a^2-ab}+\frac{a-2b}{a^2-2ab+b^2}-\frac{2a-b}{a^2-b^2}$$
$$=\frac{(a+b)(a+b)(a-b)}{(a^2-ab)(a-b)(a+b)}+\frac{(a-2b)(a+b)a}{(a^2-2ab+b^2)(a+b)a}-\frac{(2a-b)(a-b)a}{(a^2-b^2)(a-b)a}$$
$$=\frac{-4ab^2+3a^2b-b^3}{a^4-a^3b-a^2b^2+ab^3}.$$

28. $$\frac{m-2n}{m^2+mn+n^2}-\frac{m^2-3n^2}{m^3-n^3}+\frac{3m-n}{m-n}$$
$$=\frac{(m-2n)(m-n)}{(m^2+mn+n^2)(m-n)}-\frac{m^2-3n^2}{m^3-n^3}+\frac{(3m-n)(m^2+mn+n^2)}{(m-n)(m^2+mn+n^2)}$$
$$=\frac{3m^3+2m^2n+2mn^2-3mn+5n^2-n^3}{m^3-n^3}.$$

29. $$\frac{c^2+cd+d^2}{c^2-cd+d^2}-\frac{c-d}{2c+2d}-\frac{2c^2+5cd}{c^3+d^3}$$
$$=\frac{(c^2+cd+d^2)(c+d)2}{(c^2-cd+d^2)(c+d)2}-\frac{(c-d)(c^2-cd+d^2)}{(2c+2d)(c^2-cd+d^2)}-\frac{(2c^2+5cd)2}{(c^3+d^3)2}$$
$$=\frac{c^3+6c^2d+2cd^2+3d^3-4c^2-10cd}{2c^3+2d^3}.$$

30. $$\frac{x+y}{x^3-y^3}-\frac{x-2y}{(x-y)^3}=\frac{(x+y)(x-y)^2}{(x^3-y^3)(x-y)^2}-\frac{(x-2y)(x^2+xy+y^2)}{(x-y)^3(x^2+xy+y^2)}$$
$$=\frac{3y^3}{x^5-2x^4y+x^3y^2-x^2y^3+2xy^4-y^5}.$$

31. $R^2+\frac{R^2}{4}=\frac{5R^2}{4}.$

32. $x-3-\frac{x+3}{5}=\frac{(x-3)5}{5}-\frac{x+3}{5}=\frac{4x-18}{5}.$

33. $\frac{m^2+n^2}{m-n}+n+m=\frac{m^2+n^2}{m-n}+\frac{(n+m)(m-n)}{1(m-n)}=\frac{2m^2}{m-n}.$

34. $3a+b-\frac{a^2-3b^2}{3a-b}=\frac{(3a+b)(3a-b)}{1(3a-b)}-\frac{a^2-3b^2}{3a-b}=\frac{8a^2+2b^2}{3a-b}.$

35. $x^2+y^2-\frac{3x^3-y^3}{x+y}-xy=\frac{(x^2-xy+y^2)(x+y)}{1(x+y)}-\frac{3x^3-y^3}{x+y}=\frac{2y^3-2x^3}{x+y}.$

36. $$m^2 - \frac{m^4 - 2m^2}{m^2 + m + 1} + 1 - m = \frac{(m^2 - m + 1)(m^2 + m + 1)}{1(m^2 + m + 1)} - \frac{m^4 - 2m^2}{m^2 + m + 1}$$
$$= \frac{3m^2 + 1}{m^2 + m + 1}.$$

37. $$\frac{m^3 + 8n^3}{m^2 + 2mn + 4n^2} + m - 2n$$
$$= \frac{m^3 + 8n^3}{m^2 + 2mn + 4n^2} + \frac{(m - 2n)(m^2 + 2mn + 4n^2)}{m^2 + 2mn + 4n^2}$$
$$= \frac{2m^3}{m^2 + 2mn + 4n^2}.$$

38. $$a^2 - \frac{a^4 + 2a^2b^2 - b^4}{a^2 + ab + b^2} - ab + b^2$$
$$= \frac{(a^2 - ab + b^2)(a^2 + ab + b^2)}{a^2 + ab + b^2} - \frac{a^4 + 2a^2b^2 - b^4}{a^2 + ab + b^2}$$
$$= \frac{2b^4 - a^2b^2}{a^2 + ab + b^2}.$$

39. $$x^3 + x^2y - \frac{x^4 + 3y^4}{x - y} + xy^2 + y^3$$
$$= \frac{(x^3 + x^2y + xy^2 + y^3)(x - y)}{x - y} - \frac{x^4 + 3y^4}{x - y}$$
$$= \frac{4y^4}{y - x}.$$

40. $$\frac{2c^2 + d^2}{c^2 - d^2} - \frac{c}{c + d} + 5 = \frac{2c^2 + d^2}{c^2 - d^2} - \frac{c(c - d)}{(c + d)(c - d)} + \frac{5(c^2 - d^2)}{c^2 - d^2}$$
$$= \frac{6c^2 + cd - 4d^2}{c^2 - d^2}.$$

41. $$6 + \frac{6r^2 - s^2}{r^2 - 9rs + 14s^2} - \frac{3r - 5s}{r - 7s} + 5$$
$$= \frac{11(r^2 - 9rs + 14s^2)}{r^2 - 9rs + 14s^2} + \frac{6r^2 - s^2}{r^2 - 9rs + 14s^2} - \frac{(3r - 5s)(r - 2s)}{(r - 7s)(r - 2s)}$$
$$= \frac{14r^2 - 88rs + 143s^2}{r^2 - 9rs + 14s^2}.$$

42. $$\left(x - 3 - \frac{3}{x + 3}\right) - \left(2x - 7 - \frac{4}{x + 4}\right)$$
$$= -x + 4 - \frac{3}{x + 3} + \frac{4}{x + 4}$$
$$= \frac{(-x + 4)(x + 3)(x + 4)}{(x + 3)(x + 4)} - \frac{3(x + 4)}{(x + 3)(x + 4)} + \frac{4(x + 3)}{(x + 4)(x + 3)}$$
$$= \frac{48 + 17x - 3x^2 - x^3}{x^2 + 7x + 12}.$$

43. $\left(5m+\frac{6m}{2n}\right)-\left(m-\frac{m-3n}{n}\right)=4m+\frac{3m}{n}+\frac{m-3n}{n}$

$=\frac{4mn+4m-3n}{n}.$

44. $\left(7c-\frac{2d}{3de}\right)-\left(5c+\frac{c}{3de}\right)=2c-\frac{2d+c}{3de}=\frac{6cde-2d-c}{3de}.$

45. $\left(\frac{2a}{a-b}-5a\right)-\left(2a-\frac{4a}{a+b}\right)=-7a+\frac{2a}{a-b}+\frac{4a}{a+b}$

$=-\frac{7a(a-b)(a+b)}{(a-b)(a+b)}+\frac{2a(a+b)}{(a-b)(a+b)}+\frac{4a(a-b)}{(a+b)(a-b)}$

$=\frac{6a^2-2ab+7ab^2-7a^3}{a^2-b^2}.$

46. $\left(2a-3b+\frac{3a}{2a+b}\right)-\left(3a+2b-\frac{5a}{2a-b}\right)$

$=-a-5b+\frac{3a}{2a+b}+\frac{5a}{2a-b}$

$=\frac{(-a-5b)(2a-b)(2a+b)}{(2a-b)(2a+b)}+\frac{3a(2a-b)}{(2a+b)(2a-b)}+\frac{5a(2a+b)}{(2a-b)(2a+b)}$

$=\frac{-4a^3+16a^2-20a^2b+ab^2+2ab+5b^3}{4a^2-b^2}.$

47. $\left(-\frac{4r^2}{2r+s}+2r-s\right)-\left(-\frac{4s^2}{2s-3r}+2s+3r\right)$

$=-r-3s-\frac{4r^2}{2r+s}+\frac{4s^2}{2s-3r}$

$=\frac{(-r-3s)(2r+s)(2s-3r)}{(2r+s)(2s-3r)}-\frac{4r^2(2s-3r)}{(2r+s)(2s-3r)}+\frac{4s^2(2r+s)}{(2s-3r)(2r+s)}$

$=\frac{18r^3+9r^2s+3rs^2-2s^3}{2s^2+rs-6r^2}.$

48. $\frac{x-1}{2-x}+\frac{x^2-3}{x^2-4}=\frac{(1-x)(x+2)}{(x-2)(x+2)}+\frac{x^2-3}{(x+2)(x-2)}=\frac{x+1}{4-x^2}.$

49. $\frac{5}{x-3}+\frac{3}{3-x}=\frac{5}{x-3}-\frac{3}{x-3}=\frac{2}{x-3}.$

50. $\frac{6}{x^2-25}-\frac{3}{5-x}=\frac{6}{(x+5)(x-5)}+\frac{3(x+5)}{(x-5)(x+5)}=\frac{3x+21}{x^2-25}.$

51. $\frac{3a}{a^2-4}+\frac{2a-1}{2-a}=\frac{3a}{(a+2)(a-2)}-\frac{(2a-1)(a+2)}{(a-2)(a+2)}=\frac{2-2a^2}{a^2-4}.$

52. $\frac{3c}{9-c^2}-\frac{4c-2}{c-3}=\frac{3c}{(3+c)(3-c)}+\frac{(4c-2)(3+c)}{(3-c)(3+c)}=\frac{4c^2+13c-6}{9-c^2}.$

53. $\frac{x}{2x-1}-\frac{x}{1+2x}-\frac{x}{1-4x^2}$

$$=\frac{x(2x+1)}{(2x-1)(2x+1)}-\frac{x(2x-1)}{(2x+1)(2x-1)}+\frac{x}{(2x+1)(2x-1)}$$

$$=\frac{3x}{4x^2-1}.$$

54. $$\frac{7}{x^2-13x+42}+\frac{4x-1}{7-x}-\frac{2x+3}{6-x}$$

$$=\frac{7}{(x-6)(x-7)}-\frac{(4x-1)(x-6)}{(x-7)(x-6)}+\frac{(2x+3)(x-7)}{(x-6)(x-7)}$$

$$=\frac{-2x^2+14x-20}{x^2-13x+42}.$$

55. $$\frac{3x-1}{x^2+7x-8}-\frac{4x-1}{1-x}+\frac{x+2}{8+x}$$

$$=\frac{3x-1}{(x+8)(x-1)}+\frac{(4x-1)(x+8)}{(x-1)(x+8)}+\frac{(x+2)(x-1)}{(x+8)(x-1)}$$

$$=\frac{5x^2+35x-11}{x^2+7x-8}.$$

56. $$\frac{1}{(a-b)(a-c)}+\frac{1}{(b-c)(b-a)}-\frac{1}{(c-a)(c-b)}$$

$$=\frac{1}{(a-b)(a-c)}-\frac{1}{(b-c)(a-b)}-\frac{1}{(a-c)(b-c)}$$

$$=\frac{b-c}{(a-b)(a-c)(b-c)}-\frac{a-c}{(b-c)(a-b)(a-c)}-\frac{a-b}{(a-c)(b-c)(a-b)}$$

$$=\frac{-2}{(a-c)(b-c)}.$$

57. $$\frac{2}{(x-y)(x-z)}-\frac{1}{(x-y)(z-x)}=\frac{2}{(x-y)(x-z)}+\frac{1}{(x-y)(x-z)}$$

$$=\frac{3}{(x-y)(x-z)}.$$

58. $$\frac{3a}{(a-4)(a-3)}-\frac{2a}{(3-a)(4-a)}=\frac{3a}{(a-4)(a-3)}-\frac{2a}{(a-3)(a-4)}$$

$$=\frac{a}{(a-3)(a-4)}.$$

59. $$\frac{2}{(m-n)(m+n)}+\frac{3}{(n-m)(m-7)}-\frac{4}{(n-m)(7-m)}$$

$$=\frac{2}{(m-n)(m+n)}-\frac{3}{(m-n)(m-7)}-\frac{4}{(m-n)(m-7)}$$

$$=\frac{2(m-7)-3(m+n)-4(m+n)}{(m-n)(m+n)(m-7)}$$

$$=\frac{-5m-7n-14}{(m-n)(m+n)(m-7)}.$$

60.
$$\frac{c}{c^2-10c+24}-\frac{1}{6c-c^2-8}+\frac{3}{(6-c)(2-c)}$$
$$=\frac{c}{(c-4)(c-6)}+\frac{1}{(c-4)(c-2)}+\frac{3}{(c-6)(c-2)}$$
$$=\frac{c(c-2)+1(c-6)+3(c-4)}{(c-4)(c-6)(c-2)}$$
$$=\frac{c^2+2c-18}{(c-6)(c-4)(c-2)}.$$

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Performing the indicated division:

1. $$\frac{15x^2-10x+2}{5x}=3x-2+\frac{2}{5x}.$$
2. $$\frac{24a^3-6a^2-14}{6a}=4a^2-a-\frac{14}{6a}.$$
3. $$\frac{c}{c+1}=1-\frac{1}{c+1}.$$
4. $$\frac{d^3+1}{d-1}=d^2+d+1+\frac{2}{d-1}.$$
5. $$\frac{27x^3-y^3}{3x+y}=9x^2-3xy+y^2-\frac{2y^3}{3x+y}.$$
6. $$\frac{y^4+y^2+1}{y^2-y-1}=y^2+y+3+\frac{4y+4}{y^2-y-1}.$$
7. $$\frac{a^4+a^2b^2+b^4}{a^2+ab-b^2}=a^2-ab+3b^2+\frac{4b^4-4ab^3}{a^2+ab-b^2}.$$
8. $$\frac{3y^3-11}{y+3}=3y^2-9y+27-\frac{92}{y+3}.$$
9. $$\frac{(a+b)^3}{a^3+b^3}=1+\frac{3a^2b+3ab^2}{a^3+b^3}=1+\frac{3ab}{a^2-ab+b^2}.$$
10. $$\frac{16a^4+b^4}{2a-1}=8a^3+4a^2+2a+1+\frac{1+b^4}{2a-1}.$$
11. $$\frac{5a^3+3a^2-6}{5a^2+3a+2}=a+\frac{-2a-6}{5a^2+3a+2}.$$
12. $$\frac{x^6+y^6}{x^2-y^2}=x^4+x^2y^2+y^4+\frac{2y^6}{x^2-y^2}.$$
13. $$\frac{x^4}{x+1}=x^3-x^2+x-1+\frac{1}{x+1}.$$
14. $$\frac{(a^2+b)^2}{a^2+b^2}=a^2-b^2+2b+\frac{b^2-2b^3+b^4}{a^2+b^2}.$$

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1. $\dfrac{3x^2y^3}{4a^4}\cdot\dfrac{8a^3}{12xy}=\dfrac{xy^2}{2a}.$

2. $\dfrac{10a^3b^2}{15c^3}\cdot\dfrac{18c}{7a^2}=\dfrac{12ab^2}{7c^2}.$

3. $\dfrac{3x}{8y}\cdot\dfrac{6ax}{9b}\cdot\dfrac{12b^2y}{16x}=\dfrac{3abx}{16}.$

4. $\dfrac{14m^2n^4}{5n}\cdot\dfrac{25n^3}{6m^2}\cdot\dfrac{4x}{7m}=\dfrac{20n^6x}{3m}.$

5. $\left(\dfrac{2x}{3ay}\right)^2\cdot\dfrac{12a^3}{16b^2xy^3}\cdot 8b^4y^2=\dfrac{8ab^2x}{3y^3}.$

6. $\dfrac{a}{4c^2}\cdot\left(\dfrac{c}{2a}\right)^3\cdot 16=\dfrac{c}{2a^2}.$

7. $\left(\dfrac{2a}{c}\right)^2\cdot\left(\dfrac{2c}{a}\right)^3\cdot\left(\dfrac{1}{4}\right)^2=\dfrac{2c}{a}.$

8. $\left(\dfrac{-3a}{3}\right)^2\cdot\dfrac{c^5}{(2a)^5}\cdot\left(\dfrac{2}{3}\right)^2=\dfrac{c^5}{72a^3}.$

9. $\dfrac{6c}{a}\cdot\left(\dfrac{9c^2}{4a}\right)^2\cdot\left(\dfrac{-2a}{3c}\right)^3=-9c^2.$

10. $\dfrac{(4ax)^2}{225c^4}\cdot\dfrac{(5ac^2)^3}{(2x^2)^3}\cdot\left(\dfrac{-x}{c}\right)^5=\dfrac{-10a^5x}{9c^3}.$

11. $\dfrac{2a+4x}{5y^2}\cdot\dfrac{15y}{a+2x}=\dfrac{6}{y}.$

12. $\dfrac{3c}{9-c^2}\cdot\dfrac{c^2+5c+6}{18cd}=\dfrac{3c}{(3+c)(3-c)}\cdot\dfrac{(c+2)(c+3)}{18cd}=\dfrac{c+2}{18d-6cd}.$

13. $\dfrac{3a-6y}{4a+2y}\cdot\dfrac{8(2a+y)^2}{4a^2-24ay+24y^2}=\dfrac{3(a-2y)}{2(2a+y)}\cdot\dfrac{8(2a+y)^2}{4(a^2-6ay+6y^2)}$

$=\dfrac{6a^2-9ay-6y^2}{a^2-6ay+6y^2}.$

14. $\dfrac{c^2+6ce+9e^2}{6(c+d)^2}\cdot\dfrac{3c^2e-12d^2e}{c^2-2cd+3ce-6de}=\dfrac{(c+3e)^2\,3e(c+2d)(c-2d)}{6(c+d)^2(c-2d)(c+3e)}$

$=\dfrac{c^2e+2cde+3ce^2+6de^2}{2(c+d)^2}.$

15. $\dfrac{a^2+4ab+4b^2}{9-a^2}\cdot\dfrac{a^2-5a+6}{a^2-4b^2}=\dfrac{(a+2b)^2}{(3+a)(3-a)}\cdot\dfrac{(a-2)(a-3)}{(a-2b)(a+2b)}$

$=\dfrac{(a+2b)(a-2)}{(a+3)(2b-a)}$

$=\dfrac{2a-2ab+4b-a^2}{a^2-2ab+3a-6b}.$

16. $\dfrac{5c^2-20d^2}{c^3+8d^3}\cdot\dfrac{c^2-2cd+4d^2}{25cd^4}$

$=\dfrac{5(c+2d)(c-2d)}{(c+2d)(c^2-2cd+4d^2)}\cdot\dfrac{c^2-2cd+4d^2}{25cd^4}$

$=\dfrac{c-2d}{5cd^4}.$

17. $$\frac{(x+2)^2}{3x^3+6x^2+12x}\cdot(15-9x^2)\cdot\frac{x^3-8}{4-x^2}$$
$$=\frac{(x+2)^2}{3x(x^2+2x+4)}\cdot\frac{3(5-3x^2)}{1}\cdot\frac{(x-2)(x^2+2x+4)}{(2-x)(2+x)}$$
$$=\frac{3x^3+6x^2-5x-10}{x}.$$

18. $$\frac{8c-24d}{c^2-6cd+9d^2}\cdot\left(4c-\frac{d^2}{c}\right)\cdot\frac{4c^2+d^2}{64c^4-4d^4}$$
$$=\frac{8(c-3d)}{(c-3d)^2}\cdot\frac{(2c+d)(2c-d)}{c}\cdot\frac{4c^2+d^2}{4(4c^2+d^2)(4c^2-d^2)}$$
$$=\frac{2}{c^2-3cd}.$$

19. $$\left(\frac{6m-9}{2m-3}+2m\right)\left(2m-9+\frac{36}{2m+3}\right)=\frac{(2m+3)(2m-3)}{2m-3}\cdot\frac{(2m-3)^2}{(2m+3)}$$
$$=4m^2-12m+9.$$

20. $$\left(\frac{3}{2a}-1\right)^2\cdot\frac{8a^2x}{9-4a^2}\cdot\frac{3+2a}{2a-3}=\frac{(3-2a)^2}{4a^2}\cdot\frac{8a^2x}{(3-2a)(3+2a)}\cdot\frac{3+2a}{2a-3}$$
$$=-2x.$$

21. $$\left(1+\frac{2}{x}-\frac{3}{x^2}\right)\cdot\frac{9x^3}{3(x^2-18x+17)}=\frac{(x+3)(x-1)}{x^2}\cdot\frac{9x^3}{3(x-1)(x-17)}$$
$$=\frac{3x^2+9x}{x-17}.$$

22. $$\frac{x^3-8}{6xy}\cdot\left(3-\frac{4}{2-x}\right)\cdot\frac{18x^2y^2}{x^2+2x+4}$$
$$=\frac{(x-2)(x^2+2x+4)}{6xy}\cdot\frac{2-3x}{2-x}\cdot\frac{18x^2y^2}{x^2+2x+4}$$
$$=9x^2y-6xy.$$

23. $$\frac{a^3-b^3}{(a-b)^3}\cdot\frac{6a+6b}{2a^2+2ab+2b^2}\cdot\frac{(a-b)^2}{a^2-b^2}$$
$$=\frac{(a-b)(a^2+ab+b^2)}{(a-b)^3}\cdot\frac{6(a+b)}{2(a^2+ab+b^2)}\cdot\frac{(a-b)^2}{(a+b)(a-b)}$$
$$=\frac{3}{a-b}.$$

24. $$\left(4-\frac{20-9x}{5-x}-x\right)\left(\frac{1}{x}+\frac{1}{x^2}-\frac{30}{x^3}\right)=\frac{x^2}{5-x}\cdot\frac{(x+6)(x-5)}{x^3}=-\frac{x+6}{x}.$$

25. $$\left(x+2y-\frac{5x+10y}{x+y}\right)\left(\frac{y^2+x^2+2yx}{x^2-3xy-4y^2}\right)\cdot\frac{1}{x+2y}$$
$$=\frac{(x+2y)(x+y-5)}{x+y}\cdot\frac{(x+y)^2}{(x-4y)(x+y)}\cdot\frac{1}{(x+2y)}=\frac{x+y-5}{x-4y}.$$

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1. $\frac{6a}{3b} \div \frac{4a}{12b^2} = \frac{6a}{3b} \cdot \frac{12b^2}{4a} = 6b.$

2. $\frac{9a^2}{6b^2} \div \frac{14ab}{8x} = \frac{9a^2}{6b^2} \cdot \frac{8x}{14ab} = \frac{6ax}{7b^3}.$

3. $\left(\frac{2ax}{3c}\right)^2 \div \frac{6ax^2}{9c^4} = \left(\frac{2ax}{3c}\right)^2 \cdot \frac{9c^4}{6ax^2} = \frac{2ac^2}{3}.$

4. $\frac{a}{b} \div \frac{c}{d} \div \frac{ad^2}{bc^2} = \frac{a}{b} \cdot \frac{d}{c} \cdot \frac{bc^2}{ad^2} = \frac{c}{d}.$

5. $2a \cdot \left(\frac{2a^2}{3x}\right)^2 \div \frac{(4a^2)^2}{18x^2} = 2a \cdot \frac{(2a^2)^2}{(3x)^2} \cdot \frac{18x^2}{(4a^2)^2} = a.$

6. $\frac{5x}{5y} \div \frac{10x^2}{12y^3} \div \frac{4ax}{3by^2} = \frac{5x}{5y} \cdot \frac{12y^3}{10x^2} \cdot \frac{3by^2}{4ax} = \frac{9by^4}{10ax^2}.$

7. $\frac{6}{(2x)^2} \div \frac{12}{(4x^2)^3} \cdot \left(\frac{2}{x}\right)^2 = \frac{6}{(2x)^2} \cdot \frac{(4x^2)^3}{12} \cdot \frac{2^2}{x^2} = 32x^2.$

8. $\frac{3m^2}{5n} \div \frac{21m^3}{10mn^2} \div 14m^4n = \frac{3m^2}{5n} \cdot \frac{10mn^2}{21m^3} \cdot \frac{1}{14m^4n} = \frac{1}{49m^4}.$

9. $\left(\frac{4x^2}{3a}\right)^2 \div \left(\frac{2x}{3a}\right)^3 \cdot \left(\frac{a}{x}\right)^2 = \frac{(4x^2)^2}{(3a)^2} \cdot \frac{(3a)^3}{(2x)^3} \cdot \frac{a^2}{x^2} = \frac{6a^3}{x}.$

10. $\frac{6a-3}{5x} \div \frac{2a-1}{15bx^3} = \frac{3(2a-1)}{5x} \cdot \frac{15bx^3}{2a-1} = 9bx^2.$

11. $\frac{4-x^2}{3x^2+x^3} \div \frac{4-4x+x^2}{x^3-x^2-12x} = \frac{(2-x)(2+x)}{x^2(3+x)} \cdot \frac{x(x-4)(x+3)}{(2-x)^2}$

$= \frac{x^2-2x-8}{2x-x^2}.$

12. $\frac{a^2-7a+12}{a-1} \div \frac{a^2-16}{1-a^2} = \frac{(a-3)(a-4)}{a-1} \cdot \frac{(1-a)(1+a)}{(a+4)(a-4)}$

$= -\frac{a^2-2a-3}{a+4}.$

13. $(-14-5c+c^2) \div \frac{ac^2-49a}{bc^2+9bc+14b} = \frac{(c-7)(c+2)}{1} \cdot \frac{b(c+2)(c+7)}{a(c+7)(c-7)}$

$= \frac{bc^2+4bc+4b}{a}.$

14. $\frac{4a^2-4ab-3b^2}{8a^3x} \div \left(a - \frac{9b^2}{4a}\right) = \frac{(2a-3b)(2a+b)}{8a^3x} \cdot \frac{4a}{(2a-3b)(2a+3b)}$

$= \frac{2a+b}{4a^3x+6a^2bx}.$

15. $\left(\frac{2a}{x}\right)^4 \div \left(\frac{-6a^2c}{5x}\right)^3 \cdot \left(\frac{3c^3}{10ax}\right)^2 = \left(\frac{2a}{x}\right)^4 \cdot \left(\frac{5x}{-6a^2c}\right)^3 \cdot \left(\frac{3c^3}{10ax}\right)^2 = \frac{-5c^3}{6a^4x^3}.$

16. $\left(\frac{-a^2}{x}\right) \div \left(\frac{-a}{x^2}\right)^5 \cdot \left(\frac{a}{x}\right)^3 \cdot \left(\frac{-1}{x^2}\right)^3 = \frac{-a^2}{x} \cdot \left(\frac{x^2}{-a}\right)^5 \cdot \frac{a^3}{x^3} \cdot \frac{-1}{x^6} = -1.$

17. $$\left(\frac{m^2}{n^2} - \frac{n^2}{m^2}\right) \div \left(\frac{m^4 + 2m^2n^2 + n^4}{m^2n - 4mn^2}\right) = \frac{m^4 - n^4}{m^2n^2} \cdot \frac{mn(m - 4n)}{(m^2 + n^2)^2}$$
$$= \frac{(m^2 - n^2)(m - 4n)}{mn(m^2 + n^2)}$$
$$= \frac{m^3 - 4m^2n - mn^2 + 4n^3}{m^3n + mn^3}.$$

18. $$\frac{9x^2 + 6xy - 8y^2}{2x + y} \div (3x - 2y)^2 \div \left(2 - \frac{x - 2y}{2x + y}\right)$$
$$= \frac{(3x + 4y)(3x - 2y)}{2x + y} \cdot \frac{1}{(3x - 2y)^2} \cdot \frac{2x + y}{3x + 4y}$$
$$= \frac{1}{3x - 2y}.$$

19. $$\left(\frac{a}{c} + \frac{c}{a}\right) \div \left(\frac{a^6 + c^6}{a^3c^3}\right)\left(a^2 - c^2 + \frac{c^4}{a^2}\right) \cdot \frac{1}{ac^2}$$
$$= \frac{a^2 + c^2}{ac} \cdot \frac{a^3c^3}{a^6 + c^6} \cdot \frac{a^4 - a^2c^2 + c^4}{a^2} \cdot \frac{1}{ac^2} = \frac{1}{a}.$$

20. $$\frac{2a - 5}{2a^2 + 2} \div \frac{4a^4 - 25a^2}{4 - 4a^4} \cdot \left(3a^2 - \frac{a^3 - 8a^2}{a - 1}\right)$$
$$= \frac{2a - 5}{2(a^2 + 1)} \cdot \frac{4(1 - a^4)}{a^2(2a + 5)(2a - 5)} \cdot \frac{a^2(2a + 5)}{a - 1}$$
$$= -2 - 2a.$$

21. $$\frac{8m^3 - 125n^3}{m^2 + mn} \div \left(\frac{2m}{5n} - \frac{5n}{2m}\right) \div \left[3mn\left(2m + 5n + \frac{25n^2}{2m}\right)\right]$$
$$= \frac{8m^3 - 125n^3}{m(m + n)} \cdot \frac{10mn}{(2m + 5n)(2m - 5n)} \cdot \frac{2m}{3mn(4m^2 + 10mn + 25n^2)}$$
$$= \frac{20}{6m^2 + 21mn + 15n^2}.$$

22. $$\left(\frac{4y}{x} - \frac{15y^2}{x^2} + 4\right) \div \left(4 - \frac{16y}{x} + \frac{15y^2}{x^2}\right)\left(3 - \frac{4x + 20y}{2x + 5y}\right)$$
$$= \frac{(2x + 5y)(2x - 3y)}{x^2} \cdot \frac{x^2}{(2x - 3y)(2x - 5y)} \cdot \frac{2x - 5y}{2x + 5y}$$
$$= 1.$$

23. $$\left(\frac{9c^2 - 4d^2}{6c^2}\right) \div \left(d + 4c + \frac{15c^2}{4d}\right) \cdot \frac{15c^2d + 6cd^2}{9c^2d + 24cd^2 - 20d^3}$$
$$= \frac{(3c + 2d)(3c - 2d)}{6c^2} \cdot \frac{4d}{(2d + 5c)(2d + 3c)} \cdot \frac{3cd(5c + 2d)}{d(3c + 10d)(3c - 2d)}$$
$$= \frac{2d}{3c^2 + 10cd}.$$

24. $\left(6x - 11 - \frac{7}{x}\right) \div \left(2 + \frac{11}{x} + \frac{5}{x^2}\right) \div \left(\frac{1}{3x^3 - 75x}\right)$

$= \frac{(3x-7)(2x+1)}{x} \cdot \frac{x^2}{(x+5)(2x+1)} \cdot \frac{3x(x+5)(x-5)}{1}$

$= 9x^4 - 66x^3 + 105x^2.$

25. $\left(x^2 - y^2 + \frac{4xy(y+x)}{x-y}\right) \div \frac{x^2 + y(y+2x)}{2x^2 - 3xy + y^2}$

$= \frac{(x+y)^3}{x-y} \cdot \frac{(x-y)(2x-y)}{(x+y)^2}$

$= 2x^2 + xy - y^2.$

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1. $\frac{4 - \frac{1}{4}}{2 + \frac{1}{2}} = \frac{\frac{15}{4}}{\frac{5}{2}} = \frac{15}{4} \cdot \frac{2}{5} = \frac{3}{2}.$

2. $\frac{\frac{4}{9} + 1}{\frac{7}{2} - 2} = \frac{\frac{13}{9}}{\frac{3}{2}} = \frac{13}{9} \cdot \frac{2}{3} = \frac{26}{27}.$

3. $\frac{9 - (\frac{1}{5})^2}{4 - \frac{4}{5}} = \frac{\frac{224}{25}}{\frac{16}{5}} = \frac{224}{25} \cdot \frac{5}{16} = \frac{14}{5}.$

4. $\frac{\frac{3}{5} + \frac{4}{7}}{2 - \frac{3}{5} \cdot \frac{4}{7}} = \frac{\frac{41}{35}}{\frac{58}{35}} = \frac{41}{35} \cdot \frac{35}{58} = \frac{41}{58}.$

5. $\frac{(\frac{8}{5})^2 - 2}{3 \cdot \frac{8}{5}} = \frac{\frac{14}{25}}{\frac{24}{5}} = \frac{14}{25} \cdot \frac{5}{24} = \frac{7}{60}.$

6. $\frac{4 - \frac{3}{2} + \frac{2}{3}}{3 - \frac{2}{3} + \frac{3}{2}} = \frac{\frac{19}{6}}{\frac{23}{6}} = \frac{19}{23}.$

7. $\frac{\frac{2^3}{27} + 1}{\frac{9}{2} - 1} = \frac{\frac{35}{27}}{\frac{7}{2}} = \frac{35}{27} \cdot \frac{2}{7} = \frac{10}{27}.$

8. $\frac{2\frac{1}{2} - 3\frac{1}{3} + 4\frac{3}{4}}{2\frac{1}{2} \cdot 3\frac{1}{3} - 4\frac{3}{4}} = \frac{\frac{47}{12}}{\frac{43}{12}} = \frac{47}{43}.$

9. $\frac{2 - \frac{9}{2}}{(\frac{1}{2})^2 - (\frac{1}{2})^3 - 12(\frac{1}{2})^4} = \frac{-\frac{5}{2}}{-\frac{5}{8}} = 4.$

10. $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$

11. $\frac{\frac{1+x}{x}}{1 - \frac{1}{x^2}} = \frac{1+x}{x} \cdot \frac{x^2}{x^2 - 1} = \frac{x}{x-1}.$

12. $\frac{c + \frac{c}{d}}{1 + \frac{1}{d}} = \frac{\frac{cd + c}{d}}{\frac{d+1}{d}} = \frac{cd + c}{d} \cdot \frac{d}{d+1} = c.$

13. $\frac{9 - \frac{b^2}{4a^2}}{1 - \frac{b}{6a}} = \frac{\frac{36a^2 - b^2}{4a^2}}{\frac{6a - b}{6a}} = \frac{36a^2 - b^2}{4a^2} \cdot \frac{6a}{6a - b} = \frac{18a + 3b}{2a}.$

14. $\frac{\frac{a^2}{b^2} - \frac{b^2}{a^2}}{a + \frac{b^2}{a}} = \frac{a^4 - b^4}{a^2b^2} \cdot \frac{a}{a^2 + b^2} = \frac{a^2 - b^2}{ab^2}.$

15. $\dfrac{a-\dfrac{9}{a}}{\dfrac{1}{a^2}-\dfrac{1}{a^3}-\dfrac{12}{a^4}}=\dfrac{a^2-9}{a}\cdot\dfrac{a^4}{(a+3)(a-4)}=\dfrac{a^4-3a^3}{a-4}.$

16. $\dfrac{\dfrac{m^3}{n^3}+1}{m+\dfrac{n^2}{m}-n}=\dfrac{m^3+n^3}{n^3}\cdot\dfrac{m}{m^2-mn+n^2}=\dfrac{m^2+mn}{n^3}.$

17. $\dfrac{2+\dfrac{1}{a-2}+a}{a+\dfrac{1}{a+2}-2}=\dfrac{a^2-3}{a-2}\cdot\dfrac{a+2}{a^2-3}=\dfrac{a+2}{a-2}.$

18. $\dfrac{\frac{a}{b}}{c}-\dfrac{a}{\frac{b}{c}}=\dfrac{a}{b}\cdot\dfrac{1}{c}-\dfrac{a}{1}\cdot\dfrac{c}{b}=\dfrac{a-ac^2}{bc}.$

19. $\dfrac{1+\dfrac{b}{a}-\dfrac{20a}{b}}{\dfrac{b}{a}-\dfrac{8a}{b}-2}=\dfrac{(b+5a)(b-4a)}{ab}\cdot\dfrac{ab}{(b-4a)(b+2a)}=\dfrac{b+5a}{b+2a}.$

20. $\dfrac{\dfrac{16}{x}-x}{\dfrac{24}{x^4}+\dfrac{10}{x^3}+\dfrac{1}{x^2}}=\dfrac{16-x^2}{x}\cdot\dfrac{x^4}{x^2+10x+24}=\dfrac{x^3(4-x)}{6+x}.$

21. $\dfrac{x+\dfrac{x^2+y^2}{y}}{\dfrac{x}{y}-1}=\dfrac{x^2+xy+y^2}{y}\cdot\dfrac{y}{x-y}=\dfrac{x^2+xy+y^2}{x-y}.$

22. $\dfrac{2+\dfrac{a}{b}}{\dfrac{(a-2b)^2}{4ab}+2}=\dfrac{2b+a}{b}\cdot\dfrac{4ab}{a^2+4ab+4b^2}=\dfrac{4a}{a+2b}.$

23. $\dfrac{\left(\dfrac{2x^2}{x-y}\right)^2}{\dfrac{8x^3}{(x-y)^3}}=\dfrac{4x^4}{(x-y)^2}\cdot\dfrac{(x-y)^3}{8x^3}=\dfrac{x(x-y)}{2}.$

24. $\dfrac{\dfrac{a-b}{a}-\dfrac{a+b}{b}}{\dfrac{a-b}{b}+\dfrac{a+b}{a}}=\dfrac{-(a^2+b^2)}{ab}\cdot\dfrac{ab}{a^2+b^2}=-1.$

25. $$1-\frac{\left(1-\frac{c^4}{9}\right)-\left(1-\frac{c^4}{16}\right)}{1-\frac{7c^4}{144}}=1-\frac{\frac{-7c^4}{144}}{\frac{144-7c^4}{144}}$$

$$=1+\frac{7c^4}{144-7c^4}$$

$$=\frac{144}{144-7c^4}.$$

26. $$\frac{\frac{a}{1+a}+\frac{1-a}{a}}{\frac{a}{1+a}-\frac{1-a}{a}}=\frac{\frac{1}{a(a+1)}}{\frac{2a^2-1}{a(a+1)}}=\frac{1}{2a^2-1}.$$

27. $$\frac{\frac{9x}{8}-\frac{9y}{8}-\frac{7y^2}{2x}}{\frac{3x}{4}+\frac{9y}{4}+\frac{5y^2}{3x}}=\frac{\frac{9x^2-9xy-28y^2}{8x}}{\frac{9x^2+27xy+20y^2}{12x}}$$

$$=\frac{(3x-7y)(3x+4y)}{8x}\cdot\frac{12x}{(3x+5y)(3x+4y)}$$

$$=\frac{9x-21y}{6x+10y}.$$

28. $$\frac{\frac{x}{2ax+3bx+6ab+x^2}}{\frac{1}{x+2a}-\frac{1}{x+3b}}=\frac{x}{(x+2a)(x+3b)}\cdot\frac{(x+2a)(x+3b)}{3b-2a}$$

$$=\frac{x}{3b-2a}.$$

29. $$\frac{\left(\frac{3a+4b}{3a}\right)^2-\frac{6b}{a}}{8a-\frac{(3a+2b)^2}{3b}}=\frac{(3a-8b)(3a-2b)}{9a^2}\cdot\frac{3b}{(2b-3a)(3a-2b)}$$

$$=\frac{b(3a-8b)}{3a^2(2b-3a)}.$$

30. $$\frac{1-\frac{x^2-1}{5x^2-6x+1}}{1+\frac{3x+2}{5x-1}}=\frac{\frac{4x^2-6x+2}{5x^2-6x+1}}{\frac{8x+1}{5x-1}}$$

$$=\frac{2(x-1)(2x-1)}{(5x-1)(x-1)}\cdot\frac{5x-1}{8x+1}$$

$$=\frac{4x-2}{8x+1}.$$

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1. $\frac{x}{2}+\frac{x}{3}=10.$ (1)

(1) · 6, $3x+2x=60.$

Whence $x=12.$

2. $\frac{4}{3}x+\frac{2}{5}x=5\frac{1}{5}.$ (1)

(1) · 15, $20x+6x=78.$

Whence $x=3.$

3. $\frac{x+5}{4}-\frac{2x+4}{9}=1.$ (1)

(1) · 36, $9(x+5)-4(2x+4)=36.$

Whence $x=7.$

4. $\frac{2x+3}{5}-\frac{1}{3}(x-3)=2.$ (1)

(1) · 15, $3(2x+3)-5(x-3)=30.$

Whence $x=6.$

5. $\frac{4x+2}{11}-\frac{1}{5}(x+5)=0.$ (1)

(1) · 55, $5(4x+2)-11(x+5)=0.$

Whence $x=5.$

6. $\frac{3}{4}(x+1)-\frac{5x-7}{6}=\frac{7}{3}.$ (1)

(1) · 12, $9(x+1)-2(5x-7)=28.$

Whence $x=-5.$

7. $\frac{x+6}{10}+\frac{3}{2}(x+4)=-3.$ (1)

(1) · 10, $x+6+15(x+4)=-30.$

Whence $x=-6.$

8. $\frac{5x-12}{6}-\frac{4}{11}(2x-7)=\frac{1}{3}.$ (1)

(1) · 66, $11(5x-12)-24(2x-7)=22.$

Whence $x=-2.$

9. $2x-1-\frac{12x-7}{6}-\frac{1}{6}=0.$ (1)

(1) · 6, $6(2x-1)-(12x-7)-1=0.$

Whence $0=0$, or any number will satisfy the equation.

10. $\frac{10x-7}{6}+\frac{5}{2}\left(\frac{2}{5}-x\right)=\frac{15x-11}{3}.$ (1)

(1) · 6, $(10x-7)+15(\frac{2}{5}-x)=2(15x-11).$

Whence $x=\frac{3}{5}.$

11. $\frac{5x}{6}-\frac{1}{2}-\frac{3}{8}\left(x-\frac{5}{3}\right)+\frac{7}{32}=0.$ (1)

(1) · 96, $80x-48-12(3x-5)+21=0.$

Whence $x=-\frac{3}{4}.$

12. $\frac{5}{x}+\frac{4}{3}=\frac{9}{x}.$ (1)

(1) · $3x$, $15+4x=27.$

Whence $x=3.$

13. $\frac{9x}{4}-\frac{3}{4}-\frac{3x-7}{3}+4=-\frac{17}{24}.$ (1)

(1) · 24, $54x-18-8(3x-7)+96=-17.$

Whence $x=-\frac{151}{30}.$

14. $\frac{c}{3x}-\frac{c}{5x}=\frac{1}{15}.$ **15.** $\frac{1}{2x}-\frac{13}{24}=\frac{8}{3x}.$

Whence $x=2c.$ **Whence** $x=-4.$

16. $\frac{2a-3x}{6a}+\frac{5a-2x}{5a}+\frac{41}{30}=0.$

Whence $x=3a.$

17. $\frac{x}{a}-\frac{1}{3}(a-3x)+\frac{19a}{3}=-6.$

Whence $x=-6a.$

18. $\frac{cx}{3}-\frac{n}{5}(3x-5cn)=cn\left(\frac{2c}{3}-\frac{n}{5}\right).$ (1)

(1) · 15, $5cx-9nx+15cn^2=10c^2n-3cn^2.$

Whence $x(5c-9n)=2cn(5c-9n),$

or $x=2cn.$

19. $2x-b-\frac{b}{a}(3x-4b)+2a=\frac{2(2a^2-b^2)}{a}.$

Whence $2ax-3bx=2a^2+ab-6b^2,$

or $x=a+2b.$

20. $(x+5)(x-6)=x(x-\tfrac{5}{2}).$

Whence $x^2-x-30=x^2-\frac{5x}{2},$

or $x=20.$

21. $\left(x+\frac{3}{4}\right)\left(3+\frac{x}{2}\right)=\frac{x}{2}(x-5)+8\tfrac{1}{8}.$ (1)

(1) · 8, $4\left(x+\frac{3}{4}\right)2\left(3+\frac{x}{2}\right)=4x(x-5)+65.$

Whence $x=1.$

22. $(x-\tfrac{1}{2})(x+\tfrac{3}{7})=(x-1)(x+2)+1\tfrac{11}{14}.$ (1)

(1) · 14, $2(x-\tfrac{1}{2})7(x+\tfrac{3}{7})=14(x^2+x-2)+25.$

Whence $x=0.$

23. $(x-\tfrac{3}{2})(x+\tfrac{2}{3})-(x-\tfrac{1}{3})^2-1\tfrac{1}{9}=0.$ (1)

(1) · 18, $6(x-\tfrac{3}{2})3(x+\tfrac{2}{3})-18(x-\tfrac{1}{3})^2-20=0.$

Whence $x=-\tfrac{40}{3}.$

24. $(x+\tfrac{2}{5})^2-(x-\tfrac{1}{2})(x+\tfrac{1}{5})+\tfrac{31}{50}=0.$ (1)

(1) · 50, $2\cdot 25(x+\tfrac{2}{5})^2-10(x-\tfrac{1}{2})(x+\tfrac{1}{5})5+31=0.$

Whence $x=-\tfrac{4}{5}.$

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1. If $n =$ the number,

then $\frac{n}{4} + \frac{n}{12} = 16.$

Whence $n = 48.$

2. If $n =$ the number,

then $\frac{n}{3} - \frac{n}{17} = 70.$

Whence $n = 255.$

3. If $n =$ the greater number,
then $38 - n =$ the less,

and $\frac{n}{10} = \frac{38 - n}{9}.$

Whence $n = 20,$
and $38 - n = 18.$

4. If $l =$ the length in centimeters,

then $\frac{4l}{5} =$ the width,

and $2l + \frac{8l}{5} = 216.$

Whence $l = 60,$

and $\frac{4l}{5} = 48.$

Therefore the area of the rectangle is 2880 square centimeters.

5. Let $n =$ the number.

Then $\frac{4 + n}{7} = \frac{n}{5}.$

Whence $n = 10.$

6. Let n, $n + 1$, and $n + 2$ represent the numbers.

Then $\frac{3n}{4} = \frac{n + 1 + n + 2}{3}.$

Whence $n = 12.$

7. Let n, $n + 2$, and $n + 4$ represent the numbers.

Then $\frac{n}{11} = \frac{n + 2 + n + 4}{24}.$

Whence $n = 33,$
$n + 2 = 35,$
and $n + 4 = 37.$

8. If $n =$ the number,

then $\frac{10 + n}{17 + n} = \frac{5}{7}.$

Whence $n = \frac{15}{2}.$

9. Let $g =$ the greater part.
Then $42 - g =$ the less,

and $\frac{g - (42 - g)}{6} = \frac{1}{3}.$

Whence $g = 22,$
and $42 - g = 20.$

10. Let $n =$ the number.

Then $\frac{3n - 4}{4} = \frac{5n - 4}{7}.$

Whence $n = 12.$

11. Let $g =$ the greater part.
Then $112 - g =$ the less,

and $\frac{112 - g}{g} = \frac{2}{9}.$

Whence $g = 91\frac{7}{11},$
and $112 - g = 20\frac{4}{11}.$

12. Let $n =$ one number.
Then $24 - n =$ the other,
and

$$\frac{n - (24 - n)}{n + 24 - n} = n - (24 - n) - 3\tfrac{5}{6}.$$

Whence $n = 14,$
and $24 - n = 10.$

13. Let $n =$ the number.

Then $\frac{27 + 7n}{2n} = \frac{90 + 5n}{3n}.$

Whence $n = 9.$

14. Let $x =$ B's age in years.

Then $\frac{5x}{2} =$ A's age in years,

and $\frac{5x}{2} + 10 = 2(x + 10)$.

Whence $x = 20$,

and $\frac{5x}{2} = 50$.

15. Let $x =$ B's age in years.

Then $\frac{2x}{3} =$ A's age in years,

and $\frac{2x}{3} - 14 = \frac{1}{2}(x - 14)$.

Whence $x = 42$,

and $\frac{2x}{3} = 28$.

16. Let $x =$ B's age in years.

Then $x + 16 =$ A's age in years,

and $x - 8 = \frac{3}{5}(x + 16 - 8)$.

Whence $x = 32$,

and $x + 16 = 48$.

17. Let $x =$ the number of moons of Uranus.

Then $x + 4 =$ the number of moons of Jupiter,

$2x + 2 =$ the number of moons of Saturn,

$x - 2 =$ the number of moons of Mars,

and $\frac{x-2}{2} =$ the number of moons of Neptune.

Then $x + x + 4 + 2x + 2 + x - 2 + \frac{x-2}{2} = 25$.

Whence $x = 4$,

$x + 4 = 8, \quad 2x + 2 = 10, \quad x - 2 = 2,$

and $\frac{x-2}{2} = 1$.

18. Let $x =$ the other base of the trapezoid in meters.

Then $\frac{8 \cdot 30}{2} = \frac{(8 + x)10}{2}$.

Whence $x = 16$.

19. Let $d =$ the distance to the target in feet.

Then $\frac{d}{1100} + \frac{d}{1925} = 3$.

Whence $d = 2100$,

and $\frac{2100}{1925} = \frac{12}{11}$.

Therefore $\frac{12}{11}$ seconds is the time the bullet was in the air.

20. Let $v =$ the velocity of the bullet in feet per second.

Then $\frac{3 \cdot 2640}{v} + \frac{3 \cdot 2640}{1100} = 9\frac{2}{5}$.

Whence $v = 3600$.

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1. $\frac{32}{x} = 5.$

Then $32 = 5x,$

or $x = \frac{32}{5}.$

2. $\frac{25}{3x} = 5.$

Then $5 = 3x,$

or $x = \frac{5}{3}.$

3. $5x + \frac{x-2}{4} = 4x + 7.$

Then $20x + x - 2 = 16x + 28,$

or $x = 6.$

4. $3x - \frac{x-1}{4} - \frac{8x}{5} = 6.$

Then $60x - 5x + 5 - 32x = 120,$

or $x = 5.$

5. $\frac{4}{x} - \frac{12+x}{3x} = \frac{4}{3}.$

Then $x = 0,$

or the equation is "impossible," for division by zero is excluded.

6. $\frac{5}{6} - \frac{3x-5}{4x} + \frac{x-2}{3x} = 0.$

Then

$10x - 9x + 15 + 4x - 8 = 0,$

or $x = -\frac{7}{5}.$

7. $\frac{x+5}{5x} - \frac{3(x+1)}{x} = 3\frac{1}{5}.$

Then

$x + 5 - 15x - 15 = 16x,$

or $x = -\frac{1}{3}.$

8. $5x - 4x\left(3 - \frac{2}{x}\right) + \frac{1}{4} = 3.$

Then

$20x - 48x + 32 + 1 = 12,$

or $x = \frac{3}{4}.$

9. $x - \frac{3x}{5}\left(\frac{10}{x} - 4\right) + 3\frac{3}{5} = 18,$

or $5x - 30 + 12x + 18 = 90.$

Then $x = 6.$

10. $\frac{8x-7}{4x} + 3\frac{1}{2} = \frac{\frac{3}{2} - x}{x}.$

Then $8x - 7 + 14x = 6 - 4x,$

or $x = \frac{1}{2}.$

11. $\frac{2x-3}{3+2x} = 4,$

or $2x - 3 = 12 + 8x.$

Then $x = -\frac{5}{2}.$

12. $\frac{x-2}{x-3} = \frac{15}{16}.$

Then $16(x-2) = 15(x-3),$

or $x = -13.$

13. $\frac{5}{7x+5} - \frac{1}{8} = 0,$

or $40 - 7x - 5 = 0.$

Then $x = 5.$

14. $\frac{3x^2 - 7x - 4}{4x^2 - 10x - 8} = \frac{3}{4}.$

Then $6x^2 - 14x - 8 = 6x^2 - 15x - 12,$

or $x = -4.$

15. $\frac{1}{x-2} = \frac{3}{x-3},$

or $x - 3 = 3(x-2).$

Then $x = \frac{3}{2}.$

16. $$\frac{1}{x-3}=\frac{x-6}{x+3}+\frac{x}{x+3}.$$

Then $$x+3=x^2-9x+18+x^2-3x.$$

Whence $$x=\tfrac{3}{2}\text{ or }5.$$

17. $$\frac{x-3}{x+4}=\frac{x-9}{x+5},$$

or $$(x-3)(x+5)=(x+4)(x-9).$$

Then $$x=-3.$$

18. $$\frac{x-2}{x+2}+\frac{4}{x+2}+2=0,$$

or $$x-2+4+2x+4=0.$$

Then $$x=-2.$$

Since $x=-2$ the equation is "impossible," for division by zero is excluded.

19. $$\frac{3x}{4}-\frac{2}{x-2}=\frac{3x-2}{4},$$

or $$3x(x-2)-8=(3x-2)(x-2).$$

Then $$x=6.$$

20. $$\frac{4}{x-3}+\frac{3x+4}{6}=\frac{x}{2},$$

or $$24+(3x+4)(x-3)=3x(x-3).$$

Then $$x=-3.$$

21. $$\frac{2x+3}{x-5}+4=\frac{x+8}{x-5},$$

or $$2x+3+4(x-5)=x+8.$$

Then $$x=5.$$

Since $x=5$, the equation is "impossible," for division by zero is excluded.

22. $$\frac{x}{4}-\frac{5}{4x-12}=\frac{2x+\frac{5}{4}}{3},$$

or $$3x(x-3)-15=4(x-3)(2x+\tfrac{5}{4}).$$

Then $$5x^2-10x=0,$$

and $$x=2\text{ or }0.$$

23. $$\frac{x-4}{x+5}+\frac{7}{5}=\frac{3}{x+5},$$

or $$5(x-4)+7(x+5)=15.$$

Then $$x=0.$$

24.
$$\frac{x+4}{15x-5}-\frac{x}{5}=\frac{2x+\frac{20}{3}}{10}.$$

Then $36x^2+42x-44=0,$

or $(3x-2)(6x+11)=0.$

Therefore $x=\frac{2}{3}$ or $-\frac{11}{6}.$

25.
$$\frac{4}{3x+6}+\frac{5}{7x+14}+43=0,$$

or $28+15+43\cdot 21\cdot (x+2)=0.$

Then $x=-\frac{43}{21}.$

26.
$$\frac{7}{4x-12}+\frac{47}{220}=\frac{-3}{5x-15},$$

or $7(55)+47(x-3)=-132.$

Then $x=-8.$

27.
$$\frac{4x}{x+3}-\frac{6}{2x+6}=\frac{10x+11}{3x+9},$$

or $24x-18=(10x+11)2.$

Then $x=10.$

28.
$$\frac{3}{x-2}=\frac{5}{x^2-25}+\frac{3x}{x^2-25},$$

or $3(x^2-25)=5(x-2)+3x(x-2).$

Then $x=65.$

29.
$$\frac{1}{x-3}+\frac{2}{x+3}=\frac{-3}{x^2-9},$$

or $x+3+2(x-3)=-3.$

Then $x=0.$

30.
$$\frac{x+2}{x-2}=\frac{10-x^2}{4-x^2}-\frac{10}{x^2-4},$$

or
$$\frac{x+2}{x-2}=\frac{x^2-20}{x^2-4},$$

or $(x+2)(x+2)=x^2-20.$

Then $x=-6.$

31.
$$\frac{x-4}{x-5}+\frac{x-15}{x+4}=\frac{2x^2-10x-1}{x^2-x-20},$$

or $(x-4)(x+4)+(x-15)(x-5)$
$$=2x^2-10x-1.$$

Then $x=6.$

32.
$$\frac{x+2}{x+3}+\frac{x+3}{x+2}=\frac{4x+9}{x^2+5x+6},$$

or $(x+2)^2+(x+3)^2=4x+9,$

or $(x+2)(x+1)=0.$

Then $x=-1,$

for -2 does not satisfy, since division by zero is excluded.

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1. $.3x + 4 = .25.$ (1)

$[(1) \cdot 100] \div 30,$ $x = -\frac{25}{2}.$

2. $.15x - .4x = 235x - 2352.5.$ (1)

$[(1) \cdot 100,$ collected$] \div -23{,}525,$ $x = 10.$

3. $1.3x + 8.24 = -5.26 - 3.2x.$ (1)

$[(1) \cdot 100,$ collected$] \div 450,$ $x = -3.$

4. $3x - 1.245x + .6x = 1.5 + .355x.$ (1)

$[(1) \cdot 1000,$ collected$] \div 2000,$ $x = .75.$

5. $3.5x + .0564 - .1x = 4.9128 - .02x.$ (1)

$[(1) \cdot 10{,}000,$ collected$] \div 34{,}200,$ $x = 1.42.$

6. $.12(2x + .05) - .15(1.5x - 2) = 0.246.$ (1)

$.24x + .006 - .225x + .30 = .246.$ (2)

$[(2) \cdot 1000,$ collected$] \div 15,$ $x = -4.$

7. $$\frac{.01x + .003}{6} + \frac{.02x + .0008}{7} = .0017. \quad (1)$$

$(1) \cdot 10{,}000,$ $$\frac{100x + 30}{6} + \frac{200x + 8}{7} = 17. \quad (2)$$

$[(2) \cdot 42] \div 1900,$ $x = .24.$

8. $$\frac{.3(x + 5)}{8} - \frac{4(.25x - .35)}{7} = \frac{14.325}{56}. \quad (1)$$

$(1) \cdot 1000,$ $$\frac{300(x + 5)}{8} - \frac{4(250x - 350)}{7} = \frac{14325}{56}. \quad (2)$$

$[(2) \div 56] \div -5900,$ $x = \frac{5}{4}.$

9. $$\frac{0.5(6 - .2x)}{.80} - \frac{.3(.4x - 3)}{.16} = 5. \quad (1)$$

100 (numerator and denominator of each fraction),

$$\frac{5(60 - 2x)}{80} - \frac{3(4x - 30)}{16} = 5. \quad (2)$$

$(2) \cdot 80$ and solved gives $x = 5.$

10. $$\frac{.32x}{.05} + \frac{.045x}{.125} = 13.52. \quad (1)$$

(First numerator and denominator) · 100, and (second numerator and denominator) · 1000,

$$\frac{32x}{5} + \frac{45x}{125} = 13.52. \quad (2)$$

$[(2) \cdot (125)] \div 845,$ $x = 2.$

11. $$\frac{33}{x+5} + \frac{3.75}{.5(x-8.5)} = 0. \quad (1)$$

(Numerator and denominator of second fraction) · 100,

$$\frac{33}{x+5} + \frac{375}{5(10x-85)} = 0. \quad (2)$$

$[(2) \cdot (x+5)(10x-85) \cdot 5] \div 2025, \qquad x = 6.$

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1. $5cx - 8c^2 = 4c^2 - cx.$

$6cx = 12c^2.$

Therefore $x = 2c.$

2. $2(x+1) - 4k = 2.$

$2x = 4k.$

Therefore $x = 2k.$

3. $3(2x - a) = 2(x - 2a).$

$6x - 3a = 2x - 4a.$

Therefore $x = -\frac{a}{4}.$

4. $ax + bx = a^2 + ab.$

$x(a+b) = a(a+b).$

Therefore $x = a.$

5. $cx + b^2 = bx + bc.$

$x(c-b) = b(c-b).$

Therefore $x = b.$

6. $mx + n^2 = m^2 - nx.$

$x(m+n) = (m^2 - n^2).$

Therefore $x = m - n.$

7. $6ac + cx + 4a^2 = 2ax + 3c^2 + 2ca.$

$x(c - 2a) = 3c(c - 2a) + 2a(c - 2a).$

Therefore $x = 3c + 2a.$

8. $5ax - 5a^2 + 6b^2 = 7ab + 3bx.$

$x(5a - 3b) = (5a - 3b)(a + 2b).$

Therefore $x = a + 2b.$

9. $\frac{x}{2a} = b.$

Therefore $x = 2ab.$

10. $\frac{3ab}{x} = a.$

Therefore $x = 3b.$

11. $\frac{a}{x} + \frac{3a}{2x} = \frac{5}{4}.$

$4a + 6a = 5x.$

Therefore $x = 2a.$

12. $\frac{4a}{3x} + \frac{4a}{x} = \frac{3}{2} + \frac{5a}{6x}.$

$8a + 24a = 9x + 5a.$

Therefore $x = 3a.$

13. $\frac{x}{a} + \frac{x}{b} = a + b.$

$x(a+b) = ab(a+b).$

Therefore $x = ab.$

14. $\frac{c^2}{x} - c = \frac{d^2}{x} + d.$

$-x(c+d) = d^2 - c^2.$

Therefore $x = c - d.$

15. $\frac{ax}{2b} - 4b^2 = \frac{2bx}{a} - a^2.$

$x(a^2 - 4b^2) = -2ab(a^2 - 4b^2).$

Therefore $x = -2ab.$

16. $$\frac{x}{c}+\frac{c-2x}{3}-3c=-4.$$

$$x(3-2c)=-4c(3-2c).$$

Therefore $$x=-4c.$$

17. $$\frac{x}{a}+\frac{x}{c}+ac=bc+ab+\frac{x}{b}.$$

$$x(bc+ab-ac)=abc(bc+ab-ac).$$

Therefore $$x=abc.$$

18. $$\frac{3b+4x}{5b}+\frac{3b+2x}{4b}=\frac{1}{20}.$$

$$26x=b-27b.$$

Therefore $$x=-b.$$

19. $$\frac{ax}{2}-\frac{3b}{5}\left(x-\frac{2ab}{3}\right)=ab\left(\frac{a}{2}-\frac{b}{5}\right).$$

$$5ax-6bx=5a^2b-6ab^2.$$

Therefore $$x=ab.$$

20. $$\frac{2x-3b}{a}+\frac{2}{b}\left(\frac{3x}{2}-a\right)+5+\frac{9b}{a}=\frac{4a}{b}.$$

$$x(3a+2b)=6a^2-5ab-6b^2.$$

Therefore $$x=2a-3b.$$

21. $$\frac{x-m^2}{x-n^2}=\frac{n}{m}.$$

$$x(m-n)=m^3-n^3.$$

Therefore $$x=m^2+mn+n^2.$$

22. $$\frac{x}{c}-\frac{d}{c}=\frac{x+2c}{d}-3.$$

$$x(d-c)=d^2-3cd+2c^2.$$

Therefore $$x=d-2c.$$

23. $$\frac{b}{b(b-x)}+\frac{3}{a(b-x)}+\frac{3a+9}{2ab}=0.$$

$$2ab+6b+3ab+9b-3ax-9x=0.$$

Therefore $$x=\frac{5b}{3}.$$

24. $$\frac{c}{a(x+c)}+\frac{a}{c(x-a)}=\frac{c^2-ac+2a^2}{2ac(x-a)}.$$

$$2c^2x-2ac^2+2a^2x+2a^2c=c^2x-acx+2a^2x+c^3-ac^2+2a^2c.$$

$$x(c+a)=c^2+ac.$$

Therefore $$x=c.$$

25.
$$\frac{1}{ab} + 1 - \frac{ab}{x} + \frac{1}{abx} = 0.$$
$$x + abx - a^2b^2 + 1 = 0.$$
Therefore
$$x = ab - 1.$$

26.
$$\frac{a^2}{bx} + \frac{b^2}{ax} = \frac{1}{a} + \frac{1}{b}.$$
$$a^3 + b^3 = x(a + b).$$
Therefore
$$x = a^2 - ab + b^2.$$

27.
$$\frac{c^2}{dx} - \frac{d^2}{cx} - \frac{3c - 3d}{x} = \frac{c - d}{cd}.$$
$$x(c - d) = c^3 - 3c^2d + 3cd^2 + d^3.$$
Therefore
$$x = c^2 - 2cd + d^2.$$

28.
$$\frac{a + x}{b + x} - \frac{a - x}{b - x} = \frac{2}{x^2 - b^2}.$$
$$(a + x)(b - x) - (a - x)(b + x) = -2.$$
Whence
$$x = \frac{1}{a - b}.$$

29.
$$\frac{\dfrac{x}{a - b} - a}{\dfrac{x}{a - b} + a} + 1 = \frac{2}{a^2 + ab + 1}.$$
$$\frac{x - a^2 + ab}{x + a^2 - ab} + 1 = \frac{2}{a^2 + ab + 1}.$$
$$2a^2 + 2abx + 2x = 2x + 2a^2 - 2ab,$$
or
$$ax + bx = a - b.$$
Therefore
$$x = \frac{a - b}{a + b}.$$

30.
$$\frac{\dfrac{a}{2} + \dfrac{x}{3}}{\dfrac{a}{3} - \dfrac{x}{2}} + \frac{7a}{4a - 6x} = -7.$$
$$\frac{3a + 2x}{2a - 3x} + \frac{7a}{2(2a - 3x)} = -7,$$
or
$$(3a + 2x)2 + 7a = -14(2a - 3x).$$
Therefore
$$x = \frac{41a}{38}.$$

31.
$$\frac{a^2 + ac}{x + 3c} + \frac{2cx(a + c)}{x^2 + 5cx + 6c^2} = \frac{a^2 + 2ac + c^2}{x + 2c}.$$
$$(a^2 + ac)(x + 2c) + 2acx + 2c^2x = (a^2 + 2ac + c^2)(x + 3c).$$
Therefore
$$x = a + 3c.$$

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1. Solve for R, $K = 2\pi RH$.

$$R = \frac{K}{2\pi H}.$$

2. Solve for a, $A = \frac{ab}{2}$.

$$a = \frac{2A}{b}.$$

3. Solve for R, $C = 2\pi R$.

$$R = \frac{C}{2\pi}.$$

4. Solve for r and t, $d = rt$.

$$r = \frac{d}{t},\ t = \frac{d}{r}.$$

5. Solve for a and A, $\frac{a}{A} = \frac{D}{360}$.

$$a = \frac{AD}{360},\ A = \frac{360\,a}{D}.$$

6. Solve for C, $\frac{D}{360} = \frac{l}{C}$.

$$C = \frac{360\,l}{D}.$$

7. Solve for r, $C = \frac{E}{R + r}$.

$$r = \frac{E - CR}{C}.$$

8. Solve for r and n, $C = \frac{E}{R + nr}$.

$$r = \frac{E - CR}{Cn},\ n = \frac{E - CR}{Cr}.$$

9. Solve for r and n, $C = \frac{n \cdot e}{R + nr}$.

$$r = \frac{n \cdot e - CR}{Cn},$$

$$n = \frac{CR}{e - Cr}.$$

10. Solve for F, $C = \frac{5}{9}(F - 32)$.

$$F = \frac{9C}{5} + 32.$$

11. Solve for W_2, $\frac{W_1}{W_2} = \frac{L_1}{L_2}$.

$$W_2 = \frac{W_1 L_2}{L_1}.$$

12. Solve for r and t, $A = P(1 + rt)$.

$$r = \frac{A - P}{Pt},\ t = \frac{A - P}{Pr}.$$

13. Solve for P_2, $\frac{V_1}{V_2} = \frac{P_2}{P_1}$.

$$P_2 = \frac{V_1 P_1}{V_2}.$$

14. Solve for n and l, $s = \frac{n(a + l)}{2}$.

$$n = \frac{2s}{a + l},\ l = \frac{2s - an}{n}.$$

15. Solve for a, l, and r, $s = \frac{rl - a}{r - 1}$.

$$a = rl + s - rs,$$

$$l = \frac{a + rs - s}{r},$$

$$r = \frac{s - a}{s - l}.$$

16. Solve for θ, $\frac{D}{180} = \frac{\theta}{\pi}$.

$$\theta = \frac{\pi D}{180}.$$

17. Solve for t_1,

$$V_1 = V_0(1 + .00365\,t_1).$$

$$t_1 = \frac{V_1 - V_0}{.00365\,V_0}.$$

18. Solve for b_2, $A = \frac{(b_1 + b_2)a}{2}$.

$$b_2 = \frac{2A - ab_1}{a}.$$

19. Solve for x, $\frac{a}{b} = \frac{x}{c - x}$.

$$x = \frac{ac}{a + b}.$$

20. Solve for F, D_1, and D_0, $\frac{1}{F} = \frac{1}{D_1} + \frac{1}{D_0}$.

$$F = \frac{D_1 D_0}{D_1 + D_0},\quad D_1 = \frac{F D_0}{D_0 - F},\quad D_0 = \frac{F D_1}{D_1 - F}.$$

21. $C(t_m - t_1)H_c + W_c(t_m - t_1) = W_h(t_2 - t_m)$.

Find the value of t_m in the preceding equation, when $C = 80$, $t_1 = 20$, $t_2 = 99$, $H_c = .09$, $W_c = 1000$, and $W_h = 800$.

$t_m = 54.9+$.

22. $C(t_m - t_w)H_c + W(t_m - t_w) = I(t - t_m)H_I$.

Solve the preceding equation for t, when $C = 80$, $t_m = 54$, $t_w = 18$, $H_c = .09$, $W = 100$, $I = 440$, and $H_I = .11$.

$t = 133.7+$.

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1. Let $b =$ the weight of B in pounds.
Then $4 \cdot 96 = 6 \cdot b$.
Whence $b = 64$.

2. Let $a =$ the distance in feet of A from the fulcrum.
Then $100 \cdot a = 6\frac{2}{3} \cdot 120$.
Whence $a = 8$.

3. Let $a =$ the distance in feet of A from the fulcrum.
Then $9 - a =$ the distance in feet of B from the fulcrum,
and $125 \cdot a = 100 \cdot (9 - a)$.
Whence $a = 4$,
and $9 - a = 5$.

4. Let $a =$ the weight in pounds of A.
Then $210 - a =$ the weight in pounds of B,
and $3\frac{3}{4} \cdot a = 5(210 - a)$.
Whence $a = 120$,
and $b = 90$.

5. Let $x =$ C's distance in feet from the fulcrum.
Then $108 \cdot x = 90 \cdot 4 + 60 \cdot 3$,
or $x = 5$.

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1. Let $x =$ the greater part.
Then $300 - x =$ the less part,
and $\frac{x}{300 - x} = 5$.

Whence $x = 250,$
and $300 - x = 50.$

2. Let $x =$ the greater part.
Then $60 - x =$ the less part,
and $\frac{2x}{3} = \frac{3}{4}(60 - x).$
Whence $x = 31\frac{13}{17},$
and $60 - x = 28\frac{4}{17}.$

3. Let $x =$ the greater part.
Then $45 - x =$ the less part,
and $\frac{4}{9}x + \frac{2}{3}(45 - x) = 24.$
Whence $x = 27,$
and $45 - x = 18.$

4. Let $x =$ the less part.
Then $\frac{3}{5} - x =$ the greater part,
and $\frac{3}{4}x = \frac{1}{3}(\frac{3}{5} - x).$
Whence $x = \frac{12}{65},$
and $\frac{3}{5} - x = \frac{27}{65}.$

6. Let $g =$ the greater part.
Then $126 - g =$ the less part,
and $\frac{g}{126 - g} = 6 + \frac{7}{126 - g}.$
Whence $g = 109,$
and $126 - g = 17.$

7. Let $l =$ the less number.
Then $1906 - l =$ the greater number,
and $\frac{1906 - l}{l} = 41 + \frac{16}{l}.$
Whence $l = 45,$
and $1906 - l = 1861.$

8. Let $g =$ the greater part.
Then $\frac{8}{5} - g =$ the less part,
and $g(\frac{8}{5} - g) = g^2 - \frac{2}{5}.$
Whence $g = 1$ or $-\frac{1}{5},$
and $\frac{8}{5} - g = \frac{3}{5}$ or $\frac{9}{5}$ respectively.

9. Let $x =$ one part.
Then $71 - x =$ the other part,

and $40 - \frac{2x}{3} = 71 - x - 16.$

Whence $x = 45,$

and $71 - x = 26.$

10. Let $x =$ the boy's age in years.

Then $x = \frac{2}{5}(x + 12).$

Therefore $x = 8.$

11. Let $m =$ the man's age in years.

Then $\frac{2m}{3} = \frac{6}{5}(m - 30).$

Therefore $m = 67\frac{1}{2}.$

12. Let $m =$ the man's age in years.

Then $\frac{m - 8}{6} = \frac{m + 12}{8}.$

Therefore $m = 68.$

13. Let $x =$ the number of dollars invested at 6%.

Then $3100 - x =$ the number of dollars invested at 5%,

and $.05(3100 - x) - .06x = 18.60.$

Whence $x = 1240,$

and $3100 - x = 1860.$

14. Let $n =$ the number of dollars invested at 5%.

Then $5360 - n =$ the number of dollars invested at 6%,

and $.05n - .06(5360 - n) = 63.40.$

Whence $n = 3500,$

and $5360 - n = 1860.$

15. Let $n =$ the number of dollars invested at 4%.

Then $3880 - n =$ the number of dollars invested at 6%,

and $.04n + .06(3880 - n) = 171.20.$

Whence $n = 3080,$

and $3880 - n = 800.$

16. Let $n =$ the number of quarters.

Then $80 - n =$ the number of nickels,

and $25n + 5(80 - n) = 1600.$

Whence $n = 60,$

and $80 - n = 20.$

17. Let $n =$ the number of quarters.

Then $28 - n =$ the number of dimes,

and $25n + 10(28 - n) = 505.$
Whence $n = 15,$
and $28 - n = 13.$

18. Let n, $n + 2$, and $n + 4$ represent the required numbers.
Then $\frac{n^2}{4} = \frac{1}{4}(n + 2)(n + 4) - 11.$
Whence $n = 6,$
$n + 2 = 8,$
and $n + 4 = 10.$

19. Let n, $n + 1$, and $n + 3$ represent the required numbers.
Then $\frac{16n^2}{9} = \frac{16}{9}(n + 1)(n + 3) - 2864.$
Whence $n = 402,$
$n + 1 = 403,$
and $n + 3 = 405.$

20. Let m = the number of meters in width.
Then $4m$ = the number of meters in length,
and $4m^2 = (4m - 4)(m + \frac{3}{2}) - 11.$
Whence $m = 8\frac{1}{2},$
and $4m = 34.$

21. Let m = the number of meters in width.
Then $\frac{5m}{2}$ = the number of meters in length,
and $\frac{5m^2}{2} = \left(\frac{5m}{2} + 5\right)\left(m - 4\right) + 50.$
Whence $m = 6,$
and $\frac{5m}{2} = 15.$

22. Let s = the number of meters in the side of the square.
Then $20 \cdot s^2 = 80 \cdot 4s.$
Whence $s = 16.$

23. Let w = the width in yards.
Then $2w$ = the length in yards,
and $50 \cdot 6w = 15 \cdot 2w^2.$
Whence $w = 10,$
and $2w = 20.$

24. Let $w =$ the width in inches.

Then $\dfrac{5w}{2} =$ the length in inches,

and
$$(w+4)\left(\frac{5w}{2}+4\right)-\frac{5w^2}{2}=128.$$

Whence $w = 8,$

and $\dfrac{5w}{2} = 20.$

25. Let $s =$ the side of the square in yards.

Then $s + 2\frac{7}{9}$ and $s - 2\frac{1}{2}$ represent the length and width of the rectangle respectively.

Hence
$$s^2 = (s + 2\tfrac{7}{9})(s - \tfrac{5}{2}).$$

Therefore
$$s = 25,$$
$$s + 2\tfrac{7}{9} = 27\tfrac{7}{9},$$

and
$$s - 2\tfrac{1}{2} = 22\tfrac{1}{2}.$$

26. Let $n =$ the number of apples bought.

Then
$$\frac{n}{5}\cdot\frac{4}{3}+\frac{4n}{5}\cdot\frac{3}{4}=n\cdot\frac{3}{2}-76.$$

Whence $n = 120.$

28. Let $n =$ the number of days required for both.

Then
$$\frac{2}{5}+\frac{4}{15}=\frac{1}{n}.$$

Whence $n = \frac{3}{2}.$

29. Let $n =$ the number of days required for all three.

Then
$$\frac{1}{2}+\frac{5}{14}+\frac{2}{7}=\frac{1}{n}.$$

Whence $n = \frac{7}{8}.$

30. Let $x =$ the number of days required by B alone.

Then
$$\frac{5}{24}-\frac{1}{8}=\frac{1}{x}.$$

Whence $x = 12.$

31. Let $x =$ the number of days required by C alone.

Then
$$\frac{6}{7}-\frac{5}{14}-\frac{2}{7}=\frac{1}{x}.$$

Whence $x = 4\frac{2}{3}.$

32. Let $n =$ the number of days they work together.

Then $\frac{3}{12} + \frac{n}{12} + \frac{n}{15} = 1.$

Whence $n = 5.$

35. Let $r =$ the rate of automobile in miles per hour.

Then $10 \cdot 5\tfrac{2}{3} = r \cdot 4.$

Whence $r = 14\tfrac{1}{6}.$

36. Let $r =$ the rate of A in miles per hour.

Then $r + 3 =$ the rate of B in miles per hour,

and $\frac{54}{r} = \frac{72}{r+3}.$

Whence $r = 9,$

and $r + 3 = 12.$

37. Let $t =$ the required time in hours.

Then $4\tfrac{1}{2} \cdot t + 10\tfrac{1}{2}(t - 2\tfrac{1}{2}) = 40.$

Whence $t = 4\tfrac{5}{12}.$

38. Let $r =$ the rate of return in miles per hour.

Then $\frac{80}{8} + \frac{80}{r} = 18.$

Whence $r = 10.$

39. Let $t =$ A's time of actual traveling in hours.

Then $t - \tfrac{3}{2} =$ B's time of actual traveling in hours,

and $9t + 12(t - \tfrac{3}{2}) = 150.$

Whence $t = 8,$

and $t - \tfrac{3}{2} = 6\tfrac{1}{2}.$

Therefore A's distance is 72 miles and B's 78 miles.

40. Let $r =$ B's rate in miles per hour.

Then $\frac{7r}{2} =$ A's rate in miles per hour,

and $8r + 8 \cdot \frac{7r}{2} = 126.$

Whence $r = 3\tfrac{1}{2},$

and $\frac{7r}{2} = 12\tfrac{1}{4}.$

41. From Hint. $5(4\tfrac{1}{4} - x) = 3(4\tfrac{1}{4} + x).$

Whence $x = 1\tfrac{1}{16},$

or the rate of the current is $1\tfrac{1}{16}$ miles per hour.

42. Let $d =$ the distance in miles upstream.
Then $\dfrac{d}{4-2} + \dfrac{d}{4+2} = 12.$
Whence $d = 18.$

43. Let $t =$ the time, in hours, for the return.
Then $13 - t =$ the time, in hours, going,
and $(4\tfrac{1}{3} + 2\tfrac{1}{4})(13 - t) = (4\tfrac{1}{3} - 2\tfrac{1}{4})t.$
Whence $t = 9\tfrac{7}{8},$
and $13 - t = 3\tfrac{1}{8}.$

44. $5x^2 - 7x - 6 = 0$
becomes $(5x + 3)(x - 2) = 0.$
Whence $x = 2, \text{ or } -\tfrac{3}{5},$
and $x + 1 = 3, \text{ or } \tfrac{2}{5}.$
The second set of values is rejected.

45. Let $x =$ the number of days required by B alone.
Then $x + 2 =$ the number of days required by A alone,
and $\dfrac{1}{x} + \dfrac{1}{x+2} = \dfrac{5}{12},$
or $(x - 4)(5x + 6) = 0.$
Whence $x = 4, \text{ or } -\tfrac{6}{5},$
and $x + 2 = 6, \text{ or } \tfrac{4}{5}.$
The second set of values is rejected.

46. Let $r =$ the rate downstream in miles per hour.
Then $\dfrac{10}{2\frac{1}{2}} + \dfrac{10}{r} = 6.$
Whence $r = 5.$
Then $\dfrac{5 - 2\frac{1}{2}}{2} = 1\tfrac{1}{4},$ the rate of the current,
and $2\tfrac{1}{2} + 1\tfrac{1}{4} = 3\tfrac{3}{4},$ the rate in still water.

47. Let $x =$ the number of sheep bought.
Then $x - 2 =$ the number of sheep sold,
and $\dfrac{96}{x} = \dfrac{100}{x-2} - 2,$
or $(x - 12)(x + 8) = 0.$
Whence $x = 12, \text{ or } -8,$
and $x - 2 = 10, \text{ or } -10.$
The second set of values is rejected.

48. Let $n =$ the number of yards bought.

Then $n - 4 =$ the number of yards sold,

and $$\frac{64}{n} = \frac{72}{n-4} - 2,$$

or $(x - 16)(x + 8) = 0.$

Whence $n = 16,$

and $\frac{64}{16} = 4,$ the cost in dollars per yard.

49. Let $r =$ the rate going in miles per hour.

Then $r + 5 =$ the rate returning in miles per hour,

and $$\frac{100}{r} - \frac{100}{r+5} = 1,$$

or $(r + 25)(r - 20) = 0.$

Whence $r = 20,$ or $-25,$

and $r + 5 = 25,$ or $-20.$

The second set of values is rejected.

50. Let $r =$ the rate going in miles per hour.

Then $r + 4 =$ the rate returning in miles per hour,

and $$\frac{120}{r} - \frac{120}{r+4} = 5,$$

or $(r + 12)(r - 8) = 0.$

Whence $r = 8,$ or $-12,$

and $r + 4 = 12,$ or $-8.$

The second set of values is rejected.

51. Let $r =$ the rate of slower automobile in miles per hour.

Then $r + 2 =$ the rate of faster automobile in miles per hour,

and $$\frac{72}{r} - \frac{72}{r+2} = \frac{1}{2},$$

or $(r + 18)(r - 16) = 0.$

Whence $r = 16,$ or $-18,$

and $r + 2 = 18,$ or $-16.$

The second set of values is rejected.

52. Let $r =$ A's rate in miles per hour.

Then $r + 4 =$ B's rate in miles per hour,

and $$\frac{72}{r} = \frac{72}{r+4} + 3,$$

or $(r + 12)(r - 8) = 0.$

Whence $r = 8,$ or $-12,$

and $r + 4 = 12,$ or $-8.$

The second set of values is rejected.

53. Let $r =$ the rate of B in miles per hour.

Then $r + 1 =$ the rate of A in miles per hour,

and
$$2 + \frac{\frac{172 - 28}{2}}{r} = \frac{100}{r + 1},$$

or
$$2 + \frac{72}{r} = \frac{100}{r + 1},$$

or
$$(r - 4)(r - 9) = 0.$$

Whence $r = 9$ or 4,

and $r + 1 = 10$ or 5 respectively.

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1. $5 : 10 = \frac{5}{10} = \frac{1}{2}$.

2. $10 : 5 = \frac{10}{5} = 2$.

3. $16\,a^2 : 8\,a = \dfrac{16\,a^2}{8\,a} = 2\,a$.

4. $3\frac{1}{16} : 3\frac{1}{2} = \dfrac{\frac{49}{16}}{\frac{7}{2}} = \frac{7}{8}$.

5. $8\frac{2}{3} : 5\frac{7}{9} = \dfrac{\frac{26}{3}}{\frac{52}{9}} = \frac{3}{2}$.

6. 3 days $: 9$ hours $= \frac{72}{9} = 8$.

7. 150 pounds $: 1$ ton $= \frac{150}{2000} = \frac{3}{40}$.

8. $$(x^2 - y^2) : (x + y) = \frac{(x + y)(x - y)}{x + y} = x - y.$$

9. $$(a^3 + b^3) : (a + b) = \frac{a^3 + b^3}{a + b} = a^2 - ab + b^2.$$

10. $$\left(1 - \frac{1}{a^2}\right) : \left(1 + \frac{1}{a}\right) = \frac{(a^2 - 1)}{a^2} : \left(\frac{a + 1}{a}\right) = \frac{\frac{(a + 1)(a - 1)}{a^2}}{\frac{a + 1}{a}} = \frac{a - 1}{a}.$$

11. $$\left(2 + \frac{1}{x^2}\right) : \left(4 - \frac{1}{x^4}\right) = \frac{2\,x^2 + 1}{x^2} : \frac{4\,x^4 - 1}{x^4} = \frac{\frac{2\,x^2 + 1}{x^2}}{\frac{(2\,x^2 + 1)(2\,x^2 - 1)}{x^4}}$$
$$= \frac{x^2}{2\,x^2 - 1}.$$

12. $$\frac{1}{x - 3} : \frac{1}{x^2 - 5\,x + 6} = \frac{\frac{1}{x - 3}}{\frac{1}{(x - 2)(x - 3)}} = x - 2.$$

13. $$(x^2 - xy + y^2) : (x^3 + y^3) = \frac{x^2 - xy + y^2}{(x + y)(x^2 - xy + y^2)} = \frac{1}{x + y}.$$

14. $\left(x^2 - \frac{y^3}{x}\right) : \left(1 - \frac{y}{x}\right) = \frac{(x^3 - y^3)}{x} : \left(\frac{x - y}{x}\right) = \frac{(x - y)(x^2 + xy + y^2)}{x - y}$

$= x^2 + xy + y^2.$

15. $\left(\frac{16}{x} - x\right) : \left(\frac{24}{x^4} + \frac{10}{x^3} + \frac{1}{x^2}\right) = \left(\frac{16 - x^2}{x}\right) : \left(\frac{24 + 10x + x^2}{x^4}\right)$

$$= \frac{\frac{(4 + x)(4 - x)}{x}}{\frac{(4 + x)(6 + x)}{x^4}} = \frac{x^3(4 - x)}{6 + x}.$$

16. Let $2x =$ one part.
Then $3x =$ the other part.
$\therefore 2x + 3x = 40,$
or $x = 8,$
$2x = 16,$
and $3x = 24.$

17. Let $5x =$ one part.
Then $3x =$ the other part.
$\therefore 3x + 5x = 16,$
or $x = 2,$
$5x = 10,$
and $3x = 6.$

18. Let $3x =$ one part.
Then $11x =$ the other part.
$\therefore 3x + 11x = 84,$
or $x = 6,$
$3x = 18,$
and $11x = 66.$

19. Let $2x =$ the first part.
Then $3x =$ the second part,
and $4x =$ the third part.
$\therefore 2x + 3x + 4x = 36,$
or $x = 4,$
$2x = 8,$
$3x = 12,$
and $4x = 16.$

20. Let $4x =$ the first part.
Then $5x =$ the second part,
and $6x =$ the third part.
$\therefore 4x + 5x + 6x = 135,$
or $x = 9,$
$4x = 36,$
$5x = 45,$
and $6x = 54.$

21. Let $x =$ the number.
Then $\frac{5 + x}{8 + x} = \frac{21}{28}.$
Transposing,
$7x = 28,$
or $x = 4.$

22. Let $x =$ the number.
Then $\frac{13 - x}{27 - x} = \frac{2}{9}.$
Transposing,
$-7x = -63,$
or $x = 9.$

23. $\frac{4 + 2a}{4 + 3a} = \frac{16 + 24a + 8a^2}{(4 + 3a)(4 + 4a)},$
$\frac{4 + 3a}{4 + 4a} = \frac{16 + 24a + 9a^2}{(4 + 4a)(4 + 3a)}.$
The second is the greater.

24. $\frac{a + 4b}{a + 5b} = \frac{a^2 + 11ab + 28b^2}{(a + 5b)(a + 7b)},$
$\frac{a + 6b}{a + 7b} = \frac{a^2 + 11ab + 30b^2}{(a + 7b)(a + 5b)}.$
The second is the greater.

25. It is increased.

Page 179 (First set)

1. $\frac{3}{4}=\frac{6}{x}$. $\therefore x=8$.
2. $\frac{4}{7}=\frac{16}{x}$. $\therefore x=28$.
3. $\frac{3}{2}=\frac{x}{5}$. $\therefore x=7\frac{1}{2}$.
4. $3:x=7:9$. $\therefore x=3\frac{6}{7}$.
5. $x:4=3:6$. $\therefore x=2$.
6. $\frac{4}{3}=2:\frac{1}{x}$. $\therefore x=\frac{2}{3}$.
7. $\frac{2}{3}=\frac{\frac{1}{x}}{4}$. $\therefore x=\frac{3}{8}$.
8. $\frac{2}{\frac{1}{x}}=\frac{3\frac{1}{3}}{4}$. $\therefore x=\frac{5}{12}$.
9. $\frac{1}{x}:2=3:4\frac{1}{2}$. $\therefore x=\frac{3}{4}$.
10. $\frac{a}{b}=\frac{c}{x}$. $\therefore x=\frac{bc}{a}$.
11. $\frac{a}{b}=\frac{c}{\frac{1}{x}}$. $\therefore x=\frac{a}{bc}$.
12. $\frac{a}{x^2}=\frac{1}{x}:a^3$. $\therefore x=a^4$.
13. $4:3\frac{1}{3}=3:x-3$. $\therefore x=5\frac{1}{2}$.
14. $5:x-3=7:2x+6$. $\therefore x=-17$.

Page 179 (Second set)

1. $m^2=1\cdot 4$, or $m=\pm 2$.
2. $m^2=4\cdot 9$, or $m=\pm 6$.
3. $m^2=16\cdot 4$, or $m=\pm 8$.
4. $m^2=3\cdot 12$, or $m=\pm 6$.
5. $m^2=(a-b)^2\cdot 4$, or
   $m=\pm 2(a-b)$.
6. $m^2=\frac{1}{2}\cdot\frac{1}{8}$, or $m=\pm\frac{1}{4}$.
7. $m^2=\frac{1}{3}\cdot\frac{1}{27}$, or $m=\pm\frac{1}{9}$.
8. $m^2=\frac{4}{a^3}\cdot\frac{9}{ax^2}$, or $m=\pm\frac{6}{a^2x}$.
9. $1:4=4:t$, or $t=16$.
   $4:9=9:t$, or $t=20\frac{1}{4}$.
   $16:4=4:t$, or $t=1$.
   $3:12=12:t$, or $t=48$.
   $(a-b)^2:4=4:t$,
   or $t=\frac{16}{(a-b)^2}$.
   $\frac{1}{2}:\frac{1}{8}=\frac{1}{8}:t$, or $t=\frac{1}{32}$.
   $\frac{1}{3}:\frac{1}{27}=\frac{1}{27}:t$, or $t=\frac{1}{243}$.
10. $1:2=3:f$, or $f=6$.
11. $4:5=6:f$, or $f=\frac{15}{2}$.
12. $7:14=5:f$, or $f=10$.
13. $5:12=a:f$, or $f=\frac{12a}{5}$.
14. $7:21x=6x:f$, or $f=18x^2$.
15. $a:a^2=a^3:f$, or $f=a^4$.
16. $a^3:a^5=a^4:f$, or $f=a^6$.
17. $(a+b):(a-b)=(a^2-b^2):f$,
    or $f=(a-b)^2$.
18. $\frac{a-b}{2}:\frac{1}{a+b}=(a^2-b^2):f$,
    or $f=2$.

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1. $3:6=2:4$; $3:2=6:4$; $6:3=4:2$.
2. $5:10=3:6$; $10:5=6:3$; $10:6=5:3$.
3. $3:2=x:6$; $2:3=6:x$; $3:x=2:6$.
4. $a:b=c:d$; $a:c=b:d$; $b:a=d:c$.
5. $a-b:2=3:a+b$; $2:a-b=a+b:3$; $a-b:3=2:a+b$.

6. $a+b:a+2=a+3:a-b$; $a+2:a+b=a-b:a+3$; $a+b:a+3=a+2:a-b$.

7. $a-b:2=3:a+b$.

8. $a-b:3=6:a-b$.

9. $a-2:4=2:a-3$.

10. $a-4:a+b=a+b:a-3$.

11. $a-3:a-2=a-8:a-3$.

12. $x-2y:a+5b$
$=a+5b:x-2y$.

13. $x:2=2:y$.

14. $x:3=1:y$.

15. $a:1=1:b$.

16. $ab:d=e:c$.

17. $mn:xy=z:p$.

18. $a:a+b=1:b$.

19. $5:3=4:x$.

20. $4:9=7:x$.

21. $c:b=a:x$.

22. $p:q=r:x$.

23. $ac:b=d:x$.

24. $a:1=1:x$.

25. $c:a=b:x$.

26. $y:y+1=1:x$.

27. $\frac{2}{4}=\frac{3}{6}$.

28. $\frac{4}{x}=\frac{3}{y}$.

29. $\frac{4}{6}=\frac{5}{x}$.

30. $\frac{3}{a}=\frac{1}{b}$.

31. $\frac{8}{4}=\frac{6}{3}$.

32. $\frac{2}{3}=\frac{b}{a}$.

33. $\frac{P_1}{P}=\frac{W}{W_1}$.

34. $\frac{V_2}{V_1}=\frac{P_1}{P_2}$.

35. $\frac{\frac{1}{x}}{3}=\frac{a}{2}$.

36. $\frac{\frac{1}{y}}{\frac{1}{x}}=\frac{z}{1}$.

37. $\frac{N_2}{N_1}=\frac{\sqrt{T_1}}{\sqrt{T_2}}$.

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1. $\frac{5}{3}=\frac{10}{6}$.

2. $\frac{16}{12}=\frac{32}{24}$.

3. $\frac{a+x}{x}=\frac{3}{2}$.

4. $\frac{7}{3}=\frac{f+x}{x}$.

5. $\frac{A_1+A_2}{A_2}=\frac{S_1^2+S_2^2}{S_2^2}$.

6. $\frac{-1}{3}=\frac{-2}{6}$; $\frac{-8}{12}=\frac{-16}{24}$; $\frac{a-x}{x}=\frac{-1}{2}$; $\frac{1}{3}=\frac{f-x}{x}$.

7. $\frac{5}{-1}=\frac{10}{-2}$; $\frac{16}{-8}=\frac{32}{-16}$; $\frac{a+x}{a-x}=\frac{3}{-1}$; $\frac{7}{1}=\frac{f+x}{f-x}$.

8. If $\frac{a}{b}=\frac{c}{d}$,

then $\frac{b}{a}=\frac{d}{c}$,

and $\frac{b}{a}+1=\frac{d}{c}+1$.

Whence $\frac{b+a}{a}=\frac{d+c}{c}$.

9. If $\frac{a}{b}=\frac{c}{d}$,

then $\frac{b}{a}=\frac{d}{c}$,

and $1-\frac{b}{a}=1-\frac{d}{c}$.

Whence $\frac{a-b}{a}=\frac{c-d}{c}$.

10. If $\frac{a}{b}=\frac{c}{d}$, then $\frac{2a}{b}=\frac{2c}{d}$. Whence $\frac{2a+b}{b}=\frac{2c+d}{d}$.

11. If $$\frac{a}{b}=\frac{c}{d},$$

then $$\frac{a}{3b}=\frac{c}{3d}.$$

Whence $$\frac{a+3b}{3b}=\frac{c+3d}{3d},$$

or $$\frac{a+3b}{b}=\frac{c+3d}{d}.$$

12. $$\frac{3x+4}{3x-4}=\frac{3+2}{3-2}.$$

Then $$\frac{6x}{8}=\frac{6}{4},$$

or $$x=2.$$

13. $$\frac{6x+3}{6x-3}=\frac{2a+b}{2a-b}.$$

Then $$\frac{12x}{6}=\frac{4a}{2b},$$

or $$x=\frac{a}{b}.$$

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1. From $\frac{1}{2}=\frac{3}{6}=\frac{5}{10}$, $\frac{9}{18}=\frac{1}{2}$, etc.

2. From $\frac{1}{a}=\frac{2}{2a}=\frac{3b}{3ab}$, $\frac{3+3b}{3a+3ab}=\frac{1}{a}$, etc.

3. From $\frac{3}{4}=\frac{6}{8}=\frac{12}{16}$, $\frac{21}{28}=\frac{3}{4}$, etc.

4. From $\frac{1}{x-y}=\frac{a}{ax-ay}=\frac{b}{bx-by}$, $\frac{1+a+b}{(x-y)(1+a+b)}=\frac{1}{x-y}$, etc.

5. $\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{1}{3}$. Also $\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{12}{36}=\frac{1}{3}$. The perimeters are in the same ratio as the corresponding sides.

6. $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$. Multiply both terms of $\frac{a}{b}$ by 2, of $\frac{c}{d}$ by 3, and of $\frac{e}{f}$ by 4. Then use (10) on p. 183 in text, which gives

$$\frac{2a+3c+4e}{2b+3d+4f}=\frac{a}{b}=\frac{c}{d}=\frac{e}{f}.$$

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1. If $\frac{a}{b}=\frac{4}{x}$ and $\frac{a}{b}=\frac{4}{\frac{1}{x}}$, then $\frac{4}{x}=\frac{4}{\frac{1}{x}}$, or $x=\pm 1$.

2. If $\frac{a}{b}=\frac{9}{y}$ and $\frac{a}{b}=\frac{1}{\frac{1}{y}}$, then $\frac{9}{y}=\frac{1}{\frac{1}{y}}$, or $y=\pm 3$.

3. If $\frac{p}{r}=\frac{7}{6}$ and $\frac{q}{r}=\frac{3}{5}$, then, by dividing, $\frac{p}{q}=\frac{35}{18}$.

4. If $\frac{p}{q}=\frac{4}{a}$ and $\frac{q}{r}=\frac{a}{7}$, then, by multiplying, $\frac{p}{r}=\frac{4}{7}$.

5. Let the parts of 12 be x and $12 - x$ respectively.

Then $\frac{8}{10} = \frac{x}{12 - x}$,

or $x = 5\frac{1}{3}$, and $12 - x = 6\frac{2}{3}$.

6. Let $p =$ one part.

Then $21 - p =$ the other part,

and $\frac{p}{21 - p} = \frac{18}{24}$,

or $p = 9$, and $21 - p = 12$.

7. Let $h =$ the height of the pole in feet.

Then $\frac{70}{12} : h = \frac{35}{12} : 36$,

or $h = 72$.

8. Let x and x_1 be the required sides respectively.

Then $\frac{10}{12} = \frac{15}{x}$. Also $\frac{10}{12} = \frac{20}{x_1}$.

Clearing, $10x = 180$. Clearing, $10x_1 = 240$.

$x = 18$. $x_1 = 24$.

$\frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{45}{54} = \frac{5}{6}$.

Therefore they have the same ratio.

9. Let x, x_1, and x_2 be the required sides respectively.

Then $\frac{9}{x} = \frac{9 + 10 + 17}{108} = \frac{36}{108} = \frac{1}{3}$. $\frac{10}{x_1} = \frac{1}{3}$. $\frac{17}{x_2} = \frac{1}{3}$.

Clearing, $108 = 4x$. $30 = x_1$. $51 = x_2$.

$x = 27$.

10. (*a*) $\frac{AD}{DB} = \frac{AE}{EC}$.

Substituting, $\frac{6}{4} = \frac{10}{EC}$.

Clearing, $6\,EC = 40$.

$EC = 6\frac{2}{3}$.

(*b*) $\frac{AD}{AB} = \frac{DE}{BC}$.

Substituting, $\frac{8}{8 + 4} = \frac{6}{BC}$.

Clearing, $8\,BC = 72$. $BC = 9$.

(*c*) $\frac{AD}{AB} = \frac{AE}{AC}$.

Substituting, $\frac{8}{8 + 4} = \frac{AE}{10}$.

Clearing, $80 = 12\,AE$.

$AE = 6\frac{2}{3}$.

(*d*) $\frac{AD}{AB} = \frac{AE}{AC}$.

Substituting, $\frac{10}{16} = \frac{AE}{16}$.

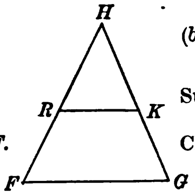
Clearing, $AE = 10$.

11. (*a*) $\frac{FG}{RK} = \frac{HF}{RH}$.

Subst., $\frac{15}{10} = \frac{HF}{8}$.

Clearing, $120 = 10\,HF$.

$HF = 12$.



(*b*) $\frac{FG - RK}{FG} = \frac{HF - HR}{HF}$.

Subst., $\frac{4}{20} = \frac{RF}{10}$.

Clearing, $40 = 20\,RF$.

$RF = 2$.

(c) $$\frac{FG}{RK}=\frac{HR+RF}{HR}.$$

Substituting, $$\frac{20}{15}=\frac{HR+6}{HR}.$$

Clearing, $20\,HR = 15\,HR + 90.$

Transposing, $5\,HR = 90.$

$HR = 18.$

(d) $$\frac{FG}{RK}=\frac{HK+KG}{HK}.$$

Substituting, $$\frac{18}{15}=\frac{KH+4}{HK}.$$

Clearing, $18\,HK = 15\,HK + 60.$

Transposing, $3\,HK = 60.$

$HK = 20.$

12. Let $x =$ the third side of the triangle.

Then $$\frac{12}{24}=\frac{12}{x}.$$

Clearing, $x = 24.$

Therefore the third side of the triangle is 24 centimeters.

13. Let x, x_1, and x_2 be the required lengths in centimeters.

Then $$\frac{5}{10}=\frac{x}{16}. \qquad \frac{6}{12}=\frac{x_1}{10}. \qquad \frac{8}{16}=\frac{x^2}{12}.$$

$$x = 8. \qquad x_1 = 5. \qquad x_2 = 6.$$

14. (*a*) $$\frac{AD}{BD}=\frac{BD}{DC}.$$

Substituting, $$\frac{9}{6}=\frac{6}{DC}.$$

Clearing, $9\,DC = 36.$

$DC = 4.$

(*b*) $$\frac{AD}{BD}=\frac{BD}{DC}.$$

Substituting, $$\frac{4}{BD}=\frac{BD}{20-4}.$$

Clearing, $64 = \overline{BD}^2.$

$BD = 8.$

15. (*a*) $$\frac{AD}{CD}=\frac{CD}{BD}.$$

Substituting, $$\frac{2}{CD}=\frac{CD}{18}.$$

Clearing, $36 = \overline{CD}^2.$

$CD = 6.$

(*b*) $$\frac{AD}{CD}=\frac{CD}{BD}.$$

Substituting, $$\frac{AD}{8}=\frac{8}{34-AD}.$$

Clearing, $34\,AD - \overline{AD}^2 = 64.$

$$\overline{AD}^2 - 34\,AD + 64 = 0.$$

$$(AD-32)(AD-2) = 0.$$

$$AD = 32,\ 2.$$

16. $$\frac{DC}{BE}=\frac{AB+BC}{AB}.$$

Substituting, $$\frac{480}{420}=\frac{8}{7}=\frac{AB+160}{AB}.$$

Clearing, $8\,AB = 7\,AB + 1120.$

$AB = 1120.$

Therefore the computed value of AB was 1120 feet.

17. Let x, x_1, and x_2 be the sides of the second triangle respectively.

Then $\frac{45}{135}=\frac{11}{x}$. $\frac{45}{135}=\frac{19}{x_1}$. $\frac{45}{135}=\frac{45-11-19}{x_2}$.

$x=33$. $x_1=57$. $x_2=45$.

18. Let $2x=$ the rate of the first man in miles per hour.

Then $3x=$ the rate of the second man in miles per hour.

Therefore $10x+15x=100$,

or $x=4$,

$2x=8$,

and $3x=12$.

19. Let $7x=$ the amount A contributed.

Then $8x=$ the amount B contributed.

Also $\frac{7x+8x}{3}$, or $5x=$ the amount each ate.

Therefore $7x-5x$, or $2x=$ the amount A should be paid for.

$8x-5x$, or $3x=$ the amount B should be paid for.

Therefore $2x+3x=30$,

or $x=6$,

$2x=12$ dollars,

and $3x=18$ dollars.

20. From February 21 to March 28 is 35 days.

Now $\frac{\text{true number of minutes in a day}}{\text{false number of minutes in a day}}=\frac{\text{true number of days}}{\text{false number of days}}$.

Therefore $\frac{1440}{1450}=\frac{x}{35}$.

Whence $x=34.758$ days.

34.758 days = 34 days, 18 hours, 12+ minutes.

Therefore the correct time was 10.12 A.M. March 28.

Page 188

1. It begins at 80° and ends at 83°.

2. The highest temperature recorded is 88°; the lowest, 54°.

3. The highest temperature was recorded at 7 P.M. Wednesday. The lowest temperature was recorded at 9.30 A.M. Thursday.

4. The instrument recorded 80° five times, 72° three times, 78° four times, and 62° twice.

5. The 80° records were made at 2 P.M. Wednesday, 10 P.M. Wednesday, 1 P.M. Thursday, 10 P.M. Thursday, 10.30 A.M. Friday.

The 72° records were made at 1 A.M. Thursday, 9 A.M. Thursday, 11 A.M. Thursday.

The 78° records were made at 10.30 P.M. Wednesday, 12.30 P.M. Thursday, 12 P.M. Thursday, 1 A.M. Friday, 9.30 A.M. Friday.

The 62° records were made at 9 A.M. Thursday, 9.30 A.M. Thursday.

6. A graph such as the one here explained might be used as a check against the waste of coal in overheating a building when not in actual use. It might be used to test the care exercised in keeping a room or a car at a uniform temperature.

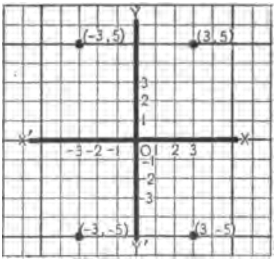
Page 190

1. The line through 8 seconds cuts the curve at a distance of 1024 feet, the line through 3 seconds cuts it at 144 feet, the one through 7 seconds cuts it at 784 feet, the one through $2\frac{1}{2}$ seconds cuts it at 100 feet, and the one through 6.2 seconds cuts it at 615 feet.

2. The line through 400 feet cuts the curve at the line for 5 seconds.
The line through 196 feet cuts the curve at the line for $3\frac{1}{2}$ seconds.
The line through 100 feet cuts the curve at the line for $2\frac{1}{2}$ seconds.
The line through 25 feet cuts the curve at the line for $1\frac{1}{4}$ seconds.
The line through 120 feet cuts the curve at the line for $2\frac{3}{4}$ seconds.
The line through 750 feet cuts the curve at the line for $6\frac{4}{5}$ seconds.

Page 194

1.



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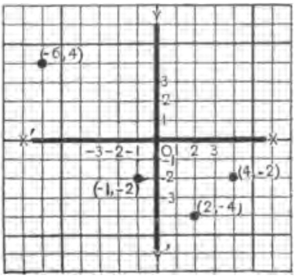

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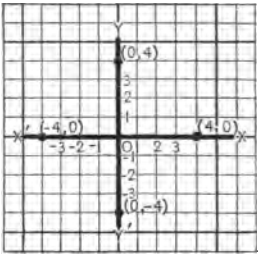

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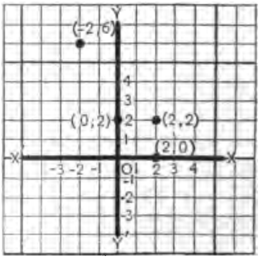

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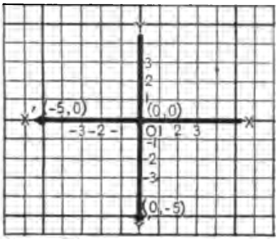


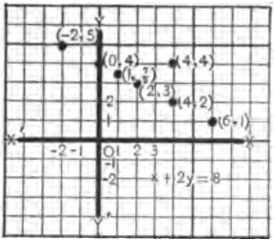
6. If one coördinate of a point is zero, the point is located on one of the axes.

If both are zero, it is located at the origin.

Page 195

1. $x + 2y = 8.$

x	-2	0	1	2	4	6
y	5	4	$\frac{7}{2}$	3	2	1

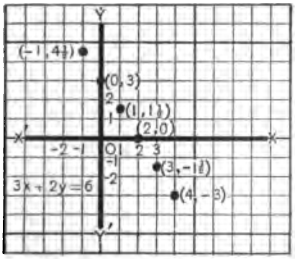


The points are in a straight line which cuts the x-axis at (8, 0) and the y-axis at (0, 4).

The graph of $x + 2y = 8$ is a straight line which passes through these points.

No, (4, 4) does not satisfy the equation. The point (4, 4) is not on the graph of the equation. If the x- and y-distances of a point satisfy the equation $x + 2y = 8$, the point is located on the graph of the equation. Moreover only such points are there located.

2.



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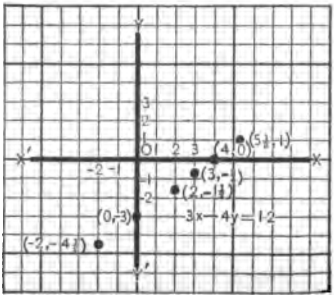

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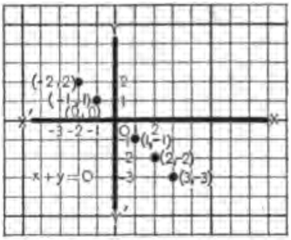

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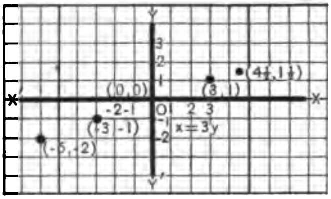

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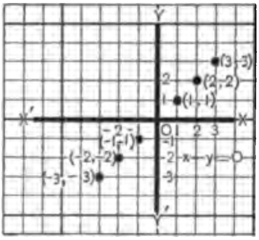

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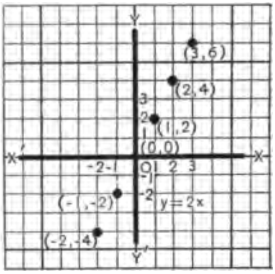
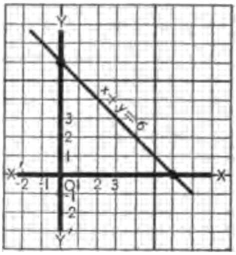


Page 197

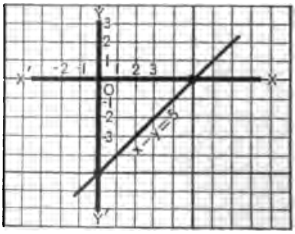
1. $x + y = 6.$

x	0	6
y	6	0



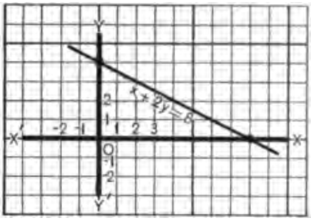
2. $x - y = 5.$

x	0	5
y	-5	0



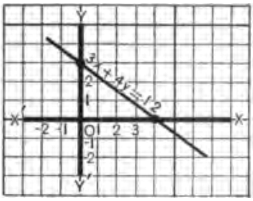
3. $x + 2y = 8.$

x	0	8
y	4	0



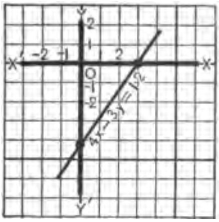
4. $3x + 4y = 12.$

x	0	4
y	3	0



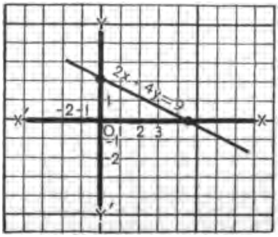
5. $4x - 3y = 12.$

x	0	3
y	-4	0



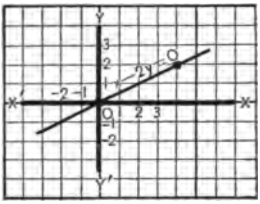
6. $2x + 4y = 9.$

x	0	$4\frac{1}{2}$
y	$2\frac{1}{4}$	0



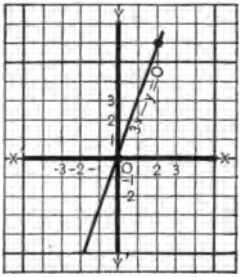
7. $x - 2y = 0.$

x	0	4
y	0	2

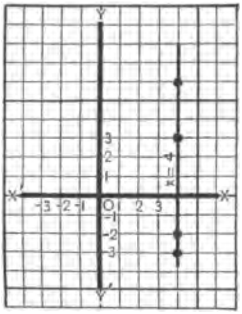


8. $3x - y = 0.$

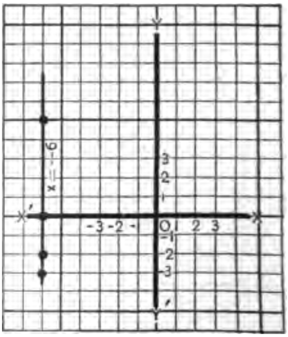
x	0	6
y	0	2



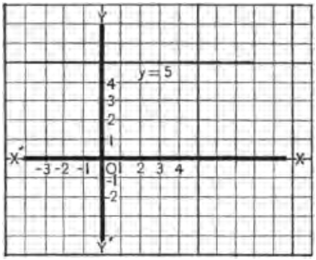
9. $x = 4$ for all values of y.



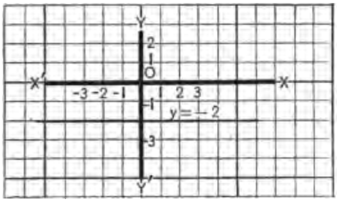
10. $x = -6$ for all values of y.



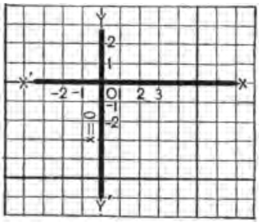
11. $y = 5$ for all values of x.



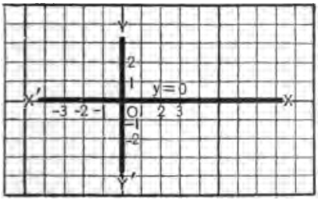
12. $y = -2$ for all values of x.



13. $x = 0$ for all values of y.



14. $y = 0$ for all values of x.



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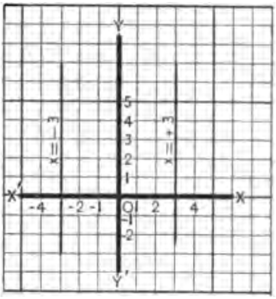
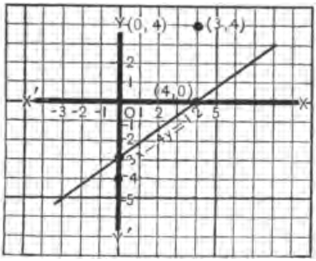


16. Yes.

17. Yes.

18. $3x - 4y = 12$.

x	0	4
y	-3	0



Point (3, 4) is not on the graph of $3x - 4y = 12$.

Point (0, 4) is not on the graph of $3x - 4y = 12$.

Point (4, 0) is on the graph of $3x - 4y = 12$.

19. Point (2, 6) will be on a graph if it satisfies the equation of that graph. It satisfies the equation of Exercise 8, and therefore lies on the graph of $3x - y = 0$.

20. The graphs of equations in Exercises 7, 8, 13, and 14 pass through the origin, for their constant terms are zero.

21. An equation whose graph passes through the origin has zero for its constant term. The constant term of an equation is never zero if the graph does not pass through the origin.

22. Yes. If the constant term of the equation is zero, and x is set equal to zero, then y equals zero also.

23. The graph of a linear equation in x and y will be parallel to the x-axis if the coefficient of y is zero.

Likewise, the graph will be parallel to the y-axis if the coefficient of x is zero.

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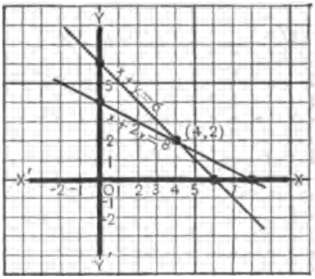
1.



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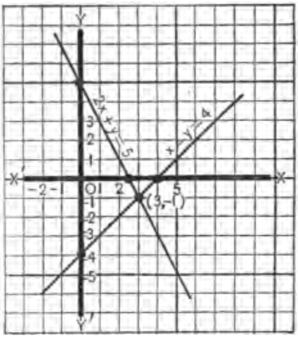

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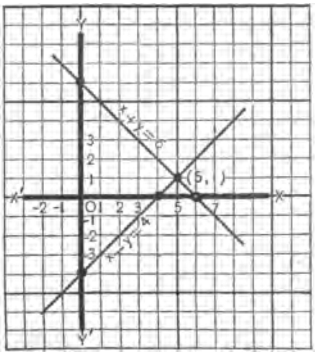

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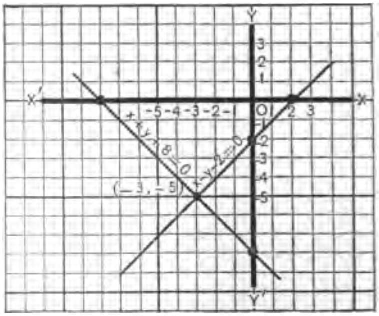

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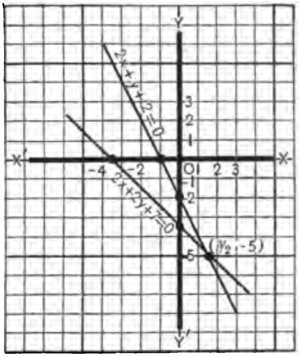
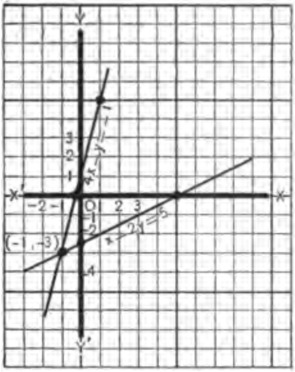
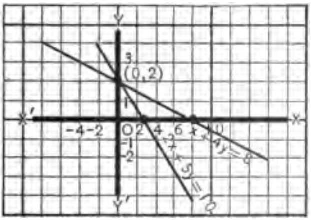

Y
X'
X
Y'
2x+y+2=0
2x+2y+7=0
(½,-5)
-4
-2
O
1
2
3
1
2
3
-1
-2
-5

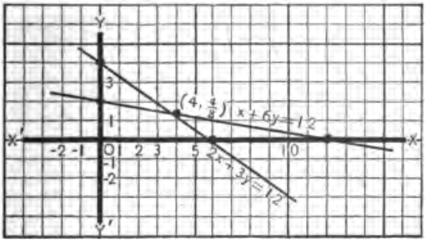
6.


Y
X'
X
Y'
4x−y=−1
x−2y=5
(−1,−3)
-2
-1
O
1
2
3
1
2
3
-1
-2
-4

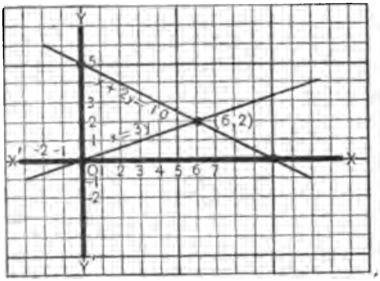
7.

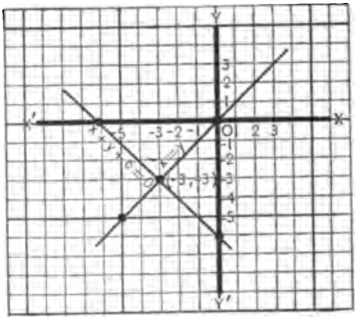

Y
X'
X
Y'
(0,2)
x+4y=8
2x+5y=10
-4
-2
O
2
4
6
10
3
1
-1
-2

8.


Y
X'
X
Y'
(4, 4/3)
x+6y=12
2x+3y=12
-2
-1
O
1
2
3
5
10
3
1
-1
-2

9.





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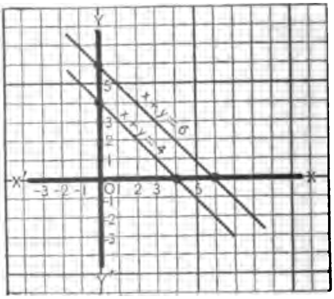
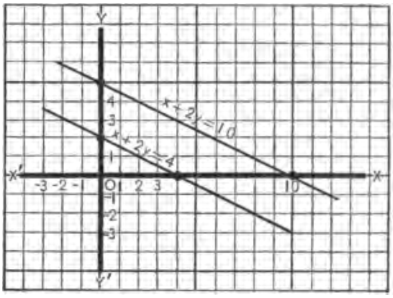


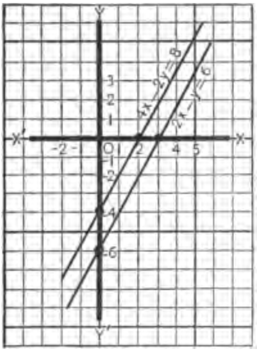
The system is inconsistent, for there is no graphical solution.

12.



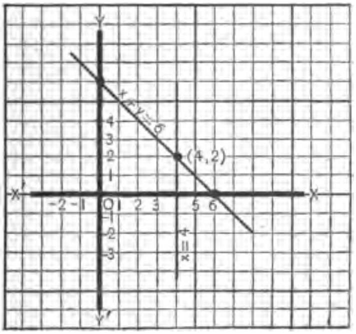
The system is inconsistent, for there is no graphical solution.

13.

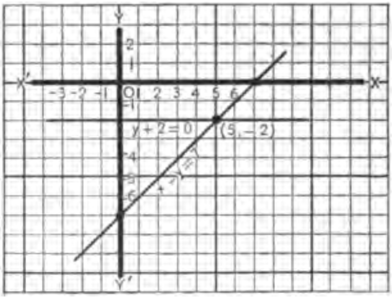


The system is inconsistent, for there is no graphical solution.

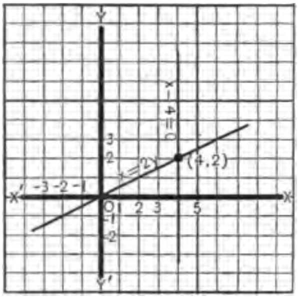
14.



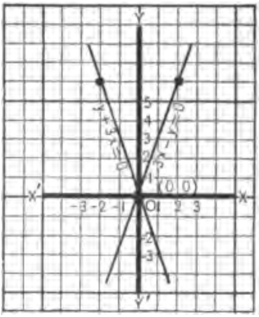
15.



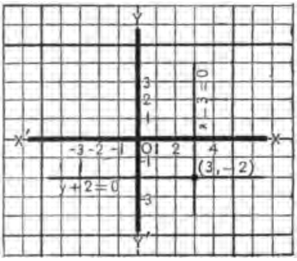
16.



17.



18.



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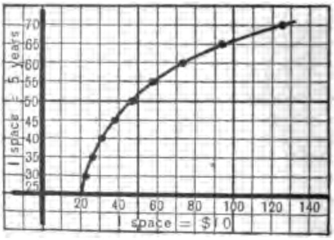
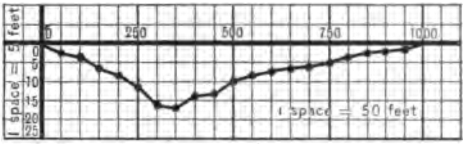
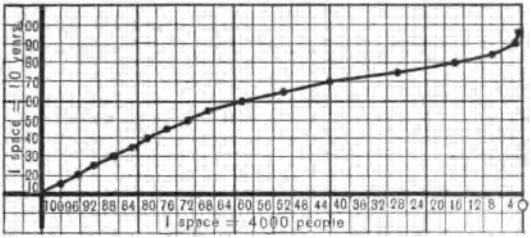
1.



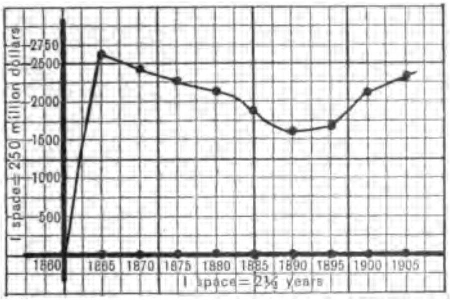
2.


1 space = 5 feet
0
250
500
750
1000
0
5
10
15
20
25
1 space = 50 feet

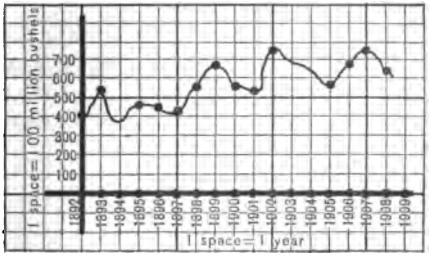
3.


1 space = 10 years
100
90
80
70
60
50
40
30
20
10
100 96 92 88 84 80 76 72 68 64 60 56 52 48 44 40 36 32 28 24 20 16 12 8 4 0
1 space = 4000 people

4.


1 space = 250 million dollars
2750
2500
2000
1500
1000
500
1860
1865
1870
1875
1880
1885
1890
1895
1900
1905
1 space = 2½ years

5.


1 space = 100 million bushels
700
600
500
400
300
200
100
1892
1893
1894
1895
1896
1897
1898
1899
1900
1901
1902
1903
1904
1905
1906
1907
1908
1909
1 space = 1 year

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1. $x + 2y = 7,$ (1)
$5x - 2y = 11.$ (2)
$[(1) + (2)] \div 6,$ $x = 3.$
Then from (1), $y = 2.$

2. $2x + y = 4,$ (1)
$3x - y = 21.$ (2)
$[(1) + (2)] \div 5,$ $x = 5.$
Then from (1), $y = -6.$

3. $7m - n = 2,$ (1)
$n - 2m = -3.$ (2)
$[(1) + (2)] \div 5,$ $m = -\frac{1}{5}.$
Then $n = -\frac{17}{5}.$

4. $10h - k = -3,$ (1)
$12h + 12k = 102.$ (2)
$[(1) \cdot 12 + (2)] \div 132,$ $h = \frac{1}{2}.$
Then $k = 8.$

5. $7r - 8s = -30,$ (1)
$r + 11s = 20.$ (2)
$[(1) - (2) \cdot 7] \div -85,$ $s = 2.$
Then $r = -2.$

6. $8x + y = 7,$ (1)
$11x + 2y = 28.$ (2)
$[(1) \cdot 2 - (2)] \div 5,$ $x = -\frac{14}{5}.$
Then $y = \frac{147}{5}.$

7. $5l + 2p = 0,$ (1)
$3l + p = 3.$ (2)
$[(1) - (2) \cdot 2] \div -1,$ $l = 6.$
Then $p = -15.$

8. $10v + 2u = 22,$ (1)
$u + 5v = 11.$ (2)
$[(1) - (2) \cdot 2]$ gives $0 = 0.$
The system has an infinite number of sets of roots.

9. $12x + 5y = 14,$ (1)
$3x - 10y = 8.$ (2)
$[(1) \cdot 2 + (2)] \div 27,$ $x = \frac{4}{3}.$
Then $y = -\frac{2}{5}.$

10. $2x - y = -1,$ (1)
$15x - 9y = 20.$ (2)
$[(1) \cdot 9 - (2)] \div 3,$ $x = -\frac{29}{3}.$
Then $y = -\frac{55}{3}.$

11. $3s - t = 12,$ (1)
$2t - 6s = 10.$ (2)
$[(1) \cdot 2 + (2)]$ gives $0 = 14.$
Therefore the system is not consistent.

12. $5x - 3w = 2,$ (1)
$15x + 12w = -5.$ (2)
$[(1) \cdot 3 - (2)] \div -21,$
$w = -\frac{11}{21}.$
Then $x = \frac{3}{35}.$

13. $x_1 - 6x_2 = 7,$ (1)
$12x_2 - x_1 = 0.$ (2)
$[(1) + (2)] \div 6,$ $x_2 = \frac{7}{6}.$
Then $x_1 = 14.$

14. $27h + 32k = 6,$ (1)
$16k - 9h = 8.$ (2)
$[(1) + (2) \cdot 3] \div 80,$ $k = \frac{3}{8}.$
Then $h = -\frac{2}{9}.$

15. $2r + 25r_1 = 15,$ (1)
$3r = 10r_1 - 44.$ (2)
$[(1) \cdot 3 - (2) \cdot 2] \div 95,$
$r_1 = \frac{7}{5}.$
Then $r = -10.$

16. $12n - 2m = 18,$ (1)
$3m = 18n + 10.$ (2)
$[(1) \cdot 3 + (2) \cdot 2]$ gives
$0 = 74.$
Therefore the system is not consistent.

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1. $x - 2y = 8,$ (1)

$3x + 2y = 7.$ (2)

Substituting $2y + 8$ for x from (1) into (2),

$3(2y + 8) + 2y = 7.$

Therefore $y = -\frac{17}{8}.$

Then from (1),

$x = \frac{15}{4}.$

2. $x - 2y = -12,$ (1)

$4x - y = 1.$ (2)

Substituting $2y - 12$ for x from (1) into (2),

$8y - 48 - y = +1.$

Therefore $y = 7,$

and $x = 2.$

3. $14m - 2n = 1,$ (1)

$n - 6m = 0.$ (2)

Substituting $6m$ for n from (2) into (1),

$m = \frac{1}{2}.$

Then $n = 3.$

4. $6h + 10k = 19,$ (1)

$2k = 3h.$ (2)

Substituting $\frac{3h}{2}$ for k from (2) into (1),

$h = \frac{19}{21}.$

Then $k = \frac{19}{14}.$

5. $3s + 12 = 3 + t,$ (1)

$t = s + 1.$ (2)

Substituting $s + 1$ for t in (1),

$s = -4,$

and $t = -3.$

6. $18 + 2p = q,$ (1)

$p + q = -9.$ (2)

Substituting $2p + 18$ for q from (1) into (2),

$p = -9.$

Then $q = 0.$

7. $3r + 15s = 7,$ (1)

$12 + 5s = -r.$ (2)

Substituting $-12 - 5s$ for r from (2) into (1),

$36 = 7.$

Therefore the system is inconsistent.

8. $20y - 3z = 1,$ (1)

$z - 6y = 0.$ (2)

Substituting $6y$ for z from (2) into (1),

$y = \frac{1}{2}.$

Therefore $z = 3.$

9. $.75p + 1.5q = 3,$ (1)

$q = p - 16.$ (2)

Substituting $p - 16$ for q into $(1) \cdot 100,$

$75p + 150p - 2400 = 300.$

Therefore $p = 12,$

and $q = -4.$

10. $\frac{3x - 20}{2} = \frac{2x + 5y}{3},$ (1)

$10 = x - y.$ (2)

Substituting $10 + y$ for x from (2) into $(1) \cdot 6,$

$y = -2.$

Then $x = 8.$

11. $$\frac{1}{7 + 2m_1} = \frac{-7}{m_2 - 1}, \qquad (1)$$

$$m_1 = m_2 + 3. \qquad (2)$$

Substituting $m_2 + 3$ for m_1 from (2) into (1) and solving,

$m_2 = -6.$

Then $m_1 = -3.$

12.
$$\frac{5R_1+2R_2}{2}=2(R_2+2), \qquad (1)$$
$$R_1-\tfrac{2}{3}R_2=0. \qquad (2)$$

Substituting $\frac{2R_2}{3}$ for R_1 from (2) into (1) and solving,

$$R_2=6.$$

Then $R_1=4.$

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1.
$$\frac{2x}{3}+4y=\frac{26}{3}, \qquad (1)$$
$$3x-\frac{7y}{2}=-4. \qquad (2)$$
(1) · 3, $2x+12y=26.$ (3)
(2) · 2, $6x-7y=-8.$ (4)
[(3) · 3 − (4)] ÷ 43, $y=2.$
Whence $x=1.$

2.
$$m-\frac{3n}{5}=\frac{18}{5}, \qquad (1)$$
$$\frac{8n}{3}+7m=-16. \qquad (2)$$
(1) · 5, $5m-3n=18.$ (3)
(2) · 3, $21m+8n=-48.$ (4)
[(3) · 8 + (4) · 3] ÷ 103,
$m=0.$
Whence $n=-6.$

3.
$$\frac{5R_1}{6}+\frac{R_2}{4}=7, \qquad (1)$$
$$\frac{2R_1}{3}-\frac{R_2}{8}=3. \qquad (2)$$
[(1) · 12 + (2) · 24] ÷ 26,
$R_1=6.$
Whence $R_2=8.$

4.
$$3k-\frac{12h}{5}=18, \qquad (1)$$
$$\frac{11k}{10}+\frac{17h}{2}=53\tfrac{1}{2}. \qquad (2)$$
(1) · 5, $15k-12h=90.$ (3)
(2) · 10, $11k+85h=535.$ (4)
[(3) · 11 + (4) · 15] ÷ − 1407,
$h=5.$
Whence $k=10.$

5.
$$\frac{2x}{9}-\frac{y}{2}=-1, \qquad (1)$$
$$x=\frac{9y}{4}-4\tfrac{1}{2}. \qquad (2)$$
(1) · 18, $4x-9y=-18,$ (3)
(2) · 4, $4x-9y=-18,$ (4)
(3) − (4), $0=0.$

Therefore the system has an infinite number of sets of roots.

6.
$$.4x+.9y=5.7, \qquad (1)$$
$$2x-y=1. \qquad (2)$$
[(1) · 10 − (2) · 2] ÷ 11,
$y=5.$
Whence $x=3.$

7.
$$12m=\frac{11y}{9}+17, \qquad (1)$$
$$\frac{2m}{3}-\frac{49}{12}=\frac{5y}{12}. \qquad (2)$$
(1) · 9, $108m-11y=153.$ (3)
(2) · 12, $8m-5y=49.$ (4)
[(3) · 5 − (4) · 11] ÷ 452,
$m=\tfrac{1}{2}.$
Whence $y=-9.$

8.
$$9s=\frac{t}{3}+\frac{23}{4}, \qquad (1)$$
$$\frac{7t}{3}-\frac{9}{4}=\frac{-3s}{4}. \qquad (2)$$

(1) · 12, $108s - 4t = 69.$ (3)

(2) · 12, $28t + 9s = 27.$ (4)

$[(3) \cdot 7 + (4) \cdot 1] \div 765,$

$s = \frac{2}{3}.$

Whence $t = \frac{3}{4}.$

9. $.04m + .75n = 10,$ (1)

$.8m - 1.25n = 5.$ (2)

(1) · 100, $4m + 75n = 1000.$ (3)

(2) · 100, $80m - 125n = 500.$ (4)

$[(3) \cdot 20 - (4) \cdot 1] \div 1625,$

$n = 12.$

Whence $m = 25.$

10. $28x - 16y = 56,$ (1)

$\frac{2x + 7}{2} - y = 4.$ (2)

$[(1) - (2) \cdot 16] \div 12,$ $x = 4.$

Whence $y = \frac{7}{2}.$

11. $\frac{r + 3}{7} - \frac{l}{5} = 0,$ (1)

$5r + 2 = 7l.$ (2)

$[(1) \cdot 35 - (2) \cdot 1]$ gives

$13 = 0.$

Therefore the system is inconsistent.

12. $\frac{x + 3}{5} + \frac{y + 4}{10} = \frac{3}{2},$ (1)

$\frac{7x + 1}{3} - \frac{11y - 4}{7} = 4.$ (2)

(1) · 10, $2x + y = 5.$ (3)

(2) · 21, $49x - 33y = 65.$ (4)

$[(3) \cdot 33 + (4) \cdot 1] \div 115,$

$x = 2.$

Whence $y = 1.$

13. $\frac{20m + 9}{7} = \frac{n}{7} + 1,$ (1)

$m + \frac{29}{5} = n.$ (2)

$[(1) \cdot 7 - (2) \cdot 20] \div - 19,$

$n = 6.$

Whence $m = \frac{1}{5}.$

14. $\frac{1}{x} + \frac{1}{y} = \frac{5}{6},$ (1)

$\frac{1}{x} - \frac{1}{y} = \frac{1}{6}.$ (2)

$[(1) + (2)] \cdot x,$ $x = 2.$

Whence $y = 3.$

15. $\frac{1}{m_1} + \frac{1}{m_2} = \frac{2}{15},$ (1)

$\frac{1}{m_1} - \frac{1}{m_2} = -\frac{1}{3}.$ (2)

$(1) + (2) \cdot 5m_1,$ $m_1 = -10.$

Whence $m_2 = \frac{30}{7}.$

16. $\frac{6}{x} + \frac{7}{y} = \frac{3}{2},$ (1)

$\frac{7}{x} - \frac{6}{y} + 5\frac{1}{3} = 0.$ (2)

$(1) \cdot 6 + (2) \cdot 7,$ $x = -3.$

Whence $y = 2.$

17. $\frac{1}{x} + \frac{1}{y} = 10,$ (1)

$\frac{3}{x} + \frac{3}{y} = 14.$ (2)

$(1) \cdot 3 - (2),$ $0 = 16.$

Therefore the system is inconsistent.

18. $\frac{5}{x} + 12 = 17,$ (1)

$\frac{2}{x} - 15y = 0.$ (2)

$[(1) \cdot 2 - (2) \cdot 5] \div 75,$ $y = \frac{2}{15}.$

Whence $x = 1.$

19. $\frac{4r + q}{6r + q} = \frac{2}{5},$ (1)

$\frac{2r}{3} - q = 5.$ (2)

$(1) \cdot 5(6r + q),$ $8r + 3q = 0.$ (3)

(2) · 3, $2r - 3q = 15.$ (4)

$[(3) + (4)] \div 10,$ $r = \frac{3}{2}.$

Whence $q = -4.$

20. $\frac{1}{t_2}+\frac{2}{t_1}=\frac{40}{t_1t_2}$, (1)

$2t_2-3t_1=0.$ (2)

$[(1)\cdot 2\cdot t_1t_2-(2)]\div 5,$

$t_1=10.$

Whence $t_2=15.$

21. $\frac{5m+3n-1}{3}=n+\frac{1}{3}$, (1)

$\frac{25m}{10m+2n}=1.$ (2)

$(1)\cdot 3\div 5,$ $m=\frac{2}{5}.$

Whence $n=3.$

22. $\frac{k+5}{k+1}=\frac{l+2}{l-2}$, (1)

$\frac{11k+4}{l+k+1}=4.$ (2)

$\frac{[(1)\cdot(k+1)(l-2)+(2)\cdot(l+k+1)]}{3},$

$k=4.$

Whence $l=7.$

23. $\frac{2x_1-3x_2}{x_2-4x_1}=3,$ (1)

$\frac{8}{x_2+x_1}+\frac{8}{x_1-x_2}=0.$ (2)

$(1)\cdot(x_2-4x_1),$

$14x_1-6x_2=0.$ (3)

$(2)\cdot(x_2+x_1)(x_1-x_2),$

$16x_1=0.$ (4)

$(4)\div 16,$ $x_1=0.$

Then $x_2=0.$

But this set makes the first member of (1) indeterminate, also the first member of (2) is not a number.

Therefore the system has no set of roots.

24. $3x-2y=7,$ (1)

$\frac{2}{x-5}=-\frac{5}{y-4}.$ (2)

$[(1)\cdot 1+(2)\cdot(x-y)(y-4)]\div 8,$

$x=5.$

Whence $x=4.$

But this set does not satisfy (2).

Hence the system has no set of roots.

25. $\frac{2}{6+s}-\frac{4}{4-t}=0,$ (1)

$\frac{t}{4}+\frac{s}{2}+2=0.$ (2)

$(1)\cdot(6+s)(4-t),$

$-2t-4s=16.$ (3)

$(2)\cdot 4,$ $t+2s=-8.$ (4)

$[(3)\cdot 1+(4)\cdot 2]$ gives $0=0.$

Therefore the system has an infinite number of sets of roots.

26. $\frac{2}{n_1+2+n_2}=\frac{1}{5n_2+3n_1-5}$, (1)

$n_1+2n_2=4.$ (2)

(1) cleared, $5n_1+9n_2=12.$ (3)

$(2)\cdot 5,$ $5n_1+10n_2=20.$ (4)

$(4)-(3),$ $n_2=8.$

Whence $n_1=-12.$

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1. Let $g=$ the greater number,

and $l=$ the less.

Then $g-l=25,$ (1)

and $g+l=46.$ (2)

$[(1)+(2)]\div 2,$

$g=\frac{71}{2}.$

Whence $l=\frac{21}{2}.$

2. Let $g=$ the greater number,

and $l=$ the less.

Then $\frac{g}{l}=6,$

or $g=6l,$ (1)

and $g+l=49.$ (2)

$[(1)-(2)]\div -7,$ $l=7.$

Whence $g=42.$

3. Let $g =$ the greater number,
and $l =$ the less.
Then $g - l = 36, \quad (1)$
and $\frac{g}{l} = 3,$
or $g - 3l = 0. \quad (2)$
$[(1) - (2)] \div 2, \quad l = 18.$
Whence $g = 54.$

4. Let $\frac{n}{d} =$ the fraction.
Then $\frac{n}{d} = \frac{3}{5},$
or $5n - 3d = 0, \quad (1)$
and $\frac{n+3}{d+1} = \frac{3}{4},$
or $4n - 3d = -9. \quad (2)$
$(1) - (2), \quad n = 9.$
Whence $d = 15,$
and $\frac{n}{d} = \frac{9}{15}.$

5. Let $g =$ the greater number,
and $l =$ the less.
Then $\frac{g}{l} = 3 + \frac{6}{l},$
or $g - 3l = 6, \quad (1)$
and $\frac{l}{g} = .3,$
or $10l - 3g = 0. \quad (2)$
$(1) \cdot 3 + (2), \quad l = 18.$
Whence $g = 60.$

6. Let $n =$ the number of nickels,
and $q =$ the number of quarters.
Then $n + q = 77, \quad (1)$
and $5n + 25q = 985. \quad (2)$
$[(1) \cdot 5 - (2) \cdot 1] \div -20,$
$q = 30.$
Whence $n = 47.$

7. Let $\frac{n}{d} =$ the fraction.
Then $\frac{n - \frac{4}{3}}{d + \frac{1}{9}} = \frac{3}{5},$
or $5n - 3d = 7, \quad (1)$
and $n + d = 19. \quad (2)$
$[(1) \cdot 1 + (2) \cdot 3] \div 8,$
$n = 8.$
Whence $d = 11,$
and $\frac{n}{d} = \frac{8}{11}.$

8. Let $\frac{n}{d} =$ the fraction.
Then $d - n = 24, \quad (1)$
and $\frac{n + \frac{3}{8}}{d - \frac{1}{4}} = \frac{3}{14},$
or $14n - 3d = -6. \quad (2)$
$[(1) \cdot 3 + (2) \cdot 1] \div 11,$
$n = 6.$
Whence $d = 30,$
and $\frac{n}{d} = \frac{6}{30}.$

9. Let w_1 and w_2 represent the weights in pounds.
Then $12w_1 = 8w_2, \quad (1)$
and $10(w_1 + 2) = 8w_2. \quad (2)$
$[(1) - (2)] \div 2, \quad w_1 = 10,$
and $w_2 = 15.$

10. Let w_1 and w_2 represent the weights in pounds.
Then $12w_1 = 18w_2, \quad (1)$
and $12(w_1 - 12) = 15w_2. \quad (2)$
$[(1) - (2)] \div -3,$
$w_2 = 48.$
Whence $w_1 = 72.$

11. Let w_1 and w_2 represent the weights in pounds.
Then $15w_1 = 10w_2, \quad (1)$
and $14(w_1 - 6) = 10(w_2 - 12). (2)$
$(1) - (2), \quad w_1 = 36.$
Whence $w_2 = 54.$

12. Let $x =$ A's age in years,
and $y =$ B's age in years.
Then $x - 2y = 0,$ (1)
and $y - 7 = \dfrac{x-7}{3}.$ (2)
$[(1) + (2)],$ $y = 14.$
Whence $x = 28.$

13. Let $x =$ A's age now in years,
and $y =$ B's age now in years.
Then $x + 5 = 2(y + 5),$ (1)
and $x - 5 = 3(y - 5).$ (2)
$(1) - (2),$ $y = 15.$
Whence $x = 35.$

14. Let $l =$ the length,
and $w =$ the width.
Then $2l + 2w = 232,$ (1)
$l - 2w = 8.$ (2)
$[(1) + (2)] \div 3,$ $l = 80.$
Whence $w = 36.$

15. Let $p_1 =$ the number of dollars at 6%,
and $p_2 =$ the number of dollars at 5%.
Then $p_1 + p_2 = 1000,$ (1)
and $.06\,p_1 + .05\,p_2 = 54.$ (2)
$[(1) \cdot 5 - (2) \cdot 100] \div -1,$
$p_1 = 400.$
Whence $p_2 = 600.$

16. Let $x =$ the number of dollars at $4\frac{1}{2}$%,
and $y =$ the number of dollars at $3\frac{1}{2}$%.
Then $x + y = 2000,$ (1)
and $.035y - .045x = 10.$ (2)
$[(1) \cdot 35 - (2) \cdot 1000] \div 80,$
$x = 750.$
Whence $y = 1250.$

17. Let $n_1 =$ the number of dollars at 4%,
and $n_2 =$ the number of dollars at 6%.
Then $n_1 + n_2 = 5000,$ (1)
and
$5\left(\dfrac{4\,n_1}{100}\right) - 3\left(\dfrac{6\,n_2}{100}\right) = 126.$ (2)
$[(2) \cdot 100 + (1) \cdot 18] \div 38,$
$n_1 = 2700,$
Whence $n_2 = 2300.$

18. Let $r =$ the value of a ruble in cents,
and $m =$ the value of a mark in cents.
Then
$20m - 10r = 10,$ (1)
and $12m - 4r = 100.$ (2)
$[(1) \cdot 2 + (2) \cdot 5] \div 20,$
$m = 24.$
Whence $r = 47.$

19. Let $f =$ the value of a franc in cents,
and $F =$ the value of a florin in cents.
Then $5f - 2F = 19,$ (1)
and $3f + F = 95.$ (2)
$[(1) + (2) \cdot 2] \div 11,$
$f = 19.$
Whence $F = 38.$

21. Let $t =$ the digit in tens' place,
and $u =$ the digit in units' place.
Then $t + u = 7,$ (1)
and $10t + u + 27 = 10u + t,$
or $t - u = -3.$ (2)
$[(1) + (2)] \div 2,$ $t = 2,$
and $u = 5.$
Therefore the number is 25.

22. Let $t =$ the digit in tens' place,
and $u =$ the digit in units' place.

Then $t - 2u = 0,$ (1)
and $10t + u - 36 = 10u + t,$
or $t - u = 4.$ (2)
$[(1) - (2)] \div -1,$
$u = 4,$
and $t = 8.$
Therefore the number is 84.

23. Let $t =$ the digit in tens' place,
and $u =$ the digit in units' place.

Then $\dfrac{10t + u}{t + u} = 4,$
or $6t - 3u = 0;$ (1)
and $10t + u + 36 = 10u + t,$
or $3t - 3u = -12.$ (2)
$[(1) - (2)] \div 3,$ $t = 4,$
and $u = 8.$
Therefore the number is 48.

24. Let $t =$ the digit in tens' place,
and $u =$ the digit in units' place.

Then
$\dfrac{10t + u + 3}{t + u} = 9,$
or $t - 8u = -3;$ (1)
and $\dfrac{10t + u}{3u} = 17,$
or $t - 5u = 0.$ (2)
$[(1) - (2)] \div -3,$
$u = 1,$
and $t = 5.$
Therefore the number is 51.

25. Let $t =$ the digit in tens' place,
and $u =$ the digit in units' place.

Then $\dfrac{10t + u}{t + u} = 7,$
or $t - 2u = 0;$ (1)
and $\dfrac{10u + t}{t + u + 3} = 3,$
or $-2t + 7u = 9.$ (2)
$[(1) \cdot 2 + (2)] \div 3,$
$u = 3,$
and $t = 6.$
Therefore the number is 63.

26. Let $n_1 =$ the first number,
and $n_2 =$ the second number.

Then $\dfrac{1}{n_1} + \dfrac{1}{n_2} = \dfrac{5}{24},$ (1)
and $\dfrac{1}{n_1} - \dfrac{1}{n_2} = \dfrac{1}{24}.$ (2)
$[(1) + (2)]$ gives $n_1 = 8.$
Whence $n_2 = 12.$

27. Let $n_1 =$ the first number,
and $n_2 =$ the second number.

Then $\dfrac{1}{n_1} - \dfrac{1}{n_2} = \dfrac{7}{6},$ (1)
and $\dfrac{n_2}{n_1} = \dfrac{15}{8}.$ (2)

Substituting $\dfrac{15\,n_1}{8}$ for n_2 from (2) into (1),
$\dfrac{1}{n_1} - \dfrac{8}{15\,n_1} = \dfrac{7}{6}.$ (3)
$[(3) \cdot 30\,n_1] \div 35,$
$n_1 = \frac{2}{5}.$
Then $n_2 = \frac{3}{4}.$

28. Let $w_1 =$ the number of grams in first pan,
and $w_2 =$ the number of grams in second pan.

Then $w_1 - 15 = \dfrac{w_2 + 15}{2},$
or $2w_1 - w_2 = 45;$ (1)
and $w_2 - 85 = w_1 + 85,$
or $w_2 - w_1 = 170.$ (2)
$(1) + (2),$ $w_1 = 215.$
Then $w_2 = 385.$

29. Let $a =$ the number of dollars A had,
and $b =$ the number of dollars B had.
Then $2(a-20) = b + 20,$
or $2a - b = 60;$ (1)
and $a - 20 + 75 = 5(b + 20 - 75),$
or $a - 5b = -330.$ (2)
$[(1) - (2) \cdot 2] \div 4,$ $b = 80.$
Then $a = 70.$

30. Let $c_1 =$ the number of feet in the circumference of the fore wheel,
and $c_2 =$ the number of feet in the circumference of the rear wheel.
Then $c_2 - c_1 = 2,$ (1)
and $\frac{155}{c_1} = \frac{186}{c_2},$
or $5c_2 - 6c_1 = 0.$ (2)
$[(1) \cdot 5 - (6)],$ $c_1 = 10.$
Then $c_2 = 12.$

31. Let $l =$ the number of feet in the length of the rectangle,
and $w =$ the number of feet in the width of the rectangle.
Then $(l + 1)(w + 1) = lw + 18,$
or $w + l = 17;$ (1)
and $(l - 1)(w - 1) = lw - 16,$
or $w + l = 17.$ (2)
$(1) - (2),$ $0 = 0.$

Therefore an infinite number of sets of dimensions satisfy the problem. The sum of each set is 17 feet.

32. Let $a =$ the number of days A requires,
and $b =$ the number of days B requires.
Then $\frac{1}{a} + \frac{1}{b} = \frac{5}{12},$ (1)
and $a - \frac{2b}{3} = 0.$ (2)

Substituting $\frac{2b}{3}$ for a from (2) into (1),

$$\frac{3}{2b} + \frac{1}{b} = \frac{5}{12}.$$

Whence $b = 6.$
Then $a = 4.$

33. Let $a =$ the number of days A requires,
and $b =$ the number of days B requires.
Then $\frac{1}{a} + \frac{1}{b} = \frac{5}{36},$ (1)
and $\frac{5}{a} + \frac{5}{b} + \frac{11}{3a} = 1.$ (2)

$[(1) \cdot 5 - (2)]$ gives

$$-\frac{11}{3a} = -\frac{11}{36},$$

or $a = 12.$
Whence $b = 18.$

34. Let $l =$ the length in feet,
and $w =$ the width in feet.

Then
$$(l + 4)(w - 2) = lw + 8,$$
or $2w - l = 8;$ (1)
and $(l - 1)(w + 3) = lw + 33,$
or $-w + 3l = 36.$ (2)
$[(1) + (2) \cdot 2] \div 5,$
$$l = 16.$$
Whence $w = 12.$

$$\text{Area} = \frac{16 \cdot 12}{9} = 21\tfrac{1}{3} \text{ square yards.}$$

35. Let $l =$ the length in feet,
and $w =$ the width in feet.

Then $lw = (l+10)(w-6),$
or $5w - 3l = 30;$ (1)
and $lw = (l+4)(w-3),$
or $4w - 3l = 12.$ (2)
$(1) - (2),$ $w = 18.$
Whence $l = 20.$

36. Let $x =$ the first number of the first pair,
and $y =$ the second number of the first pair.
Then $x + 2 =$ the first number of the second pair,
$y - 15 =$ the second number of the second pair,
$x + 3 =$ the first number of the third pair,
and $y - 18 =$ the second number of the third pair.
Therefore $xy = (x + 2)(y - 15),$
or $2y - 15x = 30;$ (1)
and $xy = (x + 3)(y - 18),$
or $3y - 18x = 54.$ (2)
$[(1) \cdot 3 - (2) \cdot 2] \div -9,$ $x = 2.$
Then $y = 30.$

The pairs are 2, 30; 4, 15; and 5, 12 respectively.

37. Let $x =$ the number of men,
and $y =$ the number of dollars each paid.
Then $(x + 3)(y - 200) = xy,$
or $3y - 200x = 600;$ (1)
and $(x - 4)(y + 500) = xy,$
or $-4y + 500x = 2000.$ (2)
$[(1) \cdot 4 + (2) \cdot 3] \div 700,$ $x = 12.$
Then $y = 1000,$
and $xy = 12{,}000.$

38. Let $x =$ the rate of the man in still water in miles per hour,
and $y =$ the rate of the river in miles per hour.

Then $$\frac{10}{x + y} = 2,$$
or $x + y = 5;$ (1)

and $$\frac{10}{x - y} = 2\tfrac{1}{2},$$

or $$x - y = 4. \qquad (2)$$
[(1) + (2)] 2, $$x = \tfrac{9}{2}.$$
Then $$y = \tfrac{1}{2}.$$

39. Let x = the rate of the boat in still water in miles per hour,
and y = the rate of the current in miles per hour.

Then $$\frac{36}{x+y} = 3,$$
or $$x + y = 12; \qquad (1)$$
and $$\frac{24}{x-y} = 3,$$
or $$x - y = 8. \qquad (2)$$
[(1) + (2)] ÷ 2, $$x = 10.$$
Then $$y = 2.$$

40. Let x = the rate of the current in miles per hour,
and y = the number of miles from A to B.

Then $$\frac{y}{x + 8\frac{1}{2}} = 14,$$
or $$14x - y = -119; \qquad (1)$$
and $$\frac{\frac{y}{2}}{8\frac{1}{2} - x} = 10,$$
or $$20x + y = 170. \qquad (2)$$
[(1) + (2)] ÷ 34, $$x = \tfrac{3}{2}.$$
Then $$y = 140.$$

41. Let x = the rate of the stream in miles per hour,
and y = the number of miles from A to C.

Then $$\frac{y}{12 + x} = 7,$$
or $$7x - y = -84; \qquad (1)$$
and $$\frac{y - 36}{12 - x} = 5,$$
or $$5x + y = 96. \qquad (2)$$
[(1) + (2)] ÷ 12, $$x = 1.$$
Then $$y = 91.$$

42. Let d = the distance from A to B in miles,
and r = the usual rate in miles per hour.

Then $$\frac{d}{\frac{5r}{4}} = \frac{d}{r} - 1,$$

or $$d - 5r = 0; \tag{1}$$

and $$\frac{d}{24} = \frac{d}{r} - \frac{5}{6},$$

or $$rd - 24d + 20r = 0. \tag{2}$$

Substituting $5r$ for d from (1) into (2),

$$5r^2 - 100r = 0.$$

Whence $r = 0$ or 20.

Then, from (1), $d = 0$ or 100.

The set $\begin{cases} r = 0 \\ d = 0 \end{cases}$ is rejected.

43. Let $d =$ the distance in miles from A to B,
and $r =$ the usual rate in miles per hour.

Then $$\frac{d}{\frac{6r}{5}} = \frac{d}{r} - \frac{2}{5},$$

or $$5d - 12r = 0; \tag{1}$$

and $$\frac{15}{r} + \frac{d - 15}{\frac{6r}{5}} = \frac{d}{r} - \frac{3}{20},$$

or $$5d - 9r = 75. \tag{2}$$

$[(1) - (2)] \div -3,$ $r = 25.$

Whence $d = 60.$

44. Let $x =$ the number of feet in the length of the passenger train,
and $y =$ the number of feet in the length of the freight train.

Then $$2x - y = 0, \tag{1}$$

and $$\frac{x + y}{66 + 44} = 15,$$

or $$x + y = 1650. \tag{2}$$

$[(1) + (2)] \div 3,$ $x = 550.$

Whence $y = 1100.$

45. Let $p =$ the number of feet in the length of the passenger train,
and $f =$ the number of feet in the length of the freight train.

Then $$f - p = 350, \tag{1}$$

and $$\frac{p + f}{5280\,(45 - 15)} = \frac{75}{60 \cdot 60},$$

or $$p + f = 1650. \tag{2}$$

$[(1) + (2)] \div 2,$ $f = 1000.$

Whence $p = 650.$

46. Let $p =$ the rate of the passenger train in feet per minute,
and $f =$ the rate of the freight train in feet per minute.

Then $$\frac{1540+660}{p+f}=\frac{1}{3},$$

or $$p+f=6600; \qquad (1)$$

and $$\frac{1540+660}{p-f}=\frac{5}{3},$$

or $$p-f=1320. \qquad (2)$$

$[(1)+(2)]\div 2$, $p=3960$, feet per minute or 45 miles per hour,

and $f=2640$, feet per minute or 30 miles per hour.

47. Let R_1 = the rate of faster in miles per hour,

and R_2 = the rate of slower in miles per hour.

Then $$\tfrac{220}{3600}(R_1+R_2)=\tfrac{1}{4},$$

or $$R_1+R_2=\tfrac{900}{22}; \qquad (1)$$

and $$\frac{220\,R_1-220\,R_2}{3600}=\frac{1}{4},$$

or $$R_1-R_2=\tfrac{90}{22}. \qquad (2)$$

$[(1)+(2)]\div 2$, $$R_1=22\tfrac{1}{2}.$$

Whence $$R_2=18\tfrac{9}{22}.$$

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1. $$3x+7y=17a, \qquad (1)$$
$$10x-4y=2a. \qquad (2)$$

$[(1)\cdot 10-(2)\cdot 3]\div 82$,
$$y=2a.$$

Whence $$x=a.$$

2. $$3x-y=10b, \qquad (1)$$
$$4x+9y=3b. \qquad (2)$$

$[(1)\cdot 9+(2)]\div 31$, $x=3b$.

Whence $$y=-b.$$

3. $$5m-4n=10a-4, \qquad (1)$$
$$m-2na=0. \qquad (2)$$

$[(1)-(2)\cdot 5]\div(10a-4)$,
$$n=1.$$

Whence $$m=2a.$$

4. $$11h+5k=33c, \qquad (1)$$
$$\frac{h}{c}-\frac{k}{2c}=3. \qquad (2)$$

$[(1)+(2)\cdot 10c]\div 21$,
$$h=3c.$$

Whence $$k=0.$$

5. $$12R_1-11R_2=a+12b, \qquad (1)$$
$$R_1+R_2=2a+b. \qquad (2)$$

$[(1)+(2)\cdot 11]\div 23$,
$$R_1=a+b.$$

Whence $$R_2=a.$$

6. $$8p+9q=4a+9a_1, \qquad (1)$$
$$\frac{p}{2}-3q=\frac{a-12a_1}{4}. \qquad (2)$$

$[(1)-(2)\cdot 16]\div 57$,
$$q=a_1.$$

Whence $$p=\frac{a}{2}.$$

7. $$\frac{7cx}{3}-\frac{5y}{2}=-3c, \qquad (1)$$
$$x+\frac{11y}{4}=11c+3. \qquad (2)$$

$[(1)\cdot 12-(2)\cdot 28c]\div(-30-77c)$,
$$y=4c.$$

Whence $$x=3.$$

8. $7.5x + 3y = 6a, \qquad (1)$

$.25x + .5y = 0. \qquad (2)$

$[(1) \cdot 10 - (2) \cdot 300] \div -120,$

$$y = \frac{-a}{2}.$$

Whence $x = a.$

9. $\frac{h}{2a} - \frac{2k}{a} = -5, \qquad (1)$

$\frac{3h}{a} - \frac{7k}{4a} = \frac{3}{4}. \qquad (2)$

$[(1) \cdot 24a - (2) \cdot 4a] \div -41,$

$$k = 3a.$$

Whence $h = 2a.$

10. $dr + 3s = 1 - d, \qquad (1)$

$7dr + 36s = 7 - 12d. \qquad (2)$

$[(1) \cdot 7 - (2)] \div -15,$

$$s = -\frac{d}{3}.$$

Whence $r = \frac{1}{d}.$

11. $h - k = 0, \qquad (1)$

$4 - \frac{h+k}{5c} = \frac{h-k}{2c}. \qquad (2)$

Substituting h for k from (1) into (2),

$$h = 10c.$$

Whence $k = 10c.$

12. $(a+b)n = 1 - cm, \qquad (1)$

$(a+b)m - 1 = -cn. \qquad (2)$

$[(1) \cdot (a+b) - (2) \cdot c] \div [(a+b)^2 - c^2],$

$$n = \frac{1}{a+b+c}.$$

Whence $m = \frac{1}{a+b+c}.$

13. $\frac{m}{b_1 - b_2} + \frac{n}{b_1 + b_2} = 2, \qquad (1)$

$m + n = 2b_1. \qquad (2)$

Substituting $2b_1 - n$ for m from (2) into (1) and solving for n,

$$n = b_1 + b_2,$$

and $m = b_1 - b_2.$

14. $\frac{1}{x} + \frac{1}{y} = 2b, \qquad (1)$

$\frac{2}{x} - \frac{3}{y} = 5c - b. \qquad (2)$

$[(1) \cdot 3 + (2)]$ solved for x gives

$$x = \frac{1}{b+c}.$$

Whence $y = \frac{1}{b-c}.$

15. $\frac{1}{x+a_1} + \frac{1}{y} = \frac{a_1 + a_2}{2a_1a_2}, \qquad (1)$

$\frac{a_1}{x+a_1} - \frac{a_2}{y} = 0. \qquad (2)$

$[(1) \cdot a_1 - (2)]$ solved gives

$$y = 2a_2.$$

Whence $x = a_1.$

16. $\frac{m+2a}{n-a} = 1,$

or $m - n = -3a; \qquad (1)$

$\frac{\frac{6}{5}n - a}{3m - 2a} - \frac{2n}{5m} = 0,$

or $-5am + 4an = 0. \qquad (2)$

$[(1) \cdot 5a + (2)] \div -a,$

$$n = 15a.$$

Whence $m = 12a.$

17. $kx - ry = 0, \qquad (1)$

$x + y - h = 0. \qquad (2)$

$[(1) + (2) \cdot r] \div (k + r),$

$$x = \frac{rh}{r+k}.$$

Whence $y = \frac{hk}{r+k}.$

18. $x + dy = 3, \qquad (1)$

$d(x-3) - y = 0. \qquad (2)$

$[(1) + (2) \cdot d] \div (d^2 + 1),$

$$x = 3.$$

Whence $y = 0.$

19. $ax - by = c, \qquad (1)$

$x + y = b. \qquad (2)$

$[(1) + (2) \cdot b] \div (a + b),$

$$x = \frac{b^2 + c}{a + b}.$$

Whence $$y = \frac{ab - c}{a + b}.$$

20. $ax + by = c, \qquad (1)$

$dx + ey = f. \qquad (2)$

$[(1) \cdot d - (2) \cdot a] \div (bd - ae),$

$$y = \frac{cd - af}{bd - ae}.$$

Whence $$x = \frac{bf - ec}{bd - ae}.$$

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1. ac dollars.

2. $\frac{b}{a}$ dollars; $\frac{bc}{a}$ dollars.

3. (a) $2a + 2b,\ ab$;
(b) $6a + 6b,\ 9ab$;
(c) 3;
(d) 9.

4. $\frac{8 \cdot 10}{2} = 40.$

5. $\frac{8 \cdot b}{2} = 4b.$

6. $\frac{a \cdot b}{2}.$

7. $\frac{(a + b)(a - b)}{2} = \frac{a^2 - b^2}{2}.$

8. $\frac{(x - 2y)(x + 2y)}{2} = \frac{x^2}{2} - 2y^2.$

9. $\frac{2k}{b}.$

10. Let x be the required number of inches the base must be increased.

Then $$\frac{(a - 2)(10 + x)}{2} = \frac{a \cdot 10}{2}.$$

Whence $$x = \frac{20}{a - 2}.$$

11. Let x be the required number of feet the base must be decreased.

Then $$\frac{(a + h)(b - x)}{2} = \frac{ab}{2}.$$

Whence $$x = \frac{bh}{a + h}.$$

12. Let $x =$ the greater number,
and $y =$ the less number.

Then $x + y = s, \qquad (1)$
and $x - y = d. \qquad (2)$

$[(1) + (2)] \div 2,\quad x = \frac{s + d}{2}.$

Whence $$y = \frac{s - d}{2}.$$

13. Let $f =$ the first number,
and $s =$ the second number.

Then $f - as = 0, \qquad (1)$
and $f - s = b. \qquad (2)$

$[(1) - (2)] \div (1 - a),$

$$s = \frac{b}{a - 1}.$$

Whence $$f = \frac{ab}{a - 1}.$$

14. Let $x =$ the first number,
and $y =$ the second number.

Then $x + y = b, \qquad (1)$
and $\frac{x}{y} = a. \qquad (2)$

$[(1) - (2) \cdot y] \div (a + 1),$

$$y = \frac{b}{a + 1}.$$

Whence $$x = \frac{ab}{a + 1}.$$

15. Let $\frac{n}{d} =$ the fraction.

Then $$\frac{n + a}{d} = 2,$$

or $n - 2d = -a;$ (1)

and $\dfrac{n}{d+b} = 1,$

or $n - d = b.$ (2)

$[(1) - (2)] \div -1,$

$$d = b + a.$$

Whence $n = a + 2b.$

The fraction is $\dfrac{a+2b}{a+b}.$

16. Let $n =$ the numerator,

and $d =$ the denominator.

Then $\dfrac{n+1}{d} = x,$

or $n - dx = -1;$ (1)

and $\dfrac{n}{d-2} = y,$

or $n - dy + 2y = 0.$ (2)

$[(1) - (2)] \div (y - x),$

$$d = \frac{2y-1}{y-x}.$$

Whence $n = \dfrac{2xy - y}{y - x}.$

17. Let $n =$ the numerator,

and $d =$ the denominator.

Then $\dfrac{n}{d} = b,$

or $n - db = 0;$ (1)

and $\dfrac{n+2}{d} = c,$

or $n - cd = -2.$ (2)

$[(1) - (2)] \div (c - b),$

$$d = \frac{2}{c-b}.$$

Whence $n = \dfrac{2b}{c-b}.$

18. Let $x =$ the number of feet the a-pound boy is from the fulcrum,

and $y =$ the number of feet the b-pound boy is from the fulcrum.

Then $ax - by = 0,$ (1)

and $x + y = l.$ (2)

$[(1) + (2) \cdot b] \div (a + b),$

$$x = \frac{bl}{a+b}.$$

Whence $y = \dfrac{al}{a+b}.$

19. Let $x =$ the number of books at 80 cents each,

and $y =$ the number of books at 110 cents each.

Then

$80x + 110y = 100h,$ (1)

and $110x + 80y = 100k.$ (2)

$[(1) \cdot 11 - (2) \cdot 8] \div 570,$

$$y = \frac{110h - 80k}{57}.$$

Whence $x = \dfrac{110k - 80h}{57}.$

20. Let $x =$ the cost of the first book in cents,

and $y =$ the cost of the second book in cents.

Then $x - y = d,$ (1)

and $x + y = 100c.$ (2)

$[(1) + (2)] \div 2,$ $x = \dfrac{d + 100c}{2}.$

Whence $y = \dfrac{100c - d}{2}.$

21. Let $x =$ the number of dollars A has,

and $y =$ the number of dollars B has.

Then $x + y = k,$ (1)

and $x - h = y + h.$ (2)

$[(1) + (2)] \div 2,\quad x = \frac{2h + k}{2}.$

Whence $\quad y = \frac{k - 2h}{2}.$

22. Let $x =$ the number of dollars A has,

and $\quad y =$ the number of dollars B has.

Then $\quad x - h = y + h,$ (1)

and $\quad x + k = 2y - 2k.$ (2)

$[(1) - (2)],\quad y = 3k + 2h.$

Whence $\quad x = 3k + 4h.$

23. Let $x =$ the number of dollars A has,

and $\quad y =$ the number of dollars B has.

Then $x - 10 + h = y + 10,$ (1)

and $\quad x + k = 3(y - k).$ (2)

$[(1) - (2)] \div 2,\quad x = 30 + 2k - \frac{3h}{2}.$

Whence $\quad y = 10 + 2k - \frac{h}{2}.$

24. Let $x =$ the number of dollars A has,

and $\quad y =$ the number of dollars B has.

Then $\quad x + y = 40,$ (1)

and $\quad x - h + k = y + h - k.$ (2)

$[(1) - (2)] \div 2,\quad y = 20 - h + k.$

Whence $\quad x = 20 + h - k.$

25. Let $x =$ the number of dollars A had at first,

and $\quad y =$ the number of dollars B had at first.

Then $\quad x - r = \frac{y + r}{2},$

or $\quad 2x - y = 3r;$ (1)

and $\quad \frac{3}{4}(x - r + 8) = y + r - 8,$

or $\quad -3x + 4y = -7r + 56.$ (2)

$[(1) \cdot 3 + (2) \cdot 2] \div 5,$

$$y = \frac{112 - 5r}{5}.$$

Whence $\quad x = \frac{56 + 5r}{5}.$

26. Let $x =$ the number of dollars at $a\%$,

and $\quad y =$ the number of dollars at $b\%$.

Then $\quad x + y = 1000,$ (1)

and $\quad \frac{ax}{100} + \frac{by}{100} = c.$ (2)

$[(1) \cdot a - (2) \cdot 100] \div (a - b),$

$$y = \frac{1000a - 100c}{a - b}.$$

Then $\quad x = \frac{1000b - 100c}{b - a}.$

27. Let $m =$ the number of dollars at 5%,

and $\quad n =$ the number of dollars at 4%.

Then $\quad m + n = x,$ (1)

and $\quad \frac{5m}{100} + \frac{4n}{100} = y.$ (2)

$[(1) \cdot 5 - (2) \cdot 100],$

$$n = 5x - 100y.$$

Whence $\quad m = 100y - 4x.$

28. Let $x =$ the number of days A requires,

and $\quad y =$ the number of days B requires.

Then $\quad 3x - y = 0,$ (1)

and $\quad \frac{1}{x} + \frac{1}{y} = \frac{1}{c}.$ (2)

Substituting $3x$ for y from (1) into (2),

$$x = \frac{4c}{3}.$$

Whence $\quad y = 4c.$

29. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $hx - y = 0,$ (1)

and $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}.$ (2)

Substituting hx for y from (1) into (2),

$$x = \frac{4h + 4}{h}.$$

Whence $y = 4h + 4.$

30. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $\frac{1}{x} + \frac{1}{y} = \frac{1}{h},$ (1)

and $\frac{6}{x} = \frac{2}{3}.$ (2)

From (2), $x = 9.$

Whence $y = \frac{9h}{9 - h}.$

31. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $\frac{1}{x} + \frac{1}{y} = \frac{1}{5},$ (1)

and $\frac{k}{x} = \frac{2}{5}.$ (2)

From (2), $x = \frac{5k}{2}.$

Whence $y = \frac{5k}{k - 2}.$

32. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $2x - y = 0,$ (1)

and $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}.$ (2)

Substituting $2x$ for y from (1) into (2),

$$x = \frac{3n}{2}.$$

Whence $y = 3n.$

33. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $\frac{1}{x} + \frac{1}{y} = \frac{1}{p},$ (1)

and $qx - y = 0.$ (2)

Substituting qx for y from (2) into (1),

$$x = \frac{pq + p}{q}.$$

Whence $y = pq + p.$

34. Let h represent the required number of hours.

Then $\frac{n}{3} + \frac{n}{4} = h,$

and $h = \frac{7n}{12}.$

35. Let h represent the required number of hours for traveling.

Then $3h =$ A's distance,

and $5h =$ B's distance.

Hence $3h + 5h = k,$

and $h = \frac{k}{8}.$

Whence $3h = \frac{3k}{8},$

and $5h = \frac{5k}{8}.$

36. Then $\frac{n}{p} + \frac{n}{q} = h,$

and $h = \frac{nq + np}{pq}.$

37. Let x represent the number of hours which must pass.

Then

$$3(x-h)+5x=k,$$

and $$x=\frac{k+3h}{8}.$$

Whence

$$\text{A's distance}=\frac{3k-15h}{8}\text{ miles,}$$

and $$\text{B's distance}=\frac{5k+15h}{8}\text{ miles.}$$

38. Let n represent the required number of hours.

Then $pn+qn=c$.

Whence $$n=\frac{c}{p+q}.$$

39. A travels $\frac{pc}{p+q}$ and B travels $\frac{qc}{p+q}$.

40. Let r represent the rate of carriage in miles per hour.

Then $$\frac{d}{r}+h=\frac{d}{3}.$$

Whence $$r=\frac{3d}{d-3h}.$$

41. Then $$\frac{d}{r}+h=\frac{d}{c}.$$

Whence $$r=\frac{cd}{d-ch}.$$

42. Let r represent the rate of riding in miles per hour.

Then $$\frac{p}{r}+\frac{p}{q}=t.$$

Whence $$r=\frac{pq}{qt-p}.$$

43. Let a = the rate of A in miles per hour,

and b = the rate of B in miles per hour.

Then $$an+bn=50, \qquad (1)$$

and $$a-b=2. \qquad (2)$$

$[(1)+(2)\cdot n]\div 2n$,

$$a=\frac{25+n}{n}.$$

Whence $$b=\frac{25-n}{n}.$$

44. Then

$$an+bn=h, \qquad (1)$$

and $$a-b=k. \qquad (2)$$

$[(1)+(2)\cdot n]\div 2n$,

$$a=\frac{h+kn}{2n}.$$

Whence $$b=\frac{h-kn}{2n}.$$

45. Let d represent the distance in miles.

Then $$\frac{d}{p}+\frac{d}{q}=t.$$

Whence $$d=\frac{tpq}{p+q}.$$

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1.

$$m+n-2p=13, \qquad (1)$$
$$m-3n-p=-3, \qquad (2)$$
$$m-n+4p=-17. \qquad (3)$$

(1) − (2), $4n-p=16$. (4)

(1) − (3), $2n-6p=30$. (5)

$[(4)-(5)\cdot 2]\div 11$, $p=-4$.

Whence $n=3$,

and $m=2$.

2.

$$x+y+3z=\tfrac{7}{2}, \qquad (1)$$
$$x-2y+4z=7, \qquad (2)$$
$$2x-11y-24z=5. \qquad (3)$$

(1) − (2), $3y-z=-\tfrac{7}{2}$. (4)

(1) · 2 − (3), $13y+30z=2$. (5)

$[(4)\cdot 30+(5)]\div 103$, $y=-1$.

Whence $z=\tfrac{1}{2}$,

and $x=3$.

3.
$$\begin{aligned}
&& x + y + z &= -1, && (1)\\
&& 3x - y - 5z &= 13, && (2)\\
&& 5x + 3y + 2z &= 1. && (3)\\
&(1) + (2), & 4x - 4z &= 12. && (4)\\
&(1) \cdot 3 - (3), & -2x + z &= -4. && (5)\\
&[(4) + (5) \cdot 2] \div -2, & z &= -2. &&\\
&\text{Whence} & x &= 1, &&\\
&\text{and} & y &= 0. &&
\end{aligned}$$

4.
$$\begin{aligned}
&& 2h + 3k - 4l &= -26, && (1)\\
&& 3h - k + 27l &= 87\tfrac{1}{2}, && (2)\\
&& h + 5k + 33l &= 74\tfrac{1}{2}. && (3)\\
&[(1) - (3) \cdot 2] \div -7, &&&&\\
&& k + 10l &= 25. && (4)\\
&[(2) \cdot 1 - (3) \cdot 3] \div -8, &&&&\\
&& 2k + 9l &= 17. && (5)\\
&[(4) \cdot 2 - (5)] \div 11, & l &= 3. &&\\
&\text{Whence} & k &= -5, &&\\
&\text{and} & h &= \tfrac{1}{2}. &&
\end{aligned}$$

5.
$$\begin{aligned}
2m + 3n - 4p &= -3, && (1)\\
m + n + 3p &= -9, && (2)\\
m + 2n - 7p &= 6. && (3)\\
(1) - (2) &= (3). &&
\end{aligned}$$

Hence the system has an infinite number of sets of roots.

6.
$$\begin{aligned}
&& x + 8y + 5z &= 1, && (1)\\
&& 3x + 10z + 4y &= -5, && (2)\\
&& x + 4z &= 0. && (3)\\
&[(1) - (2) \cdot 2], & 5x + 15z &= -11. && (4)\\
&[(4) - (3) \cdot 5] \div -5, & z &= 2\tfrac{1}{5}. &&\\
&\text{Whence} & x &= -8\tfrac{4}{5}, &&\\
&\text{and} & y &= -\tfrac{3}{20}. &&
\end{aligned}$$

7.
$$\begin{aligned}
&& 2h - 3l + 4k - 2 &= 0, && (1)\\
&& 3h - 3l - 15 &= 0, && (2)\\
&& 7h - 4k - 31 &= 0. && (3)\\
&[(1) - (2)] \div -1, &&&&\\
&& h - 4k &= 13. && (4)\\
&[(3) - (4)] \div 6, & h &= 3. &&\\
&\text{Whence} & k &= -\tfrac{5}{2}, &&\\
&\text{and} & l &= -2. &&
\end{aligned}$$

8.
$$\begin{aligned}
&& 4r - 10s &= 5, && (1)\\
&& 6r - t &= 3, && (2)\\
&& 5s + 2t &= -\tfrac{3}{2}. && (3)\\
&[(1) \cdot 3 - (2) \cdot 2], &&&&\\
&& -30s + 2t &= 9. && (4)\\
&[(4) - (3)] \div -35, & s &= -\tfrac{3}{10}. &&\\
&\text{Whence} & r &= \tfrac{1}{2}, &&\\
&\text{and} & t &= 0. &&
\end{aligned}$$

9.
$$\begin{aligned}
&& 2a_1 - 3a_2 &= 4, && (1)\\
&& 3a_1 + a_3 &= 5, && (2)\\
&& a_2 - 2a_3 &= 2. && (3)\\
&[(1) \cdot 3 - (2) \cdot 2] \div -1, &&&&\\
&& 9a_2 + 2a_3 &= -2. && (4)\\
&[(4) + (3)] \div 10, & a_2 &= 0. &&\\
&\text{Whence} & a_1 &= 2, &&\\
&\text{and} & a_3 &= -1. &&
\end{aligned}$$

10.
$$\begin{aligned}
&& 3r_1 + 5r_2 &= 74, && (1)\\
&& r_1 - 2r_3 &= -16, && (2)\\
&& 7r_3 - 4r_2 &= 44. && (3)\\
&[(1) - (2) \cdot 3], &&&&\\
&& 5r_2 + 6r_3 &= 122. && (4)\\
&[(3) \cdot 5 + (4) \cdot 4] \div 59, &&&&\\
&& r_3 &= 12. &&\\
&\text{Whence} & r_2 &= 10, &&\\
&\text{and} & r_1 &= 8. &&
\end{aligned}$$

11.
$$\begin{aligned}
&& \frac{1}{m} + \frac{1}{n} - \frac{1}{p} &= 1, && (1)\\
&& \frac{1}{m} + \frac{1}{n} + \frac{1}{p} &= \frac{2}{3}, && (2)\\
&& \frac{1}{m} - \frac{1}{n} + \frac{1}{p} &= 0. && (3)\\
&[(1) + (2)] \div 2, &&&&\\
&& \frac{1}{m} + \frac{1}{n} &= \frac{5}{6}. &&\\
&[(2) - (3)] \div 2, & \frac{1}{n} &= \frac{1}{3}. &&\\
&\text{Whence} & n &= 3. &&\\
&\text{Then} & m &= 2, &&\\
&\text{and} & p &= -6. &&
\end{aligned}$$

12. $\frac{2}{m}-\frac{3}{n}+\frac{10}{p}=-3,$ (1)

$\frac{4}{m}+\frac{5}{p}+\frac{6}{n}=15,$ (2)

$\frac{1}{m}-\frac{1}{n}+\frac{5}{p}=-\frac{1}{2}.$ (3)

$[(1)-(2)\cdot 2]\div -3,$

$\frac{2}{m}+\frac{5}{n}=11.$ (4)

$(2)-(3),$ $\frac{3}{m}+\frac{7}{n}=\frac{31}{2}.$ (5)

$[(4)\cdot 3-(5)\cdot 2],$ $\frac{1}{n}=2.$

Whence $n=\frac{1}{2}.$

Then $m=2,$

and $p=5.$

13. $\frac{1}{A}+\frac{1}{B}=2,$ (1)

$\frac{1}{A}+\frac{1}{C}=3,$ (2)

$\frac{1}{B}+\frac{1}{C}=4.$ (3)

$[(1)-(2)]+(3),$ $\frac{2}{B}=3.$

Whence $B=\frac{2}{3}.$

Then $A=2,$ and $C=\frac{2}{5}.$

14. $r+s+t+u=2.8,$ (1)

$r-s+t-u=7.2,$ (2)

$r+2s+3t-5u=7,$ (3)

$r+s-8t+u=-1.7.$ (4)

$(3)-(1),$ $s+2t-6u=4.2.$ (5)

$(3)-(2),$

$3s+2t-4u=-.2.$ (6)

$(4)-(2),$

$2s-9t+2u=-8.9.$ (7)

$[(7)-(5)\cdot 2],$

$-13t+14u=-17.3.$ (8)

$[(6)-(5)\cdot 3],$

$-4t+14u=-12.8.$ (9)

$[(9)-(8)]\div 9,$ $t=\frac{1}{2}.$

Whence $u=-\frac{54}{70},$

$s=-\frac{10}{7},$

and $r=\frac{9}{2}.$

15. $\frac{2}{x}+\frac{3}{y}=26,$ (1)

$\frac{4}{y}-\frac{10}{z}=3,$ (2)

$\frac{1}{x}+\frac{5}{z}=\frac{25}{2}.$ (3)

$[(3)\cdot 2+(2)],$ $\frac{2}{x}+\frac{4}{y}=28.$ (4)

$(4)-(1),$ $\frac{1}{y}=2.$

Whence $y=\frac{1}{2}.$

Then $x=\frac{1}{10},$

and $z=2.$

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1. Let x, y, and z represent the first, the second, and the third numbers respectively.

Then $x+y=76,$ (1)

$y+z=54,$ (2)

and $x+z=58.$ (3)

$[(1)-(2)+(3)]\div 2,$

$x=40.$

Whence $y=36,$

and $z=18.$

2. Let x, y, and z represent the first, the second, and the third numbers respectively.

Then $x+y+z=58,$ (1)

$x+y=24,$ (2)

and $\frac{x}{y}=2.$ (3)

$(1)-(2),$ $z=34.$

$[(2)-(3)\cdot y]\div 3,$ $y=8.$

Then $x=16.$

3. Let a, b, and c represent the sides of the triangle in feet.

Then $a + b + c = 64,$ (1)

$a - b = 0,$ (2)

and $a - c = -10.$ (3)

$[(1) + (2) + (3)] \div 3,$

$a = 18.$

Whence $b = 18,$

and $c = 28.$

4. Let a, b, and c represent the sides of the triangle in feet.

Then $a + b = 52,$ (1)

$a - b = 12,$ (2)

and $a + b + c = 93.$ (3)

$[(1) + (2)] \div 2,$ $a = 32.$

Whence $b = 20,$

and $c = 41.$

5. Let a, b, and c represent the sides of the triangle in feet.

Then $a + b = 41,$ (1)

$a + c = 48,$ (2)

and $b + c = 43.$ (3)

$[(1) - (2) + (3)] \div 2,$

$b = 18.$

Whence $a = 23,$

and $c = 25.$

6. Let x, y, and z represent the numbers.

Then $x + y + z = 26,$ (1)

$\frac{x}{y} = 9,$

or $x - 9y = 0.$ (2)

Also $\frac{x + y}{z} = \frac{10}{3},$

$3x + 3y - 10z = 0.$ (3)

$[(1) \cdot 3 - (3) \cdot 1] \div 13,$

$z = 6.$

Whence $x = 18,$

and $y = 2.$

7. Let x, y, and z represent the angles respectively in degrees.

Then $x - y = 0,$ (1)

$x + y - z = 0,$ (2)

$x + y + z = 180.$ (3)

$[(2) + (3) + (1) \cdot 2] \div 4,$

$x = 45.$

Whence $y = 45,$

and $z = 90.$

8. Let x, y, and z represent the angles respectively in degrees.

Then $x - y = 0,$ (1)

$2x + 2y - z = 0,$ (2)

$x + y + z = 180.$ (3)

$[(2) + (3) + (1) \cdot 3] \div 6,$

$x = 30.$

Whence $y = 30,$

and $z = 120.$

9. Let x, y, and z represent the number of degrees in A, B, and C respectively.

Then $x - y = 17,$ (1)

$y - z = 20,$ (2)

and $x + y + z = 180.$ (3)

(1) + (2), $x - z = 37.$ (4)

(3) − (2), $x + 2z = 160.$ (5)

$[(5) - (4)] \div 3,$ $z = 41.$

Whence $x = 78,$

and $y = 61.$

10. Let x, y, and z represent the angles respectively in degrees.

Then $x + y - z = 36,$ (1)

$6x - 6y - z = 0,$ (2)

and $x + y + z = 180.$ (3)

(1) + (3), $2x + 2y = 216.$ (4)

(2) + (3), $7x - 5y = 180.$ (5)

$[(4) \cdot 5 + (5) \cdot 2] \div 24,$

$x = 60.$

Whence $y = 48,$

and $z = 72.$

11. Let x, y, and z represent the time in hours required by A, B, and C respectively.

Then $$\frac{1}{x}+\frac{1}{y}=\frac{1}{2}, \quad (1)$$

$$\frac{1}{x}+\frac{1}{z}=\frac{1}{3}, \quad (2)$$

and $$\frac{1}{y}+\frac{1}{z}=\frac{1}{4}. \quad (3)$$

(1) − (2) + (3), $$\frac{2}{y}=\frac{5}{12}.$$

Whence $y=\frac{24}{5}$.

Then $x=3\frac{3}{7}$,

and $z=24$.

12. Let x, y, and z represent the number of hours required by the pumps respectively.

Then $$\frac{1}{x}+\frac{1}{y}=\frac{1}{4}, \quad (1)$$

$$\frac{1}{x}+\frac{1}{z}=\frac{1}{6}, \quad (2)$$

and $$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{13}{48}. \quad (3)$$

(3) − (1), $$\frac{1}{z}=\frac{1}{48},$$

or $z=48$.

Then $y=9\frac{3}{5}$,

and $x=6\frac{6}{7}$.

13. Let n_1 = the first numerator,

n_2 = the second numerator,

and d = each denominator.

Then $$\frac{n_1}{d}+\frac{n_2}{d}=6,$$

or $$n_1+n_2-6d=0; \quad (1)$$

$$\frac{n_1+1}{d}-\frac{n_2-1}{d}=0,$$

or $$n_1-n_2=-2, \quad (2)$$

and $$\frac{n_1}{d+22}+\frac{n_2}{d+22}=\frac{1}{2},$$

or $$2n_1+2n_2-d=22. \quad (3)$$

[(3) − (1) · 2] ÷ 11, $d=2$.

Whence $n_1=5$,

and $n_2=7$.

Therefore the fractions are $\frac{5}{2}$ and $\frac{7}{2}$.

14. Let x, y, and z be the first, second, and third digits respectively.

Whence $$x+y+z=15, \quad (1)$$

$$z-y=5, \quad (2)$$

$$100x+10y+z+396=100z+10y+x,$$

or $$x-z=-4. \quad (3)$$

[(1) + (2) − (3)] ÷ 3, $z=8$.

Whence $y=3$,

and $x=4$.

Therefore the number is 438.

15. Let x, y, and z be the hundreds', tens', and units' digits respectively.

Then $$100x+10z+y=100x+10y+z+54,$$

or $$z-y=6; \quad (1)$$

$$100y+10x+z=100x+10y+z-360,$$

$$x-y=4, \quad (2)$$

or $$x+y+z=10. \quad (3)$$

(3) − (2), $$2y+z=6. \quad (4)$$

[(4) − (1)] ÷ 3, $y=0$.

Whence $z = 6,$
and $x = 4.$

Therefore the number is 406.

16. Let a, b, c, and d be the thousands', hundreds', tens', and units' digits respectively.

Then $1000a + 100b + 10c + d + 2997 = 1000d + 100c + 10b + a,$
or $111a + 10b - 10c - 111d = -333;$ (1)
$2a - d = 0,$ (2)
$b - c = 0,$ (3)
and $a + b + c + d = 9.$ (4)
$[(1) - (3)\cdot 10] \div 111,$ $a - d = -3.$ (5)
$(2) - (5),$ $a = 3.$
Whence $b = 0,$
$c = 0,$
and $d = 6.$

Therefore the number is 3006.

17. Let a, b, c, and d be the number of degrees in the angles respectively.

Then $a + b + c + d = 360,$ (1)
$a + b = 200,$ (2)
$b + c = 180,$ (3)
and $b + c + d = 255.$ (4)
$(1) - (4),$ $a = 105.$
Then $b = 95,$
$c = 85,$
and $d = 75.$

18. Let a, b, c, and d be the number of degrees in the angles respectively.

Then $a + c = 180,$ (1)
$a - c = 30,$ (2)
$b - d = 36,$ (3)
and $a + b + c + d = 360.$ (4)
$(4) - (1),$ $b + d = 180.$ (5)
$[(5) + (3)] \div 2,$ $b = 108.$
Then $d = 72,$
$a = 105,$
and $c = 75.$

19. Let a, b, c, and d be the sides taken in order.

Then $a + c = 30,$ (1)
$b + d = 35,$ (2)
$a - b = 0,$ (3)
and $a + b - c - d + 17 = 0.$ (4)
$(1) + (2) + (3) + (4),$
$3a + b = 48.$ (5)
$[(5) + (3)] \div 4,$ $a = 12.$
Then $b = 12,$
$c = 18,$
and $d = 23.$

20. Let x, y, and z be the number of dollars A, B, and C have respectively.

Then $2(x - y - z) = 100,$ (1)
$y + y - (x - y - z) - 2z = 100,$ (2)
$2(z + z) = 100.$ (3)
$(1) \div 2,$ $x - y - z = 50.$ (4)
From (2), $-x + 3y - z = 100.$ (5)
From (3), $z = 25.$
Whence $x = 175,$
and $y = 100.$

21. Let a, b, and c be the number of dollars A, B, and C had at first respectively.

Then $a-(b+c)+(a-b-c)+(2a-2b-2c)=64,$

or $$a-b-c=16; \qquad (1)$$

$$b+b-(2c+a-b-c)+(3b-a-c)=64,$$

or $$3b-a-c=32; \qquad (2)$$

$$c+c+2c-(2a-2b-2c)-(3b-a-c)=64,$$

or $$7c-a-b=64. \qquad (3)$$

$[(1)+(2)]\div 2,$ $\quad b-c=24. \qquad (4)$

$[(1)+(3)]\div 2,$ $\quad -b+3c=40. \qquad (5)$

$[(4)+(5)]\div 2,$ $\quad c=32.$

Then $\quad b=56,$

and $\quad a=104.$

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1.

$$\begin{array}{lll}
 & a^4+2a^3+3a^2+2a+1 & \underline{\left|a^2+a+1\right.} \\
(a^2)^2= & \underline{a^4} & \\
2(a^2)=2a^2 & \left|2a^3+3a^2\right. & \\
2a^2+a & \left|\underline{2a^3+a^2}\right. & =(2a^2+a)a \\
2(a^2+a)=2a^2+2a & \left|2a^2+2a+1\right. & \\
2a^2+2a+1 & \left|\underline{2a^2+2a+1}\right. & =(2a^2+a+1)\cdot 1
\end{array}$$

Therefore the square roots are $\pm(a^2+a+1)$.

2.

$$\begin{array}{lll}
 & x^4-8x^3+24x^2-32x+16 & \underline{\left|x^2-4x+4\right.} \\
(x^2)^2= & \underline{x^4} & \\
2(x^2)=2x^2 & \left|-8x^3+24x^2\right. & \\
2x^2-4x & \left|\underline{-8x^3+16x^2}\right. & =(2x^2-4x)(-4x) \\
2(x^2-4x)=2x^2-8x & \left|8x^2-32x+16\right. & \\
2x^2-8x+4 & \left|\underline{8x^2-32x+16}\right. & =(2x^2-8x+4)\cdot 4
\end{array}$$

Therefore the square roots are $\pm(x^2-4x+4)$.

3.

$$\begin{array}{lll}
 & c^4-10c^3+21c^2+20c+4 & \underline{\left|c^2-5c-2\right.} \\
(c^2)^2= & \underline{c^4} & \\
2c^2 & \left|-10c^3+21c^2\right. & \\
2c^2-5c & \left|\underline{-10c^3+25c^2}\right. & =(2c^2-5c)(-5c) \\
2(c^2-5c)=2c^2-10c & \left|-\ 4c^2+20c+4\right. & \\
2c^2-10c-2 & \left|\underline{-\ 4c^2+20c+4}\right. & =(2c^2-10c-2)(-2)
\end{array}$$

Therefore the square roots are $\pm(c^2-5c-2)$.

4.

$$\begin{array}{lll}
 & n^6 - 6n^4 + 10n^3 + 9n^2 - 30n + 25 & \lfloor \underline{n^3 - 3n + 5} \\
(n^3)^2 = & \underline{n^6} & \\
2n^3 & -6n^4 + 10n^3 + 9n^2 & \\
2n^3 - 3n & \underline{-6n^4 \qquad + 9n^2} & = (2n^3 - 3n)(-3n) \\
2(n^3 - 3n) = 2n^3 - 6n & 10n^3 \qquad - 30n + 25 & \\
2n^3 - 6n + 5 & \underline{10n^3 \qquad - 30n + 25} & = (2n^3 - 6n + 5)5
\end{array}$$

Therefore the square roots are ± $(n^3 - 3n + 5)$.

5.

$$\begin{array}{lll}
 & 4a^6 + 4a^5 - 11a^4 + 14a^3 + 19a^2 - 30a + 25 & \lfloor \underline{2a^3 + a^2 - 3a + 5} \\
(2a^3)^2 = & \underline{4a^6} & \\
4a^3 & 4a^5 - 11a^4 & \\
4a^3 + a^2 & \underline{4a^5 + \quad a^4} & = (4a^3 + a^2)a^2 \\
4a^3 + 2a^2 & -12a^4 + 14a^3 + 19a^2 & \\
4a^3 + 2a^2 - 3a & \underline{-12a^4 - 6a^3 + 9a^2} & = (4a^3 + 2a^2 - 3a)(-3a) \\
4a^3 + 2a^2 - 6a & 20a^3 + 10a^2 - 30a + 25 & \\
4a^3 + 2a^2 - 6a + 5 & \underline{20a^3 + 10a^2 - 30a + 25} & = (4a^3 + 2a^2 - 6a + 5)5
\end{array}$$

Therefore the square roots are ± $(2a^3 + a^2 - 3a + 5)$.

6.

$$\begin{array}{ll}
 & c^4 - 4c^3d + 6c^2d^2 - 4cd^3 + d^4 \; \lfloor \underline{c^2 - 2cd + d^2} \\
(c^2)^2 = & \underline{c^4} \\
2c^2 & -4c^3d + 6c^2d^2 \\
2c^2 - 2cd & \underline{-4c^3d + 4c^2d^2} \\
2c^2 - 4cd & 2c^2d^2 - 4cd^3 + d^4 \\
2c^2 - 4cd + d^2 & \underline{2c^2d^2 - 4cd^3 + d^4}
\end{array}$$

Therefore the square roots are ± $(c^2 - 2cd + d^2)$.

7.

$$\begin{array}{ll}
 & 25y^4 + 30xy^3 - 11x^2y^2 - 12x^3y + 4x^4 \; \lfloor \underline{5y^2 + 3xy - 2x^2} \\
(5y^2)^2 = & \underline{25y^4} \\
10y^2 & 30xy^3 - 11x^2y^2 \\
10y^2 + 3xy & \underline{30xy^3 + \; 9x^2y^2} \\
10y^2 + 6xy & -20x^2y^2 - 12x^3y + 4x^4 \\
10y^2 + 6xy - 2x^2 & \underline{-20x^2y^2 - 12x^3y + 4x^4}
\end{array}$$

Therefore the square roots are ± $(5y^2 + 3xy - 2x^2)$.

8.

$$\begin{array}{ll}
 & 9a^6 - 36a^4x - 24a^3x^2 + 36a^2x^2 + 48ax^3 + 16x^4 \; \lfloor \underline{3a^3 - 6ax - 4x^2} \\
(3a^3)^2 = & \underline{9a^6} \\
6a^3 & -36a^4x - 24a^3x^2 + 36a^2x^2 \\
6a^3 - 6ax & \underline{-36a^4x \qquad + 36a^2x^2} \\
6a^3 - 12ax & -24a^3x^2 \qquad + 48ax^3 + 16x^4 \\
6a^3 - 12ax - 4x^2 & \underline{-24a^3x^2 \qquad + 48ax^3 + 16x^4}
\end{array}$$

Therefore the square roots are ± $(3a^3 - 6ax - 4x^2)$.

9.

$$\begin{array}{l|l}
 & 9c^4 - 12abc^3 - 2a^2b^2c^2 + 4a^3b^3c + a^4b^4 \;\big|\; \underline{3c^2 - 2abc - a^2b^2} \\
(3c^2)^2 = & \underline{9c^4} \\
6c^2 & -12abc^3 - 2a^2b^2c^2 \\
6c^2 - 2abc & \underline{-12abc^3 + 4a^2b^2c^2} \\
6c^2 - 4abc & -6a^2b^2c^2 + 4a^3b^3c + a^4b^4 \\
6c^2 - 4abc - a^2b^2 & \underline{-6a^2b^2c^2 + 4a^3b^3c + a^4b^4}
\end{array}$$

Therefore the square roots are ± $(3c^2 - 2abc - a^2b^2)$.

10.

$$\begin{array}{l|l}
 & c^6 - 4xc^3 + 4x^2 + 2a^2xc^3 - 4a^2x^2 + a^4x^2 \;\big|\; \underline{c^3 - 2x + a^2x} \\
(c^3)^2 = & \underline{c^6} \\
2c^3 & -4xc^3 + 4x^2 \\
2c^3 - 2x & \underline{-4xc^3 + 4x^2} \\
2c^3 - 4x & 2a^2xc^3 - 4a^2x^2 + a^4x^2 \\
2c^3 - 4x + a^2x & \underline{2a^2xc^3 - 4a^2x^2 + a^4x^2}
\end{array}$$

Therefore the square roots are ± $(c^3 - 2x + a^2x)$.

11.

$$\begin{array}{l|l}
 & 4 - \frac{4c^2}{5} + \frac{c^4}{25} \;\big|\; \underline{2 - \frac{c^2}{5}} \\
2^2 = & \underline{4} \\
4 & -\frac{4c^2}{5} + \frac{c^4}{25} \\
4 - \frac{c^2}{5} & \underline{-\frac{4c^2}{5} + \frac{c^4}{25}}
\end{array}$$

Therefore the square roots are ± $\left(2 - \frac{c^2}{5}\right)$.

12.

$$\begin{array}{l|l}
 & \frac{9}{a^2} - 3 + \frac{a^2}{4} \;\big|\; \underline{\frac{3}{a} - \frac{a}{2}} \\
\left(\frac{3}{a}\right)^2 = & \underline{\frac{9}{a^2}} \\
\frac{6}{a} - \frac{a}{2} & -3 + \frac{a^2}{4} \\
 & \underline{-3 + \frac{a^2}{4}}
\end{array}$$

Therefore the square roots are ± $\left(\frac{3}{a} - \frac{a}{2}\right)$.

13.

$$\begin{array}{ll}
 & x^4 - 4x^3 + 5x^2 - 2x + \tfrac14 \;\big|\; \underline{x^2 - 2x + \tfrac12} \\
 & \underline{x^4} \\
2x^2 & -4x^3 + 5x^2 \\
2x^2 - 2x & \underline{-4x^3 + 4x^2} \\
2x^2 - 4x & x^2 - 2x + \tfrac14 \\
2x^2 - 4x + \tfrac12 & \underline{x^2 - 2x + \tfrac14}
\end{array}$$

Therefore the square roots are $\pm(x^2 - 2x + \tfrac12)$.

15.

$$\begin{array}{ll}
 & x^4 + 6x^3 + \frac{29x^2}{3} + 2x + \frac{1}{9} \;\Big|\; \underline{x^2 + 3x + \frac{1}{3}} \\
(x^2)^2 = & \underline{x^4} \\
2x^2 & 6x^3 + \frac{29x^2}{3} \\
2x^2 + 3x & \underline{6x^3 + 9x^2} \\
2x^2 + 6x & \frac{2x^2}{3} + 2x + \frac{1}{9} \\
2x^2 + 6x + \frac{1}{3} & \underline{\frac{2x^2}{3} + 2x + \frac{1}{9}}
\end{array}$$

Therefore the square roots are $\pm(x^2 + 3x + \tfrac13)$.

16.

$$\begin{array}{ll}
 & 4a^4 + \frac{4a^3}{3} - \frac{35a^2}{9} - \frac{2a}{3} + 1 \;\Big|\; \underline{2a^2 + \frac{a}{3} - 1} \\
(2a^2)^2 = & \underline{4a^4} \\
4a^2 & \frac{4a^3}{3} - \frac{35a^2}{9} \\
4a^2 + \frac{a}{3} & \underline{\frac{4a^3}{3} + \frac{a^2}{9}} \\
4a^2 + \frac{2a}{3} & -4a^2 - \frac{2a}{3} + 1 \\
4a^2 + \frac{2a}{3} - 1 & \underline{-4a^2 - \frac{2a}{3} + 1}
\end{array}$$

Therefore the square roots are $\pm\left(2a^2 + \frac{a}{3} - 1\right)$.

17.

$$\frac{25m^4}{4}+\frac{10m^3}{3}-\frac{127m^2}{18}-2m+\frac{9}{4}\;\Big|\;\frac{5m^2}{2}+\frac{2m}{3}-\frac{3}{2}$$

$$\begin{array}{l|l}
 & \frac{25m^4}{4} \\ \hline
5m^2 & \frac{10m^3}{3}-\frac{127m^2}{18} \\
5m^2+\frac{2m}{3} & \frac{10m^3}{3}+\frac{4m^2}{9} \\ \hline
5m^2+\frac{4m}{3} & -\frac{15m^2}{2}-2m+\frac{9}{4} \\
5m^2+\frac{4m}{3}-\frac{3}{2} & -\frac{15m^2}{2}-2m+\frac{9}{4} \\ \hline
\end{array}$$

Therefore the square roots are $\pm\left(\frac{5m^2}{2}+\frac{2m}{3}-\frac{3}{2}\right)$.

18.

$$9c^4-12c^3+4c^2+6-\frac{4}{c}+\frac{1}{c^4}\;\Big|\;3c^2-2c+\frac{1}{c^2}$$

$$\begin{array}{l|l}
 & 9c^4 \\ \hline
6c^2 & -12c^3+4c^2 \\
6c^2-2c & -12c^3+4c^2 \\ \hline
6c^2-4c & +6-\frac{4}{c}+\frac{1}{c^4} \\
6c^2-4c+\frac{1}{c^2} & +6-\frac{4}{c}+\frac{1}{c^4} \\ \hline
\end{array}$$

Therefore the square roots are $\pm\left(3c^2-2c+\frac{1}{c^2}\right)$.

19.

$$\frac{m^2}{n^2}-\frac{5m}{n}+\frac{17}{4}+\frac{5n}{m}+\frac{n^2}{m^2}\;\Big|\;\frac{m}{n}-\frac{5}{2}-\frac{n}{m}$$

$$\begin{array}{l|l}
 & \frac{m^2}{n^2} \\ \hline
\frac{2m}{n} & -\frac{5m}{n}+\frac{17}{4} \\
\frac{2m}{n}-\frac{5}{2} & -\frac{5m}{n}+\frac{25}{4} \\ \hline
\frac{2m}{n}-5 & -\;2+\frac{5n}{m}+\frac{n^2}{m^2} \\
\frac{2m}{n}-5-\frac{n}{m} & -\;2+\frac{5n}{m}+\frac{n^2}{m^2} \\ \hline
\end{array}$$

Therefore the square roots are $\pm\left(\frac{m}{n}-\frac{5}{2}-\frac{n}{m}\right)$.

20.

$$\begin{array}{l|l}
 & \frac{a^4}{25c^4}+\frac{2a^2}{c^3}+\frac{117}{5c^2}-\frac{40}{a^2c}+\frac{16}{a^4}\ \Big|\ \underline{\frac{a^2}{5c^2}+\frac{5}{c}-\frac{4}{a^2}} \\
 & \frac{a^4}{25c^4} \\ \hline
\frac{2a^2}{5c^2} & \frac{2a^2}{c^3}+\frac{117}{5c^2} \\
\frac{2a^2}{5c^2}+\frac{5}{c} & \frac{2a^2}{c^3}+\frac{25}{c^2} \\ \hline
\frac{2a^2}{5c^2}+\frac{10}{c} & -\frac{8}{5c^2}-\frac{40}{a^2c}+\frac{16}{a^4} \\
\frac{2a^2}{5c^2}+\frac{10}{c}-\frac{4}{a^2} & -\frac{8}{5c^2}-\frac{40}{a^2c}+\frac{16}{a^4} \\ \hline
\end{array}$$

Therefore the square roots are $\pm\left(\frac{a^2}{5c^2}+\frac{5}{c}-\frac{4}{a^2}\right)$.

21.

$$\begin{array}{l|l}
 & \frac{a^2}{4c^4}-\frac{3a}{c^2}+9+\frac{2}{5ac}-\frac{12c}{5a^2}+\frac{4c^2}{25a^4}\ \Big|\ \underline{\frac{a}{2c^2}-3+\frac{2c}{5a^2}} \\
 & \frac{a^2}{4c^4} \\ \hline
\frac{a}{c^2} & -\frac{3a}{c^2}+9 \\
\frac{a}{c^2}-3 & -\frac{3a}{c^2}+9 \\ \hline
\frac{a}{c^2}-6 & +\frac{2}{5ac}-\frac{12c}{5a^2}+\frac{4c^2}{25a^4} \\
\frac{a}{c^2}-6+\frac{2c}{5a^2} & +\frac{2}{5ac}-\frac{12c}{5a^2}+\frac{4c^2}{25a^4} \\ \hline
\end{array}$$

Therefore the square roots are $\pm\left(\frac{a}{2c^2}-3+\frac{2c}{5a^2}\right)$.

22.

$$\begin{array}{l|l}
 & x^4-8x^3+24x^2-32x+16\ \big|\ \underline{x^2-4x+4} \\
 & x^4 \\ \hline
2x^2 & -8x^3+24x^2 \\
2x^2-4x & -8x^3+16x^2 \\ \hline
2x^2-8x & 8x^2-32x+16 \\
2x^2-8x+4 & 8x^2-32x+16 \\ \hline
\end{array}
\qquad
\begin{array}{l|l}
 & x^2-4x+4\ \big|\ \underline{x-2} \\
 & x^2 \\ \hline
2x & -4x+4 \\
2x-2 & -4x+4 \\ \hline
\end{array}$$

Therefore the fourth roots are $\pm(x-2)$.

$$\begin{array}{l|l}
 & c^4 - 4c^3d + 6c^2d^2 - 4cd^3 + d^4 \,\underline{|\, c^2 - 2cd + d^2} \\
 & \underline{c^4} \\
2c^2 & -4c^3d + 6c^2d^2 \\
2c^2 - 2cd & \underline{-4c^3d + 4c^2d^2} \\
2c^2 - 4cd & \qquad 2c^2d^2 - 4cd^3 + d^4 \\
2c^2 - 4cd + d^2 & \qquad \underline{2c^2d^2 - 4cd^3 + d^4}
\end{array}
\qquad
\begin{array}{l|l}
 & c^2 - 2cd + d^2 \,\underline{|\, c - d} \\
 & \underline{c^2} \\
2c & -2cd + d^2 \\
2c - d & \underline{-2cd + d^2}
\end{array}$$

Therefore the fourth roots are $\pm (c - d)$.

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The square root of:

1. $6241 = \pm 79$.	7. $.6279 = \pm .792+$.
2. $16129 = \pm 127$.	8. $.0451 = \pm .212+$.
3. $223{,}729 = \pm 473$.	9. $.0035 = \pm .059+$.
4. $2 = \pm 1.414+$.	10. $\frac{11}{5} = \pm 1.483+$.
5. $5 = \pm 2.236+$.	11. $1\frac{6}{7} = \pm 1.362+$.
6. $7.135 = \pm 2.671+$.	12. $\frac{2}{21} = \pm .308+$.

13. $\sqrt{(84)^2 + (13)^2} = 85$, hypotenuse; $\dfrac{84 \cdot 13}{2} = 546$, area.

14. $\sqrt{(133)^2 + (156)^2} = 205$, hypotenuse; $\dfrac{133 \cdot 156}{2} = 10{,}374$, area.

15. $\sqrt{(645)^2 + (812)^2} = 1037$, hypotenuse; $\dfrac{645 \cdot 812}{2} = 261{,}870$, area.

16. $\sqrt{(65)^2 - (56)^2} = 33$, leg; $\dfrac{56 \cdot 33}{2} = 924$, area.

17. $\sqrt{(397)^2 - (325)^2} = 228$, leg; $\dfrac{325 \cdot 228}{2} = 37{,}050$, area.

18. $\sqrt{(143)^2 + (24)^2} = 145$ feet.

19. $\sqrt{(401)^2 - (399)^2} = 40$, side; $40 \cdot 399 = 15{,}960$, area.

20. $\sqrt{(677)^2 - (52)^2} = 675$, side. Then 1454 is the perimeter.

21. Let $s =$ one side in feet.
Then $s + 21 =$ the other side in feet.
Therefore $2s + 2(s + 21) = 102$.
Whence $s = 15$,
and $s + 21 = 36$.
$\sqrt{(15)^2 + (36)^2} = 39$, the diagonal in feet.

22. Let $s =$ the number of meters in the side.
Then $2s^2 = (74)^2$.
Whence $s = 52.325+$ meters.

23. $\sqrt{(52)^2 + (52)^2} = 73.539+$ inches.

24. Let $s =$ the width.
Then $2.4\,s =$ the length.
Therefore

$$(s)^2 + (2.4\,s)^2 = (52)^2.$$

Whence $s = 20$,
and $2.4\,s = 48$, the length.

25. Let $l =$ the length.
Then $\dfrac{3\,l}{4} =$ the width.
Whence

$$l^2 + \left(\frac{3\,l}{4}\right)^2 = (100)^2.$$

Then $l = 80$,
and $\dfrac{3\,l}{4} = 60$.

The area $= 60 \cdot 80 = 4800$.

26. Let $w =$ the width.
Then

$$(2\,w)^2 - (w)^2 = (10)^2.$$

Whence $w = 5.773+$.

27. $\sqrt{6^2 - 3^2} = 5.196+$, BD.
Then
the area $= 15.588+$.

28. $\sqrt{(10)^2 - (5)^2} = 8.660+$, BD.
Then the area $= 43.30+$.

29. $(AB)^2 - \left(\dfrac{AB}{2}\right)^2 = (10)^2.$

Whence $AB = 11.546+$.
Area $= 57.734+$.

30. Let $a =$ the altitude.
Then

$$(12)^2 - (6)^2 = a^2,$$

or $a = 10.392+$.

31. Let $s =$ the side in centimeters.

Then $s^2 - \dfrac{s^2}{4} = (25)^2$,

or $s = 28.86+$.

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1. $x^{\frac{4}{3}} = \sqrt[3]{x^4}.$
2. $x^{\frac{5}{2}} = \sqrt{x^5}.$
3. $(cd)^{\frac{3}{2}} = \sqrt{(cd)^3}.$
4. $(2\,x)^{\frac{1}{3}} = \sqrt[3]{2\,x}.$
5. $2\,x^{\frac{1}{3}} = 2\sqrt[3]{x}.$
6. $5\,a^{\frac{3}{4}} = 5\sqrt[4]{a^3}.$
7. $(5\,a)^{\frac{3}{4}} = \sqrt[4]{(5\,a)^3}.$
8. $3\,rx^{\frac{2}{3}} = 3\,r\sqrt[3]{x^2}.$
9. $h^{\frac{2}{3}}k^{\frac{2}{3}} = \sqrt[3]{h^2k^2}.$
10. $7\,s^{\frac{2}{3}}(t + w)^{\frac{1}{2}} = 7\sqrt[3]{s^2} \cdot \sqrt{t + w}.$
11. $3\,a^{\frac{5}{2}}(bc)^{\frac{1}{3}} = 3\sqrt{a^5} \cdot \sqrt[3]{bc}.$
12. $5^{\frac{1}{2}}x^{\frac{2}{3}}k^{\frac{5}{4}} = \sqrt{5} \cdot \sqrt[3]{x^2} \cdot \sqrt[4]{k^5}.$
13. $4^{\frac{1}{3}}t^{\frac{1}{a}} = \sqrt[3]{4} \cdot \sqrt[a]{t}.$
14. $2\,x^{\frac{1}{b}}y^{\frac{c}{d}} = 2\sqrt[b]{x} \cdot \sqrt[d]{y^c}.$

15. $25^{\frac{1}{2}} = 5.$
16. $27^{\frac{1}{3}} = 3.$
17. $16^{\frac{1}{4}} = 2.$
18. $4^{\frac{3}{2}} = 8.$
19. $64^{\frac{2}{3}} = 16.$
20. $125^{\frac{2}{3}} = 25.$
21. $(-8)^{\frac{4}{3}} = 16.$
22. $32^{\frac{1}{5}} = 2.$
23. $81^{\frac{3}{4}} = 27.$
24. $(-216)^{\frac{2}{3}} = 36.$
25. $(\frac{1}{16})^{\frac{1}{2}} = \frac{1}{4}.$
26. $(\frac{1}{25})^{\frac{1}{2}} = \frac{1}{5}.$
27. $(\frac{1}{16})^{\frac{1}{4}} = \frac{1}{2}.$
28. $25^{\frac{1}{2}} \cdot 4^{\frac{5}{2}} = 160.$

29. $4^{\frac{1}{2}}\cdot(\frac{1}{4})^{\frac{1}{2}}=1.$
30. $(-32)^{\frac{4}{5}}(-64)^{\frac{1}{3}}=-64.$
31. $36^{\frac{1}{2}}\cdot(\frac{1}{9})^{\frac{1}{2}}=2.$
32. $9^{\frac{1}{2}}\cdot(\frac{1}{27})^{\frac{1}{3}}=1.$
33. $2(\frac{4}{9})^{\frac{1}{2}}\cdot(\frac{1}{8})^{\frac{1}{3}}=\frac{2}{3}.$
34. $\sqrt[5]{32}\cdot4^{\frac{1}{2}}=4.$
35. $\sqrt[3]{8^2}\cdot25^{\frac{3}{2}}=500.$
36. $121^{\frac{1}{2}}\cdot\sqrt{\frac{25}{121}}=5.$
37. $(-343)^{\frac{1}{3}}\cdot\sqrt{\frac{1}{49}}=-1.$
38. $\frac{2}{3}(\frac{169}{144})^{\frac{1}{2}}\div(\frac{1}{216})^{\frac{1}{3}}=\frac{13}{3}.$
39. $\sqrt[2]{a^3}=a^{\frac{3}{2}}.$
40. $\sqrt[2]{ax^4}=a^{\frac{1}{2}}x^2.$
41. $3\sqrt{2x^5}=3\cdot2^{\frac{1}{2}}x^{\frac{5}{2}}.$
42. $\sqrt{9x}=3x^{\frac{1}{2}}.$
43. $5\sqrt{16ax^2}=20a^{\frac{1}{2}}x.$
44. $\sqrt[3]{a^2}=a^{\frac{2}{3}}.$
45. $\sqrt[3]{ax^4}=a^{\frac{1}{3}}x^{\frac{4}{3}}.$
46. $2\sqrt[3]{2x^2}=2^{\frac{4}{3}}x^{\frac{2}{3}}.$
47. $3\sqrt[3]{8x^4}=6x^{\frac{4}{3}}.$
48. $4\sqrt[3]{27ax^3}=12a^{\frac{1}{3}}x.$
49. $2\sqrt[4]{a^2x^3}=2a^{\frac{1}{2}}x^{\frac{3}{4}}.$
50. $4\sqrt[4]{16x}=8x^{\frac{1}{4}}.$
51. $7\sqrt[5]{rs^7}=7r^{\frac{1}{5}}s^{\frac{7}{5}}.$
52. $5a\sqrt[5]{32m^2}=10am^{\frac{2}{5}}.$
53. $12x^2\sqrt[3]{ax^3}=12a^{\frac{1}{3}}x^3.$
54. $c\sqrt[2]{(de)^3}=cd^{\frac{3}{2}}e^{\frac{3}{2}}.$
55. $uv\sqrt[2]{(u+v)^5}=uv(u+v)^{\frac{5}{2}}.$
56. $3\sqrt[2]{a^3}\cdot\sqrt[3]{x^2}=3a^{\frac{3}{2}}x^{\frac{2}{3}}.$
57. $2a\sqrt[5]{ax^2}\cdot\sqrt[3]{2m}=2^{\frac{4}{3}}a^{\frac{6}{5}}x^{\frac{2}{5}}m^{\frac{1}{3}}.$
58. $\sqrt[n]{x^a}\cdot\sqrt[n]{y^b}=x^{\frac{a}{n}}y^{\frac{b}{n}}.$
59. $\sqrt[n]{x^a}\cdot\sqrt[n]{x^{3a}}=x^{\frac{4a}{n}}.$

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1. $\sqrt{12}=\sqrt{4\cdot3}=2\sqrt{3}.$
2. $\sqrt{32}=\sqrt{16\cdot2}=4\sqrt{2}.$
3. $\sqrt{75}=\sqrt{25\cdot3}=5\sqrt{3}.$
4. $2\sqrt{50}=2\sqrt{25\cdot2}=10\sqrt{2}.$
5. $\sqrt[3]{40}=\sqrt[3]{8\cdot5}=2\sqrt[3]{5}.$
6. $\sqrt[3]{54}=\sqrt[3]{27\cdot2}=3\sqrt[3]{2}.$
7. $\sqrt{20}=\sqrt{4\cdot5}=2\sqrt{5}.$
8. $\sqrt[3]{3000}=\sqrt[3]{1000\cdot3}=10\sqrt[3]{3}.$
9. $2\sqrt[3]{24}=2\sqrt[3]{8\cdot3}=4\sqrt[3]{3}.$
10. $\sqrt{\frac{1}{2}}=\sqrt{\frac{2}{4}}=\frac{1}{2}\sqrt{2}.$
11. $\sqrt{\frac{2}{5}}=\sqrt{\frac{10}{25}}=\frac{1}{5}\sqrt{10}.$
12. $\sqrt{\frac{3}{8}}=\sqrt{\frac{6}{16}}=\frac{1}{4}\sqrt{6}.$
13. $3\sqrt{\frac{9}{2}}=9\sqrt{\frac{2}{4}}=\frac{9}{2}\sqrt{2}.$
14. $5\sqrt{\frac{3}{5}}=5\sqrt{\frac{15}{25}}=\frac{5}{5}\sqrt{15}=\sqrt{15}.$
15. $\sqrt[3]{\frac{1}{2}}=\sqrt[3]{\frac{4}{8}}=\frac{1}{2}\sqrt[3]{4}.$
16. $4\sqrt[3]{\frac{1}{4}}=4\sqrt[3]{\frac{2}{8}}=\frac{4}{2}\sqrt[3]{2}=2\sqrt[3]{2}.$
17. $8\sqrt[3]{\frac{7}{4}}=8\sqrt[3]{\frac{14}{8}}=\frac{8}{2}\sqrt[3]{14}=4\sqrt[3]{14}.$
18. $\sqrt{1-(\frac{1}{2})^2}=\sqrt{\frac{3}{4}}=\frac{1}{2}\sqrt{3}.$
19. $\sqrt{1+(\frac{1}{3})^2}=\sqrt{\frac{10}{9}}=\frac{1}{3}\sqrt{10}.$
20. $\sqrt{2+(\frac{3}{5})^2}=\sqrt{\frac{59}{25}}=\frac{1}{5}\sqrt{59}.$
21. $\sqrt[3]{1-(\frac{1}{2})^2}=\sqrt[3]{\frac{3}{4}}=\sqrt[3]{\frac{6}{8}}=\frac{1}{2}\sqrt[3]{6}.$
22. $\sqrt[3]{4+(\frac{2}{3})^2}=\sqrt[3]{\frac{40}{9}}=2\sqrt[3]{\frac{5}{9}}=\frac{2}{3}\sqrt[3]{15}.$
23. $\sqrt[3]{1+(\frac{1}{3})^3}=\sqrt[3]{\frac{28}{27}}=\frac{1}{3}\sqrt[3]{28}.$
24. $\sqrt[4]{25a^2}=\sqrt[4]{5^2\cdot a^2}=\sqrt{5a}.$
25. $3\sqrt{4x^5}=3\sqrt{4x^4}\cdot\sqrt{x}=6x^2\sqrt{x}.$
26. $a\sqrt[6]{8x^3}=a\sqrt[6]{2^3x^3}=a\sqrt{2x}.$
27. $x\sqrt[4]{49a^2x^2}=x\sqrt[4]{(7ax)^2}=x\sqrt{7ax}.$
28. $\sqrt[6]{125x^{3a}y^{3m}}=\sqrt[6]{5^3\cdot x^{3a}\cdot y^{3m}}=\sqrt{5x^ay^m}.$

29. $\sqrt{\frac{a^3}{27}} = \sqrt{3a \cdot \frac{a^2}{81}} = \frac{a}{9}\sqrt{3a}.$

30. $a\sqrt[3]{\frac{5}{2a^2}} = a\sqrt[3]{\frac{20a}{8a^3}} = \frac{a}{2a}\sqrt[3]{20a} = \frac{1}{2}\sqrt[3]{20a}.$

31. $\sqrt{R^2 - \left(\frac{R}{2}\right)^2} = \sqrt{\frac{3R^2}{4}} = \frac{R}{2}\sqrt{3}.$

32. $\sqrt{R^2 + \left(\frac{R}{2}\right)^2} = \sqrt{\frac{5R^2}{4}} = \frac{R}{2}\sqrt{5}.$

33. $\sqrt{\left(a + \frac{a}{3}\right)\left(a - \frac{a}{3}\right)} = \sqrt{a^2 - \frac{a^2}{9}} = \sqrt{\frac{8a^2}{9}} = \frac{2a}{3}\sqrt{2}.$

34. $\sqrt[3]{R^3 - \left(\frac{R}{2}\right)^3} = \sqrt[3]{\frac{7R^3}{8}} = \frac{R}{2}\sqrt[3]{7}.$

35. $(x - y)\sqrt{\frac{x + y}{x - y}} = (x - y)\sqrt{\frac{(x^2 - y^2)}{(x - y)^2}} = \frac{(x - y)}{(x - y)}\sqrt{x^2 - y^2} = \sqrt{x^2 - y^2}.$

36. $\sqrt{4 + 4\sqrt{2}} = \sqrt{4(1 + \sqrt{2})} = 2\sqrt{1 + \sqrt{2}}.$

37. $\sqrt{8 - 4\sqrt{2}} = \sqrt{4(2 - \sqrt{2})} = 2\sqrt{2 - \sqrt{2}}.$

38. $\sqrt{18 + 9\sqrt{3}} = \sqrt{9(2 + \sqrt{3})} = 3\sqrt{2 + \sqrt{3}}.$

39. $\sqrt{25\sqrt{5} - 100} = \sqrt{25(\sqrt{5} - 4)} = 5\sqrt{\sqrt{5} - 4}.$

40. $\sqrt[3]{16 + 8\sqrt{2}} = \sqrt[3]{8(2 + \sqrt{2})} = 2\sqrt[3]{2 + \sqrt{2}}.$

41. $\sqrt[3]{81 + 3\sqrt[2]{243}} = \sqrt[3]{81 + 27\sqrt{3}} = \sqrt[3]{27(3 + \sqrt{3})} = 3\sqrt[3]{3 + \sqrt{3}}.$

42. $\sqrt{R^2 - 2R^2\sqrt{2}} = \sqrt{R^2(1 - 2\sqrt{2})} = R\sqrt{1 - 2\sqrt{2}}.$

43. $\sqrt[4]{32 - 64\sqrt{3}} = \sqrt[4]{16(2 - 4\sqrt{3})} = 2\sqrt[4]{2 - 4\sqrt{3}}.$

44. $\sqrt{\frac{R^2 + R^2\sqrt{3}}{2}} = \sqrt{\frac{2R^2(1 + \sqrt{3})}{4}} = \frac{R}{2}\sqrt{2 + 2\sqrt{3}}.$

45. $\sqrt{R^2 - \left(\frac{R}{2}\right)^2\sqrt{2}} = \sqrt{\frac{4R^2 - R^2\sqrt{2}}{4}} = \sqrt{\frac{R^2}{4}(4 - \sqrt{2})} = \frac{R}{2}\sqrt{4 - \sqrt{2}}.$

46. $\sqrt{R^2\sqrt{2} - \frac{R^2}{2}} = \sqrt{\frac{4R^2\sqrt{2} - 2R^2}{4}} = \frac{1}{2}\sqrt{R^2(4\sqrt{2} - 2)} = \frac{R}{2}\sqrt{4\sqrt{2} - 2}.$

47. $\sqrt{\frac{R^2}{3} - R^2\sqrt{3}} = \sqrt{\frac{3R^2 - 9R^2\sqrt{3}}{9}} = \frac{1}{3}\sqrt{R^2(3 - 9\sqrt{3})} = \frac{R}{3}\sqrt{3 - 9\sqrt{3}}.$

48. $2\sqrt{2} = \sqrt{4} \cdot \sqrt{2} = \sqrt{8}.$

49. $3\sqrt{5} = \sqrt{9} \cdot \sqrt{5} = \sqrt{45}.$

50. $3\sqrt[3]{4} = \sqrt[3]{27} \cdot \sqrt[3]{4} = \sqrt[3]{108}.$

51. $4\sqrt[3]{2} = \sqrt[3]{64}\sqrt[3]{2} = \sqrt[3]{128}.$

52. $3\sqrt[3]{\frac{1}{3}} = \sqrt[3]{27}\sqrt[3]{\frac{1}{3}} = \sqrt[3]{\frac{27}{3}} = \sqrt[3]{9}.$

53. $x^2\sqrt{x^3} = \sqrt{x^4}\sqrt{x^3} = \sqrt{x^7}.$

54. $2x^2\sqrt[3]{x^2} = \sqrt[3]{8x^6}\cdot\sqrt[3]{x^2} = \sqrt[3]{8x^8}.$

55. $\frac{a}{2}\sqrt[3]{\frac{4}{a^2}} = \sqrt[3]{\frac{a^3}{8}}\cdot\sqrt[3]{\frac{4}{a^2}} = \sqrt[3]{\frac{a}{2}}.$

56. $(a+2)\sqrt{\frac{1}{a^2-4}} = \sqrt{(a+2)^2}\sqrt{\frac{1}{a^2-4}} = \sqrt{\frac{a+2}{a-2}}.$

57. $\frac{x+3}{ax}\sqrt[3]{\frac{a^2x^2}{(x+3)^2}} = \sqrt[3]{\frac{(x+3)^3}{(ax)^3}}\sqrt[3]{\frac{a^2x^2}{(x+3)^2}} = \sqrt[3]{\frac{x+3}{ax}}.$

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1. $\sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}.$

2. $\sqrt{45} - \sqrt{20} = 3\sqrt{5} - 2\sqrt{5} = \sqrt{5}.$

3. $2\sqrt{200} - 3\sqrt{8} = 20\sqrt{2} - 6\sqrt{2} = 14\sqrt{2}.$

4. $\sqrt[3]{56} + 2\sqrt[3]{189} = 2\sqrt[3]{7} + 6\sqrt[3]{7} = 8\sqrt[3]{7}.$

5. $2\sqrt[3]{320} - \sqrt[2]{50} = 8\sqrt[3]{5} - 5\sqrt{2}.$

6. $\sqrt[4]{25} - \sqrt[2]{50} = \sqrt{5} - 5\sqrt{2}.$

7. $\sqrt[4]{32} + 5\sqrt[4]{162} = 2\sqrt[4]{2} + 15\sqrt[4]{2} = 17\sqrt[4]{2}.$

8. $5\sqrt{\tfrac{1}{2}} - \tfrac{3}{2}\sqrt{2} = \tfrac{5}{2}\sqrt{2} - \tfrac{3}{2}\sqrt{2} = \sqrt{2}.$

9. $\sqrt{\tfrac{1}{3}} + 2\sqrt{\tfrac{4}{3}} - 3\sqrt{\tfrac{2}{3}} = \tfrac{1}{3}\sqrt{3} + \tfrac{4}{3}\sqrt{3} - \sqrt{6} = \tfrac{5}{3}\sqrt{3} - \sqrt{6}.$

10. $\sqrt{\tfrac{3}{10}} - \sqrt{120} - 2\sqrt{\tfrac{6}{5}} = \tfrac{1}{10}\sqrt{30} - 2\sqrt{30} - \tfrac{2}{5}\sqrt{30} = -\tfrac{23}{10}\sqrt{30}.$

11. $\sqrt{\tfrac{5}{18}} + 2\sqrt{\tfrac{32}{5}} - \sqrt{\tfrac{10}{9}} = \tfrac{1}{6}\sqrt{10} + \tfrac{8}{5}\sqrt{10} - \tfrac{1}{3}\sqrt{10} = \tfrac{43}{30}\sqrt{10}.$

12. $\sqrt[3]{\tfrac{6}{8}} - \sqrt[3]{12} + \sqrt[3]{6} = \tfrac{1}{2}\sqrt[3]{6} - \sqrt[3]{12} + \sqrt[3]{6} = \tfrac{3}{2}\sqrt[3]{6} - \sqrt[3]{12}.$

13. $a\sqrt[4]{4} + \sqrt{8a^2} = a\sqrt{2} + 2a\sqrt{2} = 3a\sqrt{2}.$

14. $R - \sqrt{\frac{3R^2}{4}} = R - \frac{R}{2}\sqrt{3}.$

15. $2x\sqrt[3]{54x} - 3\sqrt[3]{16x^4} + \sqrt[6]{4x^2} = 6x\sqrt[3]{2x} - 6x\sqrt[3]{2x} + \sqrt[3]{2x} = \sqrt[3]{2x}.$

16. $\sqrt[3]{81x^7} + x\sqrt[3]{375x^4} - \sqrt[12]{16x^4} = 3x^2\sqrt[3]{3x} + 5x^2\sqrt[3]{3x} - \sqrt[3]{2x}$
$= 8x^2\sqrt[3]{3x} - \sqrt[3]{2x}.$

17. $\sqrt{a^3bc} - a\sqrt{abc} + ac\sqrt{\frac{b}{ac}} = a\sqrt{abc} - a\sqrt{abc} + \sqrt{abc} = \sqrt{abc}.$

18. $\sqrt{(m+n)^3} - n\sqrt[4]{(m+n)^2} = (m+n)\sqrt{m+n} - n\sqrt{(m+n)}$
$= m\sqrt{m+n}.$

19. $\sqrt[3]{(a+b)^4} - \sqrt[3]{8a^3(a+b)} + \sqrt[6]{a^2+2ab+b^2}$
$= (a+b)\sqrt[3]{a+b} - 2a\sqrt[3]{a+b} + \sqrt[3]{a+b}$
$= b\sqrt[3]{a+b} - a\sqrt[3]{a+b} + \sqrt[3]{a+b}$
$= (b-a+1)\sqrt[3]{a+b}.$

20. $\sqrt[2]{a^3 + 4a^2 + 4a} - \sqrt[2]{a^3} - \frac{2}{a^3}\sqrt{a^7} = (a + 2)\sqrt{a} - a\sqrt{a} - 2\sqrt{a} = 0.$

21. $\sqrt{x^3y^3} - x^2y^2\sqrt{\frac{1}{xy}} + xy\sqrt{2 + \frac{x^2 + y^2}{xy}}$

$= xy\sqrt{xy} - xy\sqrt{xy} + \sqrt{(x^2 + 2xy + y^2)xy}$

$= (x + y)\sqrt{xy}.$

22. $rs\sqrt[3]{rs} + \sqrt[3]{\frac{1}{r^2s^2}} - 2\sqrt[3]{r^4s^4} = rs\sqrt[3]{rs} + \frac{1}{rs}\sqrt[3]{rs} - 2rs\sqrt[3]{rs}$

$= \frac{1}{rs}\sqrt[3]{rs} - rs\sqrt[3]{rs}$

$= \sqrt{rs}\left(\frac{1}{rs} - rs\right).$

$= \frac{1 - r^2s^2}{rs}\sqrt{rs}.$

23. $2\sqrt{\frac{y}{x}} - \sqrt{\frac{x}{y}} + \sqrt{2 + \frac{x^2 + y^2}{xy}} = \frac{2}{x}\sqrt{xy} - \frac{1}{y}\sqrt{xy} + \frac{x + y}{xy}\sqrt{xy}$

$= \sqrt{xy}\left(\frac{2}{x} - \frac{1}{y} + \frac{x + y}{xy}\right) = \frac{3}{x}\sqrt{xy}.$

24. $\sqrt{3x^2 - 18x + 27} - \sqrt{27(x^2 + 2x + 1)} = (x - 3)\sqrt{3} - (3x + 3)\sqrt{3}$

$= -2x\sqrt{3} - 6\sqrt{3} = \sqrt{3}(-2x - 6).$

25. $\sqrt[3]{(a - 3)^2(5a - 15)} + \sqrt[3]{40} = (a - 3)\sqrt[3]{5} + 2\sqrt[3]{5}$

$= a\sqrt[3]{5} - \sqrt[3]{5} = \sqrt[3]{5}(a - 1).$

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1. $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4.$

2. $\sqrt{3} \cdot \sqrt{27} = \sqrt{81} = 9.$

3. $5^{\frac{1}{2}} \cdot 20^{\frac{1}{2}} = 100^{\frac{1}{2}} = 10.$

4. $18^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = 144^{\frac{1}{2}} = 12.$

5. $\sqrt[3]{16} \cdot \sqrt[3]{4} = \sqrt[3]{64} = 4.$

6. $\sqrt[3]{4} \cdot \sqrt[3]{12} = \sqrt[3]{48} = 2\sqrt[3]{6}.$

7. $(100)^{\frac{1}{3}}(30)^{\frac{1}{3}} = 3000^{\frac{1}{3}} = 10\sqrt[3]{3}.$

8. $\sqrt[4]{8} \cdot \sqrt[4]{32} = \sqrt[4]{2^3 \cdot 2^5} = \sqrt[4]{2^8}$

$= 2^2 = 4.$

9. $\sqrt{\frac{12}{25}} \cdot \sqrt{75} = \sqrt{36} = 6.$

10. $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{27}{8}} = \sqrt{\frac{9}{4}} = \frac{3}{2}.$

11. $\sqrt{11} \cdot \sqrt{\frac{1}{11}} = \sqrt{1} = 1.$

12. $a^{\frac{1}{2}} \cdot (bc)^{\frac{1}{2}} = \sqrt{abc}.$

13. $2\sqrt{x} \cdot \sqrt[2]{4x^3} = 2\sqrt{4x^4} = 4x^2.$

14. $5\sqrt[3]{2a} \cdot 3\sqrt[3]{16a} = 15\sqrt[3]{32a^2}$

$= 30\sqrt[3]{4a^2}.$

15. $2\sqrt{Rs} \cdot 7\sqrt{r^3s^3t^2} = 14\sqrt{r^3s^4t^2R}$

$= 14rs^2t\sqrt{Rr}.$

16. $\sqrt{\frac{a}{x}} \cdot \sqrt{\frac{4x}{a}} = \sqrt{4} = 2.$

17. $\sqrt{75a} \cdot (45a)^{\frac{1}{2}} = \sqrt{3 \cdot 5^2 \cdot 3^2 \cdot 5a^2} = 15a\sqrt{15}.$

18. $\sqrt{2u} \cdot \sqrt{4v} \cdot \sqrt{6uv} = \sqrt{48u^2v^2} = 4uv\sqrt{3}.$

19. $5\sqrt{3m}\cdot 5\sqrt{3m} = 25\sqrt{9m^2} = 75m.$

20. $(3\sqrt{3x})^2 = 9\cdot 3x = 27x.$

21. $\sqrt[3]{2}\cdot\sqrt{2} = 2^{\frac{1}{3}}\cdot 2^{\frac{1}{2}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5} = \sqrt[6]{32}.$

22. $\sqrt{3}\cdot\sqrt[3]{3} = 3^{\frac{1}{2}}\cdot 3^{\frac{1}{3}} = 3^{\frac{5}{6}} = \sqrt[6]{3^5} = \sqrt[6]{243}.$

23. $\sqrt[3]{2}\cdot\sqrt{3} = 2^{\frac{1}{3}}\cdot 3^{\frac{1}{2}} = 2^{\frac{2}{6}}\cdot 3^{\frac{3}{6}} = \sqrt[6]{2^2\cdot 3^3} = \sqrt[6]{108}.$

24. $\sqrt{2}\cdot\sqrt[3]{3} = 2^{\frac{1}{2}}\cdot 3^{\frac{1}{3}} = 2^{\frac{3}{6}}\cdot 3^{\frac{2}{6}} = \sqrt[6]{2^3\cdot 3^2} = \sqrt[6]{72}.$

25. $\sqrt[3]{4}\cdot\sqrt{2} = 2^{\frac{2}{3}}\cdot 2^{\frac{1}{2}} = 2^{\frac{7}{6}} = 2\sqrt[6]{2}.$

26. $\sqrt{8}\cdot\sqrt[3]{8} = 2^{\frac{3}{2}}\cdot 2 = 4\sqrt{2}.$

27. $\sqrt{3}\cdot\sqrt[3]{24} = 3^{\frac{1}{2}}\cdot 2\cdot 3^{\frac{1}{3}} = 2\cdot 3^{\frac{5}{6}} = 2\sqrt[6]{243}.$

28. $\sqrt{a}\cdot\sqrt[3]{a} = a^{\frac{1}{2}}\cdot a^{\frac{1}{3}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}.$

29. $\sqrt{a^3}\cdot\sqrt[3]{a^2} = a^{\frac{3}{2}}\cdot a^{\frac{2}{3}} = a^{\frac{13}{6}} = a^2\sqrt[6]{a}.$

30. $\sqrt{2a}\cdot\sqrt[3]{2a} = (2a)^{\frac{1}{2}}(2a)^{\frac{1}{3}} = (2a)^{\frac{5}{6}} = \sqrt[6]{(2a)^5} = \sqrt[6]{32a^5}.$

31. $\sqrt[3]{4x^2}\cdot\sqrt{2x} = 2^{\frac{2}{3}}\cdot x^{\frac{2}{3}}\cdot 2^{\frac{1}{2}}\cdot x^{\frac{1}{2}} = 2^{\frac{4}{6}}\cdot x^{\frac{4}{6}}\cdot 2^{\frac{3}{6}}\cdot x^{\frac{3}{6}} = 2^{\frac{7}{6}}x^{\frac{7}{6}} = 2x\sqrt[6]{2x}.$

32.
$$\sqrt[3]{\frac{5x}{a}}\cdot\sqrt[2]{\frac{a^3}{5x^2}} = \left(\frac{5x}{a}\right)^{\frac{1}{3}}\left(\frac{a}{5}\right)^{\frac{1}{2}}\cdot\frac{a}{x} = \frac{a}{x}\left(\frac{5x}{a}\right)^{\frac{2}{6}}\left(\frac{a}{5}\right)^{\frac{3}{6}} = \frac{a}{x}\sqrt[6]{\left(\frac{5x}{a}\right)^2\left(\frac{a}{5}\right)^3}$$
$$= \frac{a}{x}\sqrt[6]{\frac{x^2a}{5}} = \frac{a}{5x}\sqrt[6]{3125ax^2}.$$

33. $(x + a\sqrt{m})\sqrt{m^3} = x\sqrt{m^3} + a\sqrt{m^4} = mx\sqrt{m} + am^2.$

34. $(\sqrt{5a} - \sqrt{10a^2})(\sqrt{5a}) = 5a - \sqrt{50a^3} = 5a - 5a\sqrt{2a}.$

35. $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3}) = 2 - 3 = -1.$

36.
$$\begin{array}{l}
5\sqrt{3} - 4 \\
\underline{3\sqrt{3} + 8} \\
15\cdot 3 - 12\sqrt{3} \\
\underline{\qquad\qquad 40\sqrt{3} - 32} \\
\quad 45 + 28\sqrt{3} - 32 = 13 + 28\sqrt{3}.
\end{array}$$

37.
$$\begin{array}{llllll}
\sqrt{5} & - \sqrt{3} & & - \sqrt{2} & & \\
\sqrt{5} & + \sqrt{3} & & + \sqrt{2} & & \\
\hline
5 & - \sqrt{15} & - \sqrt{10} & & & \\
 & \sqrt{15} & & - 3 & - \sqrt{6} & \\
 & & + \sqrt{10} & & - \sqrt{6} & - 2 \\
\hline
5 & & & - 3 & - 2\sqrt{6} & - 2 = -2\sqrt{6}.
\end{array}$$

38.
$$\begin{array}{llllll}
3\sqrt{2} + 2\sqrt{3} + \sqrt{30} & & & & & \\
\sqrt{2} + \sqrt{3} - \sqrt{5} & & & & & \\
\hline
3\cdot 2 & +2\sqrt{6} & +2\sqrt{15} & & & \\
 & 3\sqrt{6} & & +2\cdot 3 & +3\sqrt{10} & \\
 & -5\sqrt{6} & -2\sqrt{15} & & -3\sqrt{10} & \\
\hline
6 & & & +6 & & =12.
\end{array}$$

39.
$$\frac{6a+b\sqrt{2}}{2}\cdot\frac{6a+b\sqrt{2}}{2}=\frac{36a^2+12ab\sqrt{2}+2b^2}{4}$$
$$=9a^2+3ab\sqrt{2}+\frac{b^2}{2}.$$

40.
$$\begin{array}{llll}
R & -\frac{R}{2}\sqrt{3} & & \\
2R & +\frac{3R}{2}\sqrt{3} & & \\
\hline
2R^2 & -R^2\sqrt{3} & & \\
 & \frac{3R^2}{2}\sqrt{3} & -\frac{3R^2}{4}\cdot 3 & \\
\hline
2R^2 & +\frac{R^2}{2}\sqrt{3} & -\frac{9R^2}{4} & =\frac{R^2}{2}\sqrt{3}-\frac{R^2}{4}=\frac{R^2}{2}\left(\sqrt{3}-\frac{1}{2}\right).
\end{array}$$

41. $$\sqrt{a+b}\cdot\sqrt{a-b}\cdot\sqrt{2a^2-2b^2}=(a^2-b^2)\sqrt{2}.$$

42.
$$\begin{array}{llll}
2\sqrt{x-a} & +\sqrt{a} & & \\
-3\sqrt{x-a} & -5\sqrt{a} & & \\
\hline
-6(x-a) & -3\sqrt{xa-a^2} & & \\
 & -10\sqrt{xa-a^2} & -5a & \\
\hline
-6x+6a & -13\sqrt{ax-a^2} & -5a & =a-6x-13\sqrt{ax-a^2}.
\end{array}$$

43. $(3\sqrt[3]{x-a})(-5\sqrt[3]{x-a})=-15\sqrt[3]{x^2-2ax+a^2}.$

44. $\sqrt{2-\sqrt{2}}\cdot\sqrt{2+\sqrt{2}}=\sqrt{4-2}=\sqrt{2}.$

45. $\sqrt{a-\sqrt{b}}\cdot\sqrt{2a-2\sqrt{b}}=\sqrt{a-\sqrt{b}}\cdot\sqrt{2(a-\sqrt{b})}=(a-\sqrt{b})\sqrt{2}.$

46. $\sqrt{\frac{R}{2}\sqrt{5}+R}\cdot\sqrt{\frac{R}{2}\sqrt{5}-R}=\sqrt{\frac{R^2\cdot 5}{4}-R^2}=\sqrt{\frac{R^2}{4}}=\frac{R}{2}.$

47. $(\sqrt[3]{2})^2=\sqrt[3]{4}.$

48. $(2\sqrt[3]{3})^2=4\sqrt[3]{9}.$

49. $(2\sqrt[3]{12})^2=4\sqrt[3]{144}=8\sqrt[3]{18}.$

50. $\left(\sqrt{2-\sqrt{2}}\right)^2=2-\sqrt{2}.$

51. $\left(3\sqrt{x+\sqrt{3}}\right)^2=9(x+\sqrt{3})=9x+9\sqrt{3}.$

52. $[(2\sqrt[3]{3})^2]^2=(4\sqrt[3]{9})^2=16\sqrt[3]{81}=48\sqrt[3]{3}.$

53. $(2a\sqrt[3]{8x})^2 = 4a^2\sqrt[3]{64x^2} = 16a^2\sqrt[3]{x^2}.$

54. $(\sqrt[3]{2}+\sqrt[3]{3})^2 = \sqrt[3]{4} + 2\sqrt[3]{6} + \sqrt[3]{9}.$

55. $\left(\sqrt[3]{4+4\sqrt{3}}\right)^2 = \sqrt[3]{16+32\sqrt{3}+48} = \sqrt[3]{8(8+4\sqrt{3})} = 2\sqrt[3]{8+4\sqrt{3}}.$

56. $\left(\frac{1}{3}\sqrt[3]{9-9\sqrt{2}}\right)^2 = \frac{1}{9}\sqrt[3]{81-162\sqrt{2}+162} = \frac{1}{9}\sqrt[3]{27(9-6\sqrt{2})}$

$$= \frac{1}{3}\sqrt[3]{9-6\sqrt{2}}.$$

57. $(\sqrt{2}+\sqrt[3]{3})^2 = 2 + 2\sqrt{2}\sqrt[3]{3} + \sqrt[3]{9} = 2 + 2\sqrt[6]{72} + \sqrt[3]{9}.$

58. $\left(2\cdot\sqrt[4]{3-\sqrt{2}}\right)^2 = 4\sqrt[2]{3-\sqrt{2}}.$

59. $(2\sqrt{3})^3 = 8\sqrt{27} = 24\sqrt{3}.$

60. $(3\sqrt{2})^3 = 27\sqrt{8} = 54\sqrt{2}.$

61. $[(\sqrt{2})^2]^3 = (2)^3 = 8.$

62. $[(\sqrt[2]{3})^3]^3 = (3^{\frac{1}{2}})^9 = 3^{\frac{9}{2}} = 81\sqrt{3}.$

63. $(\sqrt{3}-\sqrt{6})^3 = (\sqrt{3})^3 - 3(\sqrt{3})^2\sqrt{6} + 3(\sqrt{3})(\sqrt{6})^2 - (\sqrt{6})^3$

$$= 21\sqrt{3} - 15\sqrt{6}.$$

64. $(2\sqrt{2}+\sqrt{3})^3 = (2\sqrt{2})^3 + 3(2\sqrt{2})^2\sqrt{3} + 3(2\sqrt{2})(\sqrt{3})^2 + (\sqrt{3})^3$

$$= 34\sqrt{2} + 27\sqrt{3}.$$

65. $(\sqrt[3]{3}-\sqrt[3]{2})^3 = (\sqrt[3]{3})^3 - 3(\sqrt[3]{3})^2\cdot\sqrt[3]{2} + 3(\sqrt[3]{3})(\sqrt[3]{2})^2 - (\sqrt[3]{2})^3$

$$= 1 - 3\sqrt[3]{18} + 3\sqrt[3]{12}.$$

66. $\left(\sqrt{2+\sqrt{2}}\right)^3 = (2+\sqrt{2})\sqrt{2+\sqrt{2}}.$

67. $(\sqrt[3]{2}-\sqrt{3})^3 = (\sqrt[3]{2})^3 - 3(\sqrt[3]{2})^2\sqrt{3} + 3(\sqrt[3]{2})(\sqrt{3})^2 - (\sqrt{3})^3$

$$= 2 - 3\sqrt[6]{432} + 9\sqrt[3]{2} - 3\sqrt{3}.$$

68. $R^2 + \left(\frac{R}{2}\sqrt{2-\sqrt{2}}\right)^2 = R^2 + \left[\frac{R^2}{4}(2-\sqrt{2})\right]$

$$= R^2 + \frac{R^2}{2} - \frac{R^2\sqrt{2}}{4}$$

$$= \frac{3R^2}{2} - \frac{R^2\sqrt{2}}{4}$$

$$= \frac{R^2}{4}(6-\sqrt{2}).$$

69. $\sqrt{R^2 - \left(\frac{R}{2}\sqrt{5} - \frac{R}{2}\right)^2} = \sqrt{R^2 - \left(\frac{R^2\cdot 5}{4} - \frac{2R^2\sqrt{5}}{4} + \frac{R^2}{4}\right)}$

$$= \sqrt{\frac{4R^2 - 6R^2 + 2R^2\sqrt{5}}{4}}$$

$$= \sqrt{\frac{R^2}{4}(2\sqrt{5}-2)}$$

$$= \frac{R}{2}\sqrt{2\sqrt{5}-2}.$$

70. $$\left[R^2-\left(\frac{R\sqrt{5}-R}{4}\right)^2\right]^{\frac{1}{2}}=\left[R^2-\left(\frac{R^2\cdot 5-2\,R^2\sqrt{5}+R^2}{16}\right)\right]^{\frac{1}{2}}$$
$$=\tfrac{1}{4}\left[16\,R^2-6\,R^2+2\,R^2\sqrt{5}\right]^{\frac{1}{2}}$$
$$=\tfrac{1}{4}\left[R^2(10+2\sqrt{5})\right]^{\frac{1}{2}}$$
$$=\frac{R}{4}\sqrt{10+2\sqrt{5}}.$$

71. $$\left[\left(R-\frac{R}{2}\sqrt{3}\right)^2+\left(\frac{R}{2}\right)^2\right]^{\frac{1}{2}}=\left[\left(R^2-R^2\sqrt{3}+\frac{3\,R^2}{4}\right)+\frac{R^2}{4}\right]^{\frac{1}{2}}$$
$$=\left[R^2(2-\sqrt{3})\right]^{\frac{1}{2}}$$
$$=R\sqrt{2-\sqrt{3}}.$$

72. $$\left[R^2-\left(\frac{R\sqrt{2-\sqrt{3}}}{2}\right)^2\right]^{\frac{1}{2}}=\left[R^2-\left(\frac{R^2(2-\sqrt{3})}{4}\right)\right]^{\frac{1}{2}}$$
$$=\tfrac{1}{2}\left[4\,R^2-2\,R^2+R^2\sqrt{3}\right]^{\frac{1}{2}}$$
$$=\tfrac{1}{2}\left[R^2(2+\sqrt{3})\right]^{\frac{1}{2}}$$
$$=\frac{R}{2}\sqrt{2+\sqrt{3}}.$$

73. $$\sqrt{R^2-\left(\frac{R\sqrt{2-\sqrt{2}}}{2}\right)^2}=\left[R^2-\frac{R^2(2-\sqrt{2})}{4}\right]^{\frac{1}{2}}$$
$$=\left[R^2-\frac{2\,R^2}{4}+\frac{R^2\sqrt{2}}{4}\right]^{\frac{1}{2}}$$
$$=\left[\frac{2\,R^2+R^2\sqrt{2}}{4}\right]^{\frac{1}{2}}$$
$$=\frac{R}{2}\sqrt{2+\sqrt{2}}.$$

74. $$\sqrt{\left(\frac{R\sqrt{2}}{2}\right)^2+\left(R-\frac{R}{2}\sqrt{2}\right)^2}=\sqrt{\frac{2\,R^2}{4}+\left(R^2-R^2\sqrt{2}+\frac{2\,R^2}{4}\right)}$$
$$=\sqrt{2\,R^2-R^2\sqrt{2}}$$
$$=R\sqrt{2-\sqrt{2}}.$$

75. $$\left(\frac{R}{2}\sqrt{2+\sqrt{2}}\right)\left(\frac{R\sqrt{2-\sqrt{2}}}{2}\right)4=\left(\frac{R^2}{4}\sqrt{4-2}\right)4=R^2\sqrt{2}.$$

76. When $x=-2+\sqrt{3}$,
$$x^2+4x+1=(4-4\sqrt{3}+3)+(-8+4\sqrt{3})+1=0.$$

77. When $x=2+\sqrt{5}$,
$$x^2-2x-3=(4+4\sqrt{5}+5)-(4+2\sqrt{5})-3=2+2\sqrt{5}$$

78. If $x = 2 + \sqrt{5}$,

$$x^2 - 4x - 1 = (4 + 4\sqrt{5} + 5) - (8 + 4\sqrt{5}) - 1 = 0.$$

If $x = 2 - \sqrt{5}$,

$$x^2 - 4x - 1 = (4 - 4\sqrt{5} + 5) - (8 - 4\sqrt{5}) - 1 = 0.$$

79. If $x = \frac{1}{2} + \frac{1}{2}\sqrt{13}$,

$$\begin{aligned} 3x^2 + 3x - 5 &= 3(\tfrac{1}{4} + \tfrac{1}{2}\sqrt{13} + \tfrac{13}{4}) + (\tfrac{3}{2} + \tfrac{3}{2}\sqrt{13}) - 5 \\ &= \tfrac{21}{2} + \tfrac{3}{2}\sqrt{13} + \tfrac{3}{2} + \tfrac{3}{2}\sqrt{13} - 5 \\ &= 7 + 3\sqrt{13}. \end{aligned}$$

If $x = \frac{1}{2} - \frac{1}{2}\sqrt{13}$,

$$\begin{aligned} 3x^2 + 3x - 5 &= (\tfrac{3}{4} - \tfrac{3}{2}\sqrt{13} + \tfrac{39}{4}) + (\tfrac{3}{2} - \tfrac{3}{2}\sqrt{13}) - 5 \\ &= 7 - 3\sqrt{13}. \end{aligned}$$

80. If $x = 2 + \sqrt{3}$, $4 + 4\sqrt{3} + 3 - 8 - 4\sqrt{3} + 1 = 0$. Yes.

If $x = 2 - \sqrt{3}$, $4 - 4\sqrt{3} + 3 - 8 + 4\sqrt{3} + 1 = 0$. Yes.

81. If $x = -4 + \sqrt{5}$, $16 - 8\sqrt{5} + 5 - 32 + 8\sqrt{5} + 11 = 0$. Yes.

If $x = -4 - \sqrt{5}$, $16 + 8\sqrt{5} + 5 - 32 - 8\sqrt{5} + 11 = 0$. Yes.

82. That $2 \pm \sqrt{3}$ are roots of $x^2 - 4x + 1 = 0$, and that $-4 \pm \sqrt{5}$ are roots of $x^2 + 8x + 11 = 0$.

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A rationalizing factor of:

1. $\sqrt{5}$ is $\sqrt{5}$, $\sqrt{5} \cdot \sqrt{5} = 5$.
2. $3\sqrt{6}$ is $\sqrt{6}$, $3\sqrt{6} \cdot \sqrt{6} = 18$.
3. $2\sqrt{7}$ is $\sqrt{7}$, $2\sqrt{7} \cdot \sqrt{7} = 14$.
4. $\sqrt{8}$ is $\sqrt{2}$, $\sqrt{8} \cdot \sqrt{2} = 4$.
5. $\sqrt{32}$ is $\sqrt{2}$, $\sqrt{32} \cdot \sqrt{2} = 8$.
6. $\sqrt{27}$ is $\sqrt{3}$, $\sqrt{27} \cdot \sqrt{3} = 9$.
7. $\sqrt[3]{2}$ is $\sqrt[3]{4}$, $\sqrt[3]{2} \cdot \sqrt[3]{4} = 2$.
8. $\sqrt[3]{3}$ is $\sqrt[3]{9}$, $\sqrt[3]{3} \cdot \sqrt[3]{9} = 3$.
9. $2\sqrt[3]{5}$ is $\sqrt[3]{25}$, $2\sqrt[3]{5} \cdot \sqrt[3]{25} = 10$.
10. $\sqrt[3]{16}$ is $\sqrt[3]{4}$, $\sqrt[3]{16} \cdot \sqrt[3]{4} = 4$.
11. $\sqrt[3]{25}$ is $\sqrt[3]{5}$, $\sqrt[3]{25} \cdot \sqrt[3]{5} = 5$.
12. $\sqrt[3]{36}$ is $\sqrt[3]{6}$, $\sqrt[3]{36} \cdot \sqrt[3]{6} = 6$.
13. $\sqrt[3]{49}$ is $\sqrt[3]{7}$, $\sqrt[3]{49} \cdot \sqrt[3]{7} = 7$.
14. $\sqrt{2} + 3$ is $\sqrt{2} - 3$, $(\sqrt{2} + 3)(\sqrt{2} - 3) = -7$.
15. $\sqrt{3} - \sqrt{2}$ is $\sqrt{3} + \sqrt{2}$, $(\sqrt{3} - \sqrt{2})(\sqrt{3} + 2) = 1$.
16. $3 + \sqrt{7}$ is $3 - \sqrt{7}$, $(3 + \sqrt{7})(3 - \sqrt{7}) = 2$.
17. $3\sqrt{2} - 5$ is $3\sqrt{2} + 5$, $(3\sqrt{2} - 5)(3\sqrt{2} + 5) = -7$.
18. $4\sqrt{3} - \sqrt{2}$ is $4\sqrt{3} + \sqrt{2}$, $(4\sqrt{3} - \sqrt{2})(4\sqrt{3} + \sqrt{2}) = 46$.
19. $2\sqrt{5} + 7\sqrt{6}$ is $2\sqrt{5} - 7\sqrt{6}$,
$(2\sqrt{5} + 7\sqrt{6})(2\sqrt{5} - 7\sqrt{6}) = -274$.
20. $\sqrt{x} - \sqrt{a}$ is $\sqrt{x} + \sqrt{a}$, $(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a}) = x - a$.
21. $\sqrt{3a} + \sqrt{x}$ is $\sqrt{3a} - \sqrt{x}$, $(\sqrt{3a} + \sqrt{x})(\sqrt{3a} - \sqrt{x}) = 3a - x$.

22. $3\sqrt{x} - a\sqrt{2}$ is $3\sqrt{x} + a\sqrt{2}$, $(3\sqrt{x} - a\sqrt{2})(3\sqrt{x} + a\sqrt{2})$
$= 9x - 2a^2$.

23. $\sqrt{x+a} + \sqrt{x}$ is $\sqrt{x+a} - \sqrt{x}$, $(\sqrt{x+a} + \sqrt{x})(\sqrt{x+a} - \sqrt{x}) = a$.

24. $\sqrt{x} - \sqrt{a-x}$ is $\sqrt{x} + \sqrt{a-x}$, $(\sqrt{x} - \sqrt{a-x})(\sqrt{x} + \sqrt{a-x})$
$= 2x - a$.

25. $\sqrt{2ax - x^2} - \sqrt{ax}$ is $\sqrt{2ax - x^2} + \sqrt{ax}$,
$(\sqrt{2ax - x^2} - \sqrt{ax})(\sqrt{2ax - x^2} + \sqrt{ax}) = ax - x^2$.

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1. $\dfrac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$.

2. $\dfrac{(18)^{\frac{1}{2}}}{(3)^{\frac{1}{2}}} = 6^{\frac{1}{2}}$.

3. $\dfrac{\sqrt{6}}{\sqrt{18}} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{9}} = \dfrac{1}{3}\sqrt{3}$.

4. $\dfrac{6}{2\sqrt{2}} = \dfrac{3\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \dfrac{3}{2}\sqrt{2}$.

5. $\dfrac{3\sqrt{12}}{\sqrt{6}} = 3\sqrt{2}$.

6. $\dfrac{8\sqrt{15}}{4\sqrt{5}} = 2\sqrt{3}$.

7. $\dfrac{10}{2\sqrt{5}} = \dfrac{5\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \sqrt{5}$.

8. $\dfrac{4\sqrt{10}}{8\sqrt{5}} = \dfrac{\sqrt{2}}{2}$.

9. $\dfrac{8}{4\sqrt{3}} = \dfrac{2\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{2}{3}\sqrt{3}$.

10. $\dfrac{3\sqrt{2}}{15\sqrt{8}} = \dfrac{1}{5\sqrt{4}} = \dfrac{1}{10}$.

11. $\dfrac{\sqrt{6} + \sqrt{18}}{3\sqrt{2}} = \dfrac{\sqrt{3} + \sqrt{9}}{3} = \dfrac{\sqrt{3} + 3}{3} = \dfrac{1}{3}\sqrt{3} + 1$.

12. $\dfrac{\sqrt{12} - \sqrt{24}}{2\sqrt{3}} = \dfrac{\sqrt{4} - \sqrt{8}}{2} = \dfrac{2 - 2\sqrt{2}}{2} = 1 - \sqrt{2}$.

13. $\dfrac{6\sqrt{10} + 4\sqrt{15} - \sqrt{20}}{2\sqrt{5}} = 3\sqrt{2} + 2\sqrt{3} - \dfrac{\sqrt{4}}{2} = 3\sqrt{2} + 2\sqrt{3} - 1$.

14. $\dfrac{12 + \sqrt{3} + \sqrt{5}}{\sqrt{6}} = \dfrac{12\sqrt{6} + \sqrt{18} + \sqrt{30}}{6} = 2\sqrt{6} + \dfrac{1}{2}\sqrt{2} + \dfrac{1}{6}\sqrt{30}$.

15. $\dfrac{\sqrt{6} - \sqrt{9} + 18}{2\sqrt{2}} = \dfrac{\sqrt{12} - \sqrt{18} + 18\sqrt{2}}{2\cdot 2} = \dfrac{2\sqrt{3} - 3\sqrt{2} + 18\sqrt{2}}{4}$
$= \dfrac{2\sqrt{3} + 15\sqrt{2}}{4}$.

16. $\dfrac{\sqrt{5} + 2}{\sqrt{125}} = \dfrac{\sqrt{25} + 2\sqrt{5}}{\sqrt{625}} = \dfrac{5}{25} + \dfrac{2}{25}\sqrt{5} = \dfrac{1}{5} + \dfrac{2}{25}\sqrt{5}$.

17. $\dfrac{\sqrt{\frac{32}{10}}}{\sqrt{\frac{4}{5}}} = \sqrt{\frac{32}{10}\cdot\frac{5}{4}} = \sqrt{4} = 2$.

18. $\dfrac{(xy)^{\frac{1}{2}}}{x^{\frac{1}{2}}} = y^{\frac{1}{2}}.$

19. $\dfrac{a\sqrt{bc}}{d\sqrt{c}} = \dfrac{a\sqrt{b}}{d}.$

20. $\dfrac{a^2\sqrt{c}}{a\sqrt{bc}} = \dfrac{a}{\sqrt{b}} = \dfrac{a\sqrt{b}}{\sqrt{b^2}} = \dfrac{a}{b}\sqrt{b}.$

21. $\dfrac{\sqrt[3]{16}}{\sqrt[3]{4}} = \sqrt[3]{4}.$

22. $\dfrac{8\sqrt[3]{125}}{4\sqrt[3]{25}} = 2\sqrt[3]{5}.$

23. $\dfrac{2\sqrt[3]{5}}{3\sqrt[3]{4}} = \dfrac{2\sqrt[3]{5}\cdot\sqrt[3]{2}}{3\sqrt[3]{8}}$

$= \dfrac{2\sqrt[3]{10}}{6} = \dfrac{1}{3}\sqrt[3]{10}.$

24. $\dfrac{a\sqrt[3]{9}}{b\sqrt[3]{8}} = \dfrac{a\sqrt[3]{9}}{2b}.$

25. $\dfrac{\sqrt[3]{\frac{1}{8}}}{\sqrt[3]{\frac{1}{4}}} = \sqrt[3]{\tfrac{1}{8}\cdot\tfrac{4}{1}} = \sqrt[3]{\tfrac{1}{2}} = \tfrac{1}{2}\sqrt[3]{4}.$

26. $\dfrac{\sqrt[3]{4}}{\sqrt{2}} = \dfrac{\sqrt[6]{16}}{\sqrt[6]{8}} = \sqrt[6]{2}.$

27. $\dfrac{\sqrt{2}}{\sqrt[3]{4}} = \dfrac{\sqrt{2}\cdot\sqrt[3]{2}}{\sqrt[3]{8}} = \dfrac{2^{\frac{1}{2}}\cdot 2^{\frac{1}{3}}}{2} = \dfrac{1}{2}\sqrt[6]{2^5} = \dfrac{1}{2}\sqrt[6]{32}.$

28. $\dfrac{4^{\frac{1}{3}}}{6^{\frac{1}{2}}} = \dfrac{\sqrt[3]{4}}{\sqrt{6}} = \dfrac{\sqrt[3]{4}\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}} = \dfrac{2^{\frac{2}{3}}\cdot 2^{\frac{1}{2}}\cdot 3^{\frac{1}{2}}}{6} = \dfrac{2^{\frac{7}{6}}\cdot 3^{\frac{3}{6}}}{6} = \dfrac{2}{6}\sqrt[6]{2\cdot 3^3} = \dfrac{1}{3}\sqrt[6]{54}.$

29. $\dfrac{6^{\frac{1}{2}}}{4^{\frac{1}{3}}} = \dfrac{\sqrt{6}}{\sqrt[3]{4}} = \dfrac{\sqrt{6}\cdot\sqrt[3]{2}}{\sqrt[3]{4}\cdot\sqrt[3]{2}} = \dfrac{2^{\frac{1}{2}}\cdot 3^{\frac{1}{2}}\cdot 2^{\frac{1}{3}}}{2} = \dfrac{2^{\frac{5}{6}}\cdot 3^{\frac{3}{6}}}{2} = \dfrac{1}{2}\sqrt[6]{2^5\cdot 3^3} = \dfrac{1}{2}\sqrt[6]{864}.$

30. $\dfrac{\sqrt[5]{27}}{\sqrt{27}} = \dfrac{\sqrt[5]{27}\cdot\sqrt{3}}{\sqrt{27}\cdot\sqrt{3}} = \dfrac{3^{\frac{3}{5}}\cdot 3^{\frac{1}{2}}}{9} = \dfrac{3^{\frac{11}{10}}}{9} = \dfrac{1}{3}\sqrt[10]{3}.$

31. $\dfrac{\sqrt[4]{\frac{1}{8}}}{\sqrt[3]{\frac{1}{2}}} = \dfrac{\sqrt[4]{\frac{1}{8}}\cdot\sqrt[3]{\frac{1}{4}}}{\sqrt[3]{\frac{1}{2}}\cdot\sqrt[3]{\frac{1}{4}}} = \dfrac{(\frac{1}{2})^{\frac{3}{4}}(\frac{1}{2})^{\frac{2}{3}}}{\frac{1}{2}} = \dfrac{(\frac{1}{2})^{\frac{17}{12}}}{\frac{1}{2}} = \left(\dfrac{1}{2}\right)^{\frac{5}{12}} = \sqrt[12]{\dfrac{1}{32}} = \sqrt[12]{\dfrac{128}{2^{12}}} = \dfrac{1}{2}\sqrt[12]{128}.$

32. $\dfrac{\sqrt[6]{\dfrac{a^2}{72a}}}{\sqrt[3]{\dfrac{a}{81}}} = \dfrac{\sqrt[6]{\dfrac{a^2}{72a}}}{\sqrt[6]{\dfrac{a^2}{81\cdot 81}}} = \sqrt[6]{\dfrac{81\cdot 81}{72a}} = \sqrt[6]{\dfrac{729}{8a}} = 3\sqrt[6]{\dfrac{8a^5}{64a^6}} = \dfrac{3}{2a}\sqrt[6]{8a^5}.$

33. $\dfrac{5}{\sqrt{5}+2} = \dfrac{5(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)} = \dfrac{5\sqrt{5}-10}{5-4} = 5\sqrt{5}-10.$

34. $\dfrac{2}{2-\sqrt{3}} = \dfrac{2(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \dfrac{4+2\sqrt{3}}{4-3} = 4+2\sqrt{3}.$

35. $\dfrac{4}{\sqrt{3}-\sqrt{2}} = \dfrac{4(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = \dfrac{4\sqrt{3}+4\sqrt{2}}{3-2} = 4\sqrt{3}+4\sqrt{2}.$

36. $\dfrac{\sqrt{5}}{3\sqrt{2}-\sqrt{5}} = \dfrac{\sqrt{5}(3\sqrt{2}+\sqrt{5})}{(3\sqrt{2}-\sqrt{5})(3\sqrt{2}+\sqrt{5})} = \dfrac{3\sqrt{10}+5}{18-5} = \dfrac{3}{13}\sqrt{10}+\dfrac{5}{13}.$

37. $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}=\frac{5-2\sqrt{15}+3}{5-3}=4-\sqrt{15}.$

38. $\frac{4\sqrt{3}+2\sqrt{2}}{4\sqrt{3}-2\sqrt{2}}=\frac{(2\sqrt{3}+\sqrt{2})(2\sqrt{3}+\sqrt{2})}{(2\sqrt{3}-\sqrt{2})(2\sqrt{3}+\sqrt{2})}=\frac{12+4\sqrt{6}+2}{12-2}=\frac{7}{5}+\frac{2}{5}\sqrt{6}.$

39. $\frac{2\sqrt{5}-3\sqrt{3}}{5\sqrt{5}-5}=\frac{(2\sqrt{5}-3\sqrt{3})(\sqrt{5}+1)}{5(\sqrt{5}-1)(\sqrt{5}+1)}=\frac{2\cdot 5-3\sqrt{15}+2\sqrt{5}-3\sqrt{3}}{5(5-1)}$

$=\frac{1}{2}-\frac{3}{20}\sqrt{15}+\frac{1}{10}\sqrt{5}-\frac{3}{20}\sqrt{3}.$

40. $3+\sqrt{2}=3+1.414=4.414.$

41. $14-5\sqrt{7}=14-13.2285=.771+.$

42. $6+2\sqrt{5}=6+2\cdot 2.236=6+4.472=10.472.$

$6-2\sqrt{5}=6-4.472+=1.527.$

43. $\frac{7\pm\sqrt{6}}{3}=\frac{7\pm 2.449}{3}=\frac{9.449}{3}$ or $\frac{4.550}{3}$ which equal 3.149 or 1.516 respectively.

44. $\frac{3\sqrt{6}}{2\sqrt{5}}=\frac{3\sqrt{30}}{10}=\frac{16.431}{10}=1.643.$

45. $\frac{2\sqrt{5}+1}{3\sqrt{5}-\sqrt{3}}=\frac{(2\sqrt{5}+1)(3\sqrt{5}+\sqrt{3})}{(3\sqrt{5}-\sqrt{3})(3\sqrt{5}+\sqrt{3})}=\frac{6\cdot 5+3\sqrt{5}+2\sqrt{15}+\sqrt{3}}{9.5-3}$

$=\frac{30+3\sqrt{5}+2\sqrt{15}+\sqrt{3}}{42}=\frac{30+6.708+7.745+1.732}{42}=1.099.$

46. $\sqrt{2-\sqrt{3}}=\sqrt{2-1.732}=\sqrt{.267}=.517.$

47. $\frac{\sqrt{3}}{\sqrt{3}+\sqrt[3]{4}+\sqrt[4]{5}}=\frac{1.732}{1.732+1.587+1.495}=\frac{1.732}{4.814}\doteq .359.$

48. $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{(\sqrt{a}+\sqrt{b})(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}=\frac{a+2\sqrt{ab}+b}{a-b}.$

49. $\frac{2\sqrt{x}-\sqrt{a}}{\sqrt{x}+3\sqrt{a}}=\frac{(2\sqrt{x}-\sqrt{a})(\sqrt{x}-3\sqrt{a})}{(\sqrt{x}+3\sqrt{a})(\sqrt{x}-3\sqrt{a})}=\frac{2x-7\sqrt{ax}+3a}{x-9a}.$

50. $\frac{r\sqrt{3}+\sqrt{r}}{\sqrt{3}-\sqrt{r}}=\frac{(r\sqrt{3}+\sqrt{r})(\sqrt{3}+\sqrt{r})}{(\sqrt{3}-\sqrt{r})(\sqrt{3}+\sqrt{r})}=\frac{3r+\sqrt{3r}+r\sqrt{3r}+r}{3-r}$

$=\frac{4r+\sqrt{3r}+r\sqrt{3r}}{3-r}.$

51. $\frac{m\sqrt{n}+a\sqrt{b}}{m\sqrt{n}-a\sqrt{b}}=\frac{(m\sqrt{n}+a\sqrt{b})(m\sqrt{n}+a\sqrt{b})}{(m\sqrt{n}-a\sqrt{b})(m\sqrt{n}+a\sqrt{b})}$

$=\frac{m^2n+2am\sqrt{bn}+a^2b}{m^2n-a^2b}.$

52. $$\frac{\sqrt{a-2}-2}{\sqrt{a-2}+2}=\frac{(\sqrt{a-2}-2)(\sqrt{a-2}-2)}{(\sqrt{a-2}+2)(\sqrt{a-2}-2)}=\frac{a-2-4\sqrt{a-2}+4}{a-2-4}$$
$$=\frac{a-4\sqrt{a-2}+2}{a-6}.$$

53. $$\frac{2}{\sqrt[4]{2+\sqrt{2}}}=\frac{2\sqrt[4]{2-\sqrt{2}}}{\sqrt[4]{2}}=\frac{2\sqrt{16-8\sqrt{2}}}{2}=\sqrt[4]{16-8\sqrt{2}}.$$

54. $$\frac{x-\sqrt{a+b}}{x+\sqrt{a+b}}=\frac{(x-\sqrt{a+b})(x-\sqrt{a+b})}{(x+\sqrt{a+b})(x-\sqrt{a+b})}=\frac{x^2-2x\sqrt{a+b}+a+b}{x^2-a-b}.$$

55. $$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{c}+\sqrt{d}}=\frac{(\sqrt{a}+\sqrt{b})(\sqrt{c}-\sqrt{d})}{(\sqrt{c}+\sqrt{d})(\sqrt{c}-\sqrt{d})}=\frac{\sqrt{ac}+\sqrt{bc}-\sqrt{ad}-\sqrt{bd}}{c-d}.$$

56. No.

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1. $10^2+5^2=h^2.$
$h^2=125.$
$h=\sqrt{125}=5\sqrt{5},$
the hypotenuse.

2. $10^2-5^2=l^2.$
$l^2=75.$
$l=\sqrt{75}=5\sqrt{3},$
the leg.

$$\text{Area}=\frac{5\cdot 5\sqrt{3}}{2}=12\tfrac{1}{2}\sqrt{3}.$$

3. $R^2-\frac{R^2}{4}=l^2.$
$l^2=\frac{3R^2}{4}.$
$l=\sqrt{\frac{3R^2}{4}}=\frac{R}{2}\sqrt{3},$
the leg.

$$\text{Area}=\frac{\frac{R}{2}\cdot\frac{R}{2}\sqrt{3}}{2}=\frac{R^2}{8}\sqrt{3}.$$

4. $10^2+10^2=D^2.$
$D^2=200.$
$D=\sqrt{200}=10\sqrt{2},$
the diagonal.

5. $10^2=2S^2.$
$S^2=50.$
$S=\sqrt{50}=5\sqrt{2},$
the side.
$\text{Area}=S^2=50.$

6. $4R^2=2S^2.$
$S^2=2R^2.$
$S=\sqrt{2R^2}=R\sqrt{2},$
the side.
$\text{Area}=S^2=2R^2.$

7. $A^2=12^2-6^2.$
$A^2=108.$
$A=\sqrt{108}=6\sqrt{3},$
the altitude.

$$\text{Area}=\frac{12\cdot 6\sqrt{3}}{2}=36\sqrt{3}.$$

8. $A^2=S^2-\frac{S^2}{4}.$
$A^2=\frac{3S^2}{4}.$
$A=\sqrt{\frac{3S^2}{4}}=\frac{S}{2}\sqrt{3},$
the altitude.

$$\text{Area}=\frac{S\cdot S\sqrt{3}}{2\cdot 2}=\frac{S^2}{4}\sqrt{3}.$$

9. $$S^2 - \frac{S^2}{4} = 10^2.$$
$$\frac{3S^2}{4} = 100.$$
$$S^2 = \tfrac{400}{3}.$$
$$S = \sqrt{\tfrac{400}{3}} = 20\sqrt{\tfrac{1}{3}}$$
$$= 6\tfrac{2}{3}\sqrt{3}, \text{ the side.}$$
$$\text{Area} = \frac{(\frac{20}{3}\sqrt{3})^2}{4}\sqrt{3}$$
$$= \tfrac{100}{3}\sqrt{3}.$$

10. $$2l^2 = 20^2.$$
$$l^2 = 200.$$
$$l = \sqrt{200} = 10\sqrt{2},$$
the leg.
$$\text{Area} = \frac{10\sqrt{2}\cdot 10\sqrt{2}}{2}$$
$$= 100.$$

11. $$\frac{l^2}{2} = 32.$$
$$l^2 = 64.$$
$$l = 8, \text{ the leg.}$$
$$H^2 = 8^2 + 8^2 = 128.$$
$$H = \sqrt{128} = 8\sqrt{2},$$
the hypotenuse.

12. $$H^2 = \frac{R^2}{4} + \frac{9R^2}{4}.$$
$$H = \sqrt{\frac{10R^2}{4}} = \frac{R}{2}\sqrt{10},$$
the hypotenuse.

13. $$l^2 = R^2 - \frac{R^2}{25}.$$
$$l^2 = \frac{24R^2}{25}.$$
$$l = \sqrt{\frac{24R^2}{25}} = \frac{2R}{5}\sqrt{6}, \text{ the leg.}$$

14. $$H^2 = R^2 + \frac{R^2}{4}(5 - 2\sqrt{5} + 1)$$
$$= \frac{4R^2 + 6R^2 - 2R^2\sqrt{5}}{4}$$
$$= \frac{R^2}{4}(10 - 2\sqrt{5}).$$
$$H = \frac{R}{2}\sqrt{10 - 2\sqrt{5}},$$
the hypotenuse.

15. $$H^2 = \frac{R^2}{4} + \left(R^2 - R^2\sqrt{3} + \frac{3R^2}{4}\right)$$
$$= 2R^2 - R^2\sqrt{3}.$$
$$H = R\sqrt{2 - \sqrt{3}},$$
the hypotenuse.

16. $$\text{Area} = 5\cdot\frac{2R}{3}\sqrt{4-\sqrt{3}}\cdot 3\cdot\frac{9R\sqrt{4+\sqrt{3}}}{10} = 9R^2\sqrt{16-3} = 9R^2\sqrt{13}.$$

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1. $x^2 - 6 = (x + \sqrt{6})(x - \sqrt{6})$.
2. $3x^2 - 4 = (x\sqrt{3} + 2)(x\sqrt{3} - 2)$.
3. $5x^2 - 1 = (x\sqrt{5} + 1)(x\sqrt{5} - 1)$.
4. $x^4 - 4 = (x^2 + 2)(x^2 - 2) = (x^2 + 2)(x + \sqrt{2})(x - \sqrt{2})$.
5. $4x^4 - 1 = (2x^2 + 1)(2x^2 - 1) = (2x^2 + 1)(x\sqrt{2} + 1)(x\sqrt{2} - 1)$.
6. $x^3 - 2 = (x - \sqrt[3]{2})(x^2 + x\sqrt[3]{2} + \sqrt[3]{4})$.
7. $x^3 + 6 = (x + \sqrt[3]{6})(x^2 - x\sqrt[3]{6} + \sqrt[3]{36})$.
8. $3x^3 - 1 = (x\sqrt[3]{3} - 1)(x^2\sqrt[3]{9} + x\sqrt[3]{3} + 1)$.
9. $x^3 + 4 = (x + \sqrt[3]{4})(x^2 - x\sqrt[3]{4} + \sqrt[3]{16}) = (x + \sqrt[3]{4})(x^2 - x\sqrt[3]{4} + 2\sqrt[3]{2}$

10. $2x^3 + 8 = 2(x^3 + 4) = 2(x + \sqrt[3]{4})(x^2 - x\sqrt[3]{4} + \sqrt[3]{16})$
$= 2(x + \sqrt[3]{4})(x^2 - x\sqrt[3]{4} + 2\sqrt[3]{2}).$

11. $2x^3 - 8 = 2(x^3 - 4) = 2(x - \sqrt[3]{4})(x^2 + x\sqrt[3]{4} + \sqrt[3]{16})$
$= 2(x - \sqrt[3]{4})(x^2 + x\sqrt[3]{4} + 2\sqrt[3]{2}).$

12. $6x^3 + 24 = 6(x^3 + 4) = 6(x + \sqrt[3]{4})(x^2 - \sqrt[3]{4}x + 2\sqrt[3]{2}).$

13. $x^2 - 2 = 0.$
$(x + \sqrt{2})(x - \sqrt{2}) = 0.$
Whence $x = \pm\sqrt{2}.$

14. $x^2 - 6 = 0.$
$(x + \sqrt{6})(x - \sqrt{6}) = 0.$
Whence $x = \pm\sqrt{6}.$

15. $2x^2 - 1 = 0.$
$(\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0.$
Whence $x = \pm\dfrac{1}{\sqrt{2}} = \pm\dfrac{\sqrt{2}}{2}.$

16. $x^4 + 6 = 5x^2.$
$(x^2 - 2)(x^2 - 3) = 0.$
Whence $x = \pm\sqrt{2}$ or $\pm\sqrt{3}.$

17. $4x^4 + 5 = 12x^2.$
$(2x^2 - 5)(2x^2 - 1) = 0.$
Whence $x = \pm\sqrt{\frac{5}{2}}$ or $\pm\sqrt{\frac{1}{2}}$
$= \pm\dfrac{\sqrt{10}}{2}$ or $\pm\dfrac{\sqrt{2}}{2}.$

18. $3x^4 + 8 = 14x^2.$
$(3x^2 - 2)(x^2 - 4) = 0.$
Whence $x = \pm 2$ or $\pm\frac{1}{3}\sqrt{6}.$

19. $5x^4 - 16x^2 + 3 = 0.$
$(5x^2 - 1)(x^2 - 3) = 0.$
Whence $x = \pm\frac{1}{5}\sqrt{5}$ or $\pm\sqrt{3}.$

20. $x^4 + 8a = 4x^2 + 2ax^2.$
$(x^2 - 4)(x^2 - 2a) = 0.$
Whence $x = \pm 2$ or $\pm\sqrt{2a}.$

21. $ax^4 - x^2 + 3a = 3a^2x^2.$
$(ax^2 - 1)(x^2 - 3a) = 0.$
Whence $x = \pm\dfrac{\sqrt{a}}{a}$ or $\pm\sqrt{3a}.$

22. $4x^4 + a = x^2 + 4ax^2.$
$(4x^2 - 1)(x^2 - a) = 0.$
Whence $x = \pm\frac{1}{2}$ or $\pm\sqrt{a}.$

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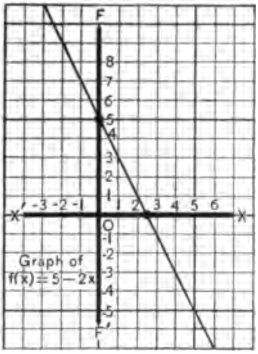
1. When $f(x) = 0$, $x = -\frac{3}{2}.$
2. If $2x + 3 = 0$, $x = -\frac{3}{2}.$
3. They are equal.
4. When $f(x) = 4$, $x = \frac{1}{2}.$
5. If $2x + 3 = 4$, $x = \frac{1}{2}.$
6. They are equal.
7. Yes. When $f(x) = 6$, $x = \frac{3}{2}.$
8. When $f(x)$, $2x + 3 = -2$, $x = -\frac{5}{2}.$

9.

x	0	$\frac{5}{2}$
$f(x)$	5	0



10. Yes. When $f(x)$, $5 - 2x = 0$, $x = \frac{5}{2}$.

11. (*a*) When $f(x)$, $5 - 2x = 9$, $x = -2$.
  (*b*) When $f(x)$, $5 - 2x = 5$, $x = 0$.
  (*c*) When $f(x)$, $5 - 2x = -3$, $x = 4$.
  (*d*) When $f(x)$, $5 - 2x = -7$, $x = 6$.

12. The roots obtained from the graph check in the equation.

13. The graph of any linear function of x should give a straight line.

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1. When $f(x)$,
$4x^2 - 4x - 15 = 0,$
$x = -\frac{3}{2}$ or $\frac{5}{2}$.

2. If $4x^2 - 4x - 15 = 0$,
then $(2x + 3)(2x - 5) = 0$.
Whence $x = -\frac{3}{2}$ or $\frac{5}{2}$.

3. They are the same.

4. When $f(x)$,
$4x^2 - 4x - 15 = 20,$
$x = -\frac{5}{2}$ or $\frac{7}{2}$.

5. The roots are $-\frac{5}{2}$ and $\frac{7}{2}$.

6. If $4x^2 - 4x - 15 = 20$,
then $(2x + 5)(2x - 7) = 0$.
Whence $x = -\frac{5}{2}$ or $\frac{7}{2}$.

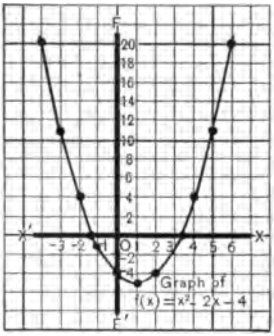
7. If $f(x)$,
$4x^2 - 4x - 15 = 9,$
then $x = 3$ or -2.
If $f(x)$,
$4x^2 - 4x - 15 = -7,$
then $x = 2$ or -1.

8. If $4x^2 - 4x - 15 = 9$,
then $(2x - 6)(2x + 4) = 0$.
Whence $x = 3$ or -2.
If $4x^2 - 4x - 15 = -7$,
then $(2x - 4)(2x + 2) = 0$.
Whence $x = 2$ or -1.

9. Values of x for $f(x) = -25$ or -20 cannot be read from the graph, for the graph does not reach these values.

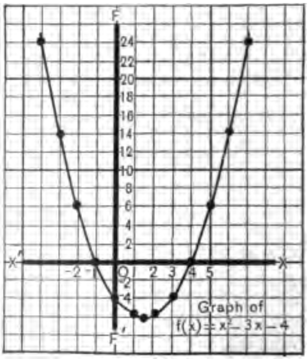
10.

When $x =$	-4	-3	-2	-1	0	1	2	3	4	5	6
fx, $x^2 - 2x - 4 =$	20	11	4	-1	-4	-5	-4	-1	4	11	20



11. Yes.
12. When $f(x)$, $x^2 - 2x - 4 = 0$, $x = -\frac{5}{4}$ or $\frac{13}{4}$, nearly.
13. The roots are $-\frac{5}{4}$ or $\frac{13}{4}$, nearly.
14. No. It can be obtained graphically.
15. Yes. Read values of x for $f(x) = 0$.
16.

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	24	14	6	0	-4	-6	-6	-4	0	6	14	24



When $f(x) = 0$, $x = 4$ or -1.
If $x^2 - 3x - 4 = 0$,
then $(x - 4)(x + 1) = 0$.
Whence $x = 4$ or -1.

17.

x	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	25	16	9	4	1	0	1	4	9	16	25

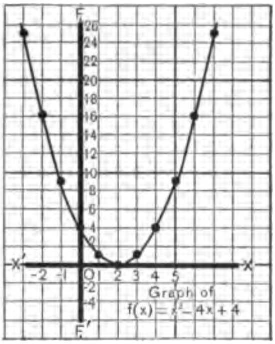
When $f(x)$,

$$x^2 - 4x + 4 = 0,$$
$$x = +2.$$

If $x^2 - 4x + 4 = 0,$

then $(x-2)(x-2) = 0.$

Whence $x = +2.$



18. It touches the x-axis. The equation $x^2 - 4x + 4 = 0$ has one root. The only value of x which satisfies the equation $x^2 - 4x + 4 = 0$ is $+2$.

If $f(x)$, $x^2 - 4x + 4 = 0$, $x = 2$.

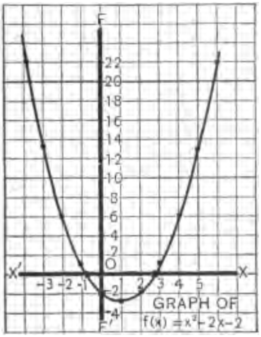
If $f(x)$, $x^2 - 4x + 4 = 4$, $x = 0$ or 4.

If $f(x)$, $x^2 - 4x + 4 = -1$, x has no real value.

If $f(x)$, $x^2 - 4x + 4 = -10$, x has no real value.

19.

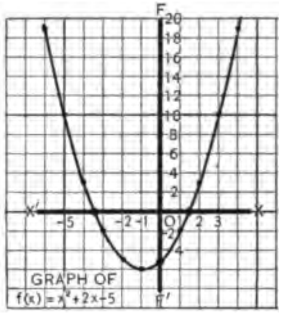
x	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$	22	13	6	1	-2	-3	-2	1	6	13	22



When $f(x)$, $x^2 - 2x - 2 = 0$, $x = 2.7$ or $-.7$.

20.

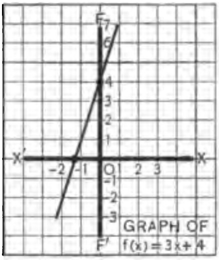
x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	19	10	3	-2	-5	-6	-5	-2	3	10	19

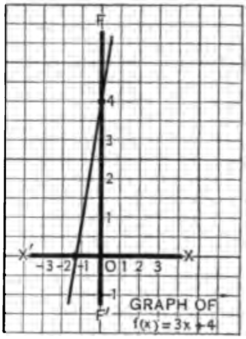


When $f(x)$, $x + 2x - 5 = 0$, $x = 1.4$ or -3.4.

21.

x	0	$-\frac{4}{3}$
$f(x)$	4	0

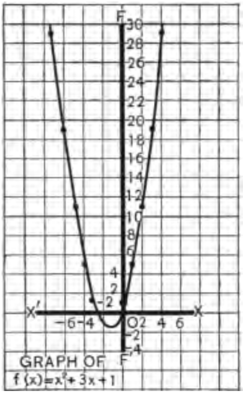


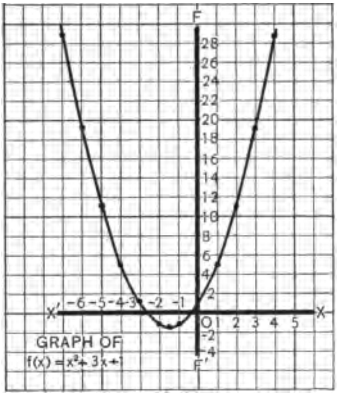


The lines cross the axes at points which have the same values respectively.

22.

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	29	19	11	5	1	-1	-1	1	5	11	19	29





The lines cross the axes at points which have the same values respectively.

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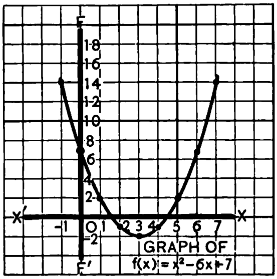
1.

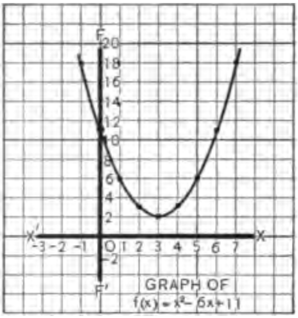
x	-1	0	1	2	3	4	5	6	7
$f(x)$	14	7	2	-1	-2	-1	2	7	14



When $f(x) = 0$, $x = 1.6$ or 4.4.

2.

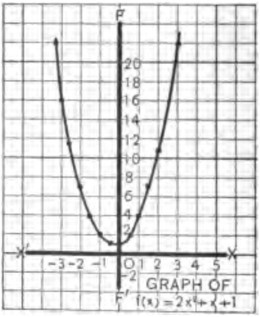
x	-1	0	1	2	3	4	5	6	7
$f(x)$	18	11	6	3	2	3	6	11	18



The equation $x^2 - 6x + 11 = 0$ has no graphical solution.

3.

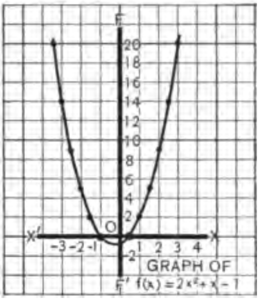
x	$-\frac{7}{2}$	-3	2	$-\frac{3}{2}$	-1	0	$\frac{1}{2}$	1	2	3
$f(x)$	22	16	7	4	2	1	2	4	11	22



The equation $2x^2 + x + 1 = 0$ has no graphical solution.

4.

x	$-\frac{7}{2}$	-3	-2	$-\frac{3}{2}$	-1	0	$\frac{1}{2}$	1	2	3
$f(x)$	20	14	5	2	0	-1	0	2	9	20



When $f(x) = 0$, $x = -1$ or $\frac{1}{2}$.

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1. $x^2 - 8x - 48 = 0,$

or $x^2 - 8x = 48.$

$(x-4)^2 = 64.$

Whence $x = +12$ or -4.

2. $x^2 - 5x - 14 = 0,$

or $x^2 - 5x = 14.$

$(x - \frac{5}{2})^2 = \frac{81}{4}.$

Whence $x = +7$ or -2.

3. $x(x+2) - 5(x+2) = 0,$

or $x^2 - 3x = 10.$

$(x - \frac{3}{2})^2 = \frac{49}{4}.$

Whence $x = -2$ or $+5$.

4. $\frac{3}{4} - y = y^2,$

or $y^2 + y = \frac{3}{4}.$

$(y + \frac{1}{2})^2 = 1.$

Whence $y = \frac{1}{2}$ or $-\frac{3}{2}$.

5. $2y^2 - 9y + 4 = 0,$

or $y^2 - \frac{9y}{2} = -2.$

$(y - \frac{9}{4})^2 = \frac{49}{16}.$

Whence $y = 4$ or $\frac{1}{2}$.

6. $2y^2 + 5y = 0,$

or $y^2 + \frac{5y}{2} = 0.$

$(y + \frac{5}{4})^2 = \frac{25}{16}.$

Whence $y = 0$ or $-\frac{5}{2}$.

7. $t^2 - 2t - 15 = 0,$

or $t^2 - 2t = 15.$

$(t-1)^2 = 16.$

Whence $t = 5$ or -3.

8. $3t^2 - 7t = 6,$

or $t^2 - \frac{7t}{3} = 2.$

$(t - \frac{7}{6})^2 = \frac{121}{36}.$

Whence $t = 3$ or $-\frac{2}{3}$.

9. $9v = 5v^2 - 2,$

or $v^2 - \frac{9v}{5} = \frac{2}{5}.$

$(v - \frac{9}{10})^2 = \frac{121}{100}.$

Whence $v = 2$ or $-\frac{1}{5}$.

10. $x^2 - 2x - 4 = 0,$
or $x^2 - 2x = 4.$
$(x-1)^2 = 5.$
Whence $x = 1 \pm \sqrt{5}$
$= 1 \pm 2.236$
$= 3.236$ or $-1.236.$

11. $v^2 - \frac{7}{12}v - 1 = 0,$
or $v^2 - \frac{7v}{12} = 1.$
$(v - \frac{7}{24})^2 = \frac{625}{576},$
or $v = \frac{4}{3}$ or $-\frac{3}{4}.$

12. $s^2 - 2s - 3\frac{1}{2} = 0,$
or $s^2 - 2s = \frac{7}{2}.$
$(s-1)^2 = \frac{9}{2}.$
Whence $s = 1 \pm \frac{3}{2}\sqrt{2}.$

13. $h^2 + 10h + 13 = 0,$
or $h^2 + 10h = -13.$
$(h+5)^2 = 12.$
Whence $h = -5 \pm 2\sqrt{3}$
$= -1.535$ or $-8.464.$

14. $12t^2 - 25t + 12 = 0,$
or $t^2 - \frac{25t}{12} = -1.$
$(t - \frac{25}{24})^2 = \frac{49}{576}.$
Whence $t = \frac{4}{3}$ or $\frac{3}{4}.$

15. $42 + 2z^2 = -19z,$
or $z^2 + \frac{19z}{2} = -21.$
$(z + \frac{19}{4})^2 = \frac{25}{16}.$
Whence $z = -\frac{7}{2}$ or $-6.$

16. $15x^2 + 4x = 4,$
or $x^2 + \frac{4x}{15} = \frac{4}{15}.$
$(x + \frac{2}{15})^2 = \frac{64}{225}.$
Whence $x = \frac{2}{5}$ or $-\frac{2}{3}.$

17. $1 - 6v^2 = 2v,$
or $v^2 + \frac{v}{3} = \frac{1}{6}.$
$(v + \frac{1}{6})^2 = \frac{7}{36}.$
Whence $v = \frac{-1 \pm \sqrt{7}}{6}.$

18. $20s^2 + s = 1,$
or $s^2 + \frac{s}{20} = \frac{1}{20}.$
$(s + \frac{1}{40})^2 = (\frac{9}{40})^2.$
Whence $s = \frac{1}{5}$ or $-\frac{1}{4}.$

19. $9x + 4 = 9x^2,$
or $x^2 - x = \frac{4}{9}.$
$(x - \frac{1}{2})^2 = \frac{25}{36}.$
Whence $x = \frac{4}{3}$ or $-\frac{1}{3}.$

20. $25x^2 - 20x - 12 = 0,$
or $x^2 - \frac{4x}{5} = \frac{12}{25}.$
$(x - \frac{2}{5})^2 = \frac{16}{25}.$
Whence $x = \frac{6}{5}$ or $-\frac{2}{5}.$

21. $4x^2 = 1 - 4x,$
or $x^2 + x = \frac{1}{4}.$
$(x + \frac{1}{2})^2 = \frac{2}{4}.$
Whence $x = \frac{-1 \mp \sqrt{2}}{2}$
$= -1.207$ or $.207.$

22. $x^2 + 4\sqrt{5}x = 25,$
$(x + 2\sqrt{5})^2 = 45.$
Whence $x = \sqrt{5}$ or $-5\sqrt{5}.$

23. $x^2 - 3\sqrt{2}x + 4 = 0,$
or $x^2 - 3\sqrt{2}x = -4.$
$(x - \frac{3}{2}\sqrt{2})^2 = \frac{2}{4}.$
Whence $x = 2\sqrt{2}$ or $\sqrt{2}.$

24. $1 - \frac{3\sqrt{3}}{2R} + \frac{3}{2R^2} = 0,$
or $R^2 - \frac{3\sqrt{3}R}{2} = -\frac{3}{2}.$
$(R - \frac{3}{4}\sqrt{3})^2 = \frac{3}{16}.$
Whence $R = \sqrt{3}$ or $\frac{1}{2}\sqrt{3}.$

25. $\frac{2x}{9}+\frac{2}{3}=\frac{7}{18x},$

or $x^2+3x=\frac{7}{4}.$

$(x+\frac{3}{2})^2=4.$

Whence $x=\frac{1}{2}$ or $-\frac{7}{2}.$

26. $\frac{1}{3}+\frac{x}{9}-\frac{2}{x}=0,$

or $x^2+3x=18.$

$(x+\frac{3}{2})^2=\frac{81}{4}.$

Whence $x=3$ or $-6.$

27. $\frac{2}{5v^2}+\frac{1}{4}-\frac{11}{10v}=0,$

or $v^2-\frac{22v}{5}=-\frac{8}{5}.$

$(v-\frac{11}{5})^2=\frac{81}{25}.$

Whence $v=4$ or $\frac{2}{5}.$

28. $\frac{3a}{2}+\frac{1}{2}-\frac{1}{3a}=0,$

or $a^2+\frac{a}{3}=\frac{2}{9}.$

$(a+\frac{1}{6})^2=\frac{1}{4}.$

Whence $a=\frac{1}{3}$ or $-\frac{2}{3}.$

29. $\frac{25c}{2}-\frac{40}{c}-40=0,$

or $c^2-\frac{16c}{5}=\frac{16}{5}.$

$(c-\frac{8}{5})^2=\frac{144}{25}.$

Whence $c=4$ or $-\frac{4}{5}.$

30. $(x-4)^2-3(x-9)=15,$

or $x^2-11x=-28.$

$(x-\frac{11}{2})^2=\frac{9}{4}.$

Whence $x=7$ or $4.$

31. $(x-2)(x+3)=x(5x-9)-2,$

or $x^2-\frac{5x}{2}=-1.$

$(x-\frac{5}{4})^2=\frac{9}{16}.$

Whence $x=2$ or $\frac{1}{2}.$

32. $x-\frac{3}{x+2}=0,$

or $x^2+2x=3.$

$(x+1)^2=4.$

Whence $x=1$ or $-3.$

33. $\frac{v^2}{v-5}+\frac{5}{2}=0,$

or $v^2+\frac{5v}{2}=\frac{25}{2}.$

$(v+\frac{5}{4})^2=\frac{225}{16}.$

Whence $v=\frac{5}{2}$ or $-5.$

34. $\frac{3}{t-7}+\frac{t}{4}=0,$

or $t^2-7t=-12.$

$(t-\frac{7}{2})^2=\frac{1}{4}.$

Whence $t=4$ or $3.$

35. $\frac{s}{s-2}+\frac{s-2}{s}=\frac{5}{2},$

or $s^2-2s=8.$

$(s-1)^2=9.$

Whence $s=4$ or $-2.$

36. $\frac{7}{2y-3}-\frac{5}{1-y}=12,$

or $y^2-\frac{77y}{24}=-\frac{58}{24}.$

$(y-\frac{77}{48})^2=\frac{361}{2304}.$

Whence $y=2$ or $\frac{29}{24}.$

37. $\frac{3+x}{4+x}-\frac{x-5}{x-6}=\frac{1}{12},$

or $x^2+22x=48.$

$(x+11)^2=169.$

$x=2$ or $-24.$

38. If $y=2,$

$x^2-xy-3y^2=-12$ becomes

$x^2-2x=0.$

Whence $(x-1)^2=1,$

and $x=2$ or $0.$

39. If $x = -3$, $x^2 - 4xy + x^3 + y^2 + 5 = 0$ becomes $y^2 + 12y = 13$.

Whence $(y + 6)^2 = 49,$

and $y = 1 \text{ or } -13.$

41. $x^4 - 13x^2 + 36 = 0,$

or $x^4 - 13x^2 = -36.$

$(x^2 - \frac{13}{2})^2 = \frac{25}{4}.$

Whence $x^2 = 9 \text{ or } 4,$

and $x = \pm 3 \text{ or } \pm 2.$

42. $4x^4 - 5x^2 + 1 = 0,$

or $$x^4 - \frac{5x^2}{4} = -\frac{1}{4}.$$

$(x^2 - \frac{5}{8})^2 = \frac{9}{64}.$

Whence $x^2 = 1 \text{ or } \frac{1}{4},$

and $x = \pm 1 \text{ or } \pm \frac{1}{2}.$

43. $9x^4 - 37x^2 + 4 = 0,$

or $$x^4 - \frac{37x^2}{9} = -\frac{4}{9}.$$

$(x^2 - \frac{37}{18})^2 = \frac{1225}{324}.$

Whence $x^2 = \frac{1}{9} \text{ or } 4,$

and $x = \pm \frac{1}{3} \text{ or } \pm 2.$

44. $4k^4 = 9k^2 - 2,$

or $$k^4 - \frac{9k^2}{4} = -\frac{1}{2}.$$

$(k^2 - \frac{9}{8})^2 = \frac{49}{64}.$

Whence $k^2 = 2 \text{ or } \frac{1}{4},$

and $k = \pm \sqrt{2} \text{ or } \pm \frac{1}{2}.$

45. $9s^4 + 12 = 31s^2,$

or $$s^4 - \frac{31s^2}{9} = -\frac{12}{9}.$$

$(s^2 - \frac{31}{18})^2 = (\frac{23}{18})^2.$

Whence $s^2 = 3 \text{ or } \frac{4}{9},$

and $s = \pm \sqrt{3} \text{ or } \pm \frac{2}{3}.$

46. $4v^4 + 5 = 21v^2,$

or $$v^4 - \frac{21v^2}{4} = -\frac{5}{4}.$$

$(v^2 - \frac{21}{8})^2 = \frac{361}{64}.$

Whence $v^2 = 5 \text{ or } \frac{1}{4},$

and $v = \pm \sqrt{5} \text{ or } \pm \frac{1}{2}.$

47.

$$9 - 30 = 49 - 70. \quad (1)$$
$$9 - 30 + 25 = 49 - 70 + 25. \quad (2)$$
$$(3 - 5)^2 = (7 - 5)^2. \quad (3)$$
$$3 - 5 = 7 - 5. \quad (4)$$
$$3 = 7. \quad (5)$$

The error occurred in passing from (3) to (4). Square roots of numerically equal expressions are equal only when the signs of the roots are properly chosen. Line (4) could have read

$$+(3 - 5) = -(7 - 5) \text{ or } -(3 - 5) = +(7 - 5),$$

but not $+(3 - 5) = +(7 - 5) \text{ or } -(3 - 5) = -(7 - 5).$

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2. $x^2 - ax - 2a^2 = 0,$

or $x^2 - ax = 2a^2.$

$$\left(x - \frac{a}{2}\right)^2 = \frac{9a^2}{4}.$$

Whence $x = 2a \text{ or } -a.$

3. $x^2 + 2ax + a^2 - 4 = 0,$

or $x^2 + 2ax = 4 - a^2.$

$(x + a)^2 = 4.$

Whence $x = -a \pm 2.$

4. $x^2+1=a+2x,$

or $x^2-2x=a-1.$

$(x-1)^2=a.$

Whence $x=1\pm\sqrt{a}.$

5. $3x^2-ax=10a^2,$

or $x^2-\frac{ax}{3}=\frac{10a^2}{3}.$

$\left(x-\frac{a}{6}\right)^2=\frac{121a^2}{36}.$

Whence $x=2a$ or $-\frac{5a}{3}.$

6. $3mx+2m^2=2x^2,$

or $x^2-\frac{3mx}{2}=m^2.$

$\left(x-\frac{3m}{4}\right)^2=\frac{25m^2}{16}.$

Whence $x=2m$ or $-\frac{m}{2}.$

7. $a^2x^2-7ax+10=0,$

or $x^2-\frac{7x}{a}=-\frac{10}{a^2}.$

$\left(x-\frac{7}{2a}\right)^2=\frac{9}{4a^2}.$

Whence $x=\frac{5}{a}$ or $\frac{2}{a}.$

8. $4x^2+4ax-3a^2=0,$

or $x^2+ax=\frac{3a^2}{4}.$

$\left(x+\frac{a}{2}\right)^2=a^2.$

Whence $x=\frac{a}{2}$ or $-\frac{3a}{2}.$

9. $x^2+4\sqrt{a}x-5a=0,$

or $x^2+4a\sqrt{x}=5a.$

$(x+2\sqrt{a})^2=9a.$

Whence $x=\sqrt{a}$ or $-5\sqrt{a}.$

10. $2x^2+9x\sqrt{h}=5h,$

or $x^2+\frac{9\sqrt{h}x}{2}=\frac{5h}{2}.$

$\left(x+\frac{9\sqrt{h}}{4}\right)^2=\frac{121h}{16}.$

Whence $x=\frac{\sqrt{h}}{2}$ or $-5\sqrt{h}.$

11. $a^2x^2+2ab=a^2+b^2,$

or $x^2=\frac{a^2-2ab+b^2}{a^2}.$

$x^2=\frac{(a-b)^2}{a^2}.$

Whence $x=\pm\frac{a-b}{a}.$

12. $5x^2+ax=x,$

or $x^2+\frac{(a-1)x}{5}=0.$

$\left(x+\frac{a-1}{10}\right)^2=\left(\frac{a-1}{10}\right)^2.$

Whence $x=0$ or $\frac{1-a}{5}.$

13. $x(x-b)=a(a+b),$

or $x^2-bx=a^2+ab.$

$\left(x-\frac{b}{2}\right)^2=\left(a+\frac{b}{2}\right)^2.$

Whence $x=a+b$ or $-a.$

14. $1-\frac{4}{x}+\frac{4}{x^2}=\frac{b}{4x^2},$

or $x^2-4x+4=\frac{b}{4}.$

$(x-2)^2=\frac{b}{4}.$

Whence $x=2\pm\frac{\sqrt{b}}{2}.$

15. $x^2+2x=ax+2a,$

or $x^2+(2-a)x=2a.$

Whence $x=a$ or $-2.$

16. $x^2-(a+1)x+a=0,$

or $$x^2-(a+1)x=-a.$$

$$\left(x-\frac{a+1}{2}\right)^2=\left(\frac{a-1}{2}\right)^2.$$

Whence $x=a$ or 1.

17. $x^2+bx+cx+bc=0,$

or $$x^2+(b+c)x=-bc.$$

$$\left(x+\frac{b+c}{2}\right)^2=\left(\frac{b-c}{2}\right)^2.$$

Whence $x=-c$ or $-b$.

18. $x^2-ax+4x-4a=0,$

or $$x^2-(a-4)x=4a.$$

$$\left(x-\frac{a-4}{2}\right)^2=\left(\frac{a+4}{2}\right)^2.$$

Whence $x=a$ or -4.

19. $$x^2+2a^2b^2=a^2x+2b^2x,$$

or $$x^2-(a^2+2b^2)x=-2a^2b^2.$$

$$\left(x-\frac{a^2+2b^2}{2}\right)^2=\left(\frac{a^2-2b^2}{2}\right)^2.$$

Whence $x=a^2$ or $2b^2$.

20. $$\frac{1}{a}-\frac{1}{5x}=1-\frac{5x}{a},$$

or $$x^2+\frac{(1-a)x}{5}=\frac{a}{25}.$$

$$\left(x+\frac{1-a}{10}\right)^2=\left(\frac{1+a}{10}\right)^2.$$

Whence $x=\frac{a}{5}$ or $-\frac{1}{5}$.

21. $$x^2+bx+c=0,$$

or $$x^2+bx=-c.$$

$$\left(x+\frac{b}{2}\right)^2=\frac{b^2-4c}{4}.$$

Whence $x=\dfrac{-b\pm\sqrt{b^2-4c}}{2}$.

22. $$ax^2+bx+c=0,$$

or $$x^2+\frac{b}{a}x=-\frac{c}{a}.$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}.$$

Whence $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Page 274 (First set)

1. $2x^2+5x+2=0.\quad\therefore x=\dfrac{-5\pm\sqrt{25-4\cdot2\cdot2}}{4}=-\frac{1}{2}$ or -2.

2. $3x^2+5x=2$, or $3x^2+5x-2=0$.

$$\therefore x=\frac{-5\pm\sqrt{25+24}}{6}=\frac{1}{3}\text{ or }-2.$$

3. $x^2-3x-10=0.\quad\therefore x=\dfrac{3\pm\sqrt{9+40}}{2}=5$ or -2.

4. $2x+2=x^2$, or $x^2-2x-2=0.\quad\therefore x=\dfrac{2\pm\sqrt{4+8}}{2}=1\pm\sqrt{3}$.

5. $x^2-x=1.\quad\therefore x=\dfrac{1\pm\sqrt{1+4}}{2}=\dfrac{1\pm\sqrt{5}}{2}$.

6. $2x^2-\dfrac{11x}{2}-\dfrac{15}{2}=0$, or $4x^2-11x-15=0$.

$$\therefore x=\frac{11\pm\sqrt{121+240}}{8}=\frac{15}{4}\text{ or }-1.$$

7. $2x^2 - 3x = 1.\quad \therefore x = \frac{3 \pm \sqrt{9+8}}{4} = \frac{3 \pm \sqrt{17}}{4}.$

8. $4x + 5 = x^2$, or $x^2 - 4x - 5 = 0.\quad \therefore x = \frac{4 \pm \sqrt{16+20}}{2} = 5$ or $-1.$

9. $x^2 + x\sqrt{5} = 10$, or $x^2 + x\sqrt{5} - 10 = 0.$

$\therefore x = \frac{-\sqrt{5} \pm \sqrt{5+40}}{2} = \sqrt{5}$ or $-2\sqrt{5}.$

10. $12x = 1 - 72x^2$, or $72x^2 + 12x - 1 = 0.$

$\therefore x = \frac{-12 \pm \sqrt{144+288}}{144} = \frac{-1 \pm \sqrt{3}}{12}.$

11. $x^2 + 2hx - 3h^2 = 0.\quad \therefore x = \frac{-2h \pm \sqrt{4h^2 + 12h^2}}{2} = h$ or $-3h.$

12. $2m^2 = 9mx + 5x^2$, or $5x^2 + 9mx - 2m^2 = 0.$

$\therefore x = \frac{-9m \pm \sqrt{81m^2 + 40m^2}}{10} = \frac{m}{5}$ or $-2m.$

13. $2x^2 + kx - 3k^2 = 0.\quad x = \frac{-k \pm \sqrt{k^2 + 24k^2}}{4} = k$ or $-\frac{3k.}{2}$

14. $x^2 + 2x\sqrt{a} - 3a = 0.\quad \therefore x = \frac{-2\sqrt{a} \pm \sqrt{4a + 12a}}{2} = \sqrt{a}$ or $-3\sqrt{a}.$

15. $mx = -m^2 + 6x^2$, or $6x^2 - mx - m^2 = 0.$

$\therefore x = \frac{m \pm \sqrt{m^2 + 24m^2}}{12} = \frac{m}{2}$ or $-\frac{m}{3}.$

16. $x^2 + \frac{kx}{2}\sqrt{2} - k^2 = 0.\quad \therefore x = \frac{\frac{-k\sqrt{2}}{2} \pm \sqrt{\frac{2k^2}{4} + 4k^2}}{2} = \frac{k}{2}\sqrt{2}$ or $-k\sqrt{2}.$

17. $n^2x^2 - 3knx - 10k^2 = 0.\quad \therefore x = \frac{3kn \pm \sqrt{9k^2n^2 + 40k^2n^2}}{2n^2} = \frac{5k}{n}$ or $-\frac{2k}{n}.$

18. $6m^2x^2 + 19mnx = 7n^2$, or $6m^2x^2 + 19mnx - 7n^2 = 0.$

$\therefore x = \frac{-19mn \pm \sqrt{361m^2n^2 + 168m^2n^2}}{12m^2} = \frac{n}{3m}$ or $-\frac{7n}{2m}.$

19. $x^2 + 2x = hx + 2h$, or $x^2 + (2-h)x - 2h = 0.\quad \therefore x = h$ or $-2.$

20. $x^2 + rx - sx - rs = 0$, or $x^2 + (r-s)x - rs = 0.$

$\therefore x = \frac{s - r \pm \sqrt{(r-s)^2 + 4rs}}{2} = s$ or $-r.$

21. $2x^2 + rs = rx + 2sx$ or $2x^2 - (r+2s)x + rs = 0.$

$\therefore x = \frac{r + 2s \pm \sqrt{(r+2s)^2 - 8rs}}{4} = s$ or $\frac{r}{2}.$

22. $mnx^2 + nx = 3mx + 3$, or $mnx^2 + (n - 3m)x - 3 = 0$.

$$\therefore x = \frac{3m - n \pm \sqrt{(n - 3m)^2 + 12mn}}{2mn} = \frac{3}{n} \text{ or } -\frac{1}{m}.$$

23. $mhx^2 + 4hx = 3mx + 12$, or $mhx^2 + (4h - 3m)x - 12 = 0$.

$$\therefore x = \frac{3m - 4h \pm \sqrt{(4h - 3m)^2 + 48hm}}{2hm} = \frac{3}{h} \text{ or } \frac{-4}{m}.$$

Page 274 (Second set)

1. Let $x =$ the number.
Then $x^2 + 2x = 15$.
Whence $x = -5$ or 3.

2. Let $x =$ the less number.
Then $x + 11 =$ the greater number.
Hence
$x(x + 11) = 42$,
or $x^2 + 11x = 42$.
Whence $x = -14$ or $+3$,
and $x + 11 = -3$ or $+14$.

3. Let $n =$ the number.
Then $2n^2 - n = 45$.
Whence $n = 5$ or $-\frac{9}{2}$.

4. Let $x =$ the less number.
Then $x + 1 =$ the greater number.
Therefore
$x(x + 1) = 462$,
or $x^2 + x = 462$.
Whence $x = 21$ or -22,
and $x + 1 = 22$ or -21.

5. Let n and $n + 2$ represent the odd numbers.
Then $n(n+2) = 255$,
or $n^2 + 2n = 255$.
Whence $n = 15$ or -17,
and $n + 2 = 17$ or -15.

6. Let n, $n + 2$, and $n + 4$ represent the even numbers.
Then
$$n + n + 2 + n + 4 = \frac{n(n + 2)}{6},$$
or $n^2 - 16n - 36 = 0$.
Whence $n = 18$ or -2,
$n + 2 = 20$ or 0,
and $n + 4 = 22$ or 2.

7. Let $n =$ the width in rods.
Then $n + 16 =$ the length in rods.
Therefore
$n(n + 16) = 32 \cdot 160$,
or $n^2 + 16n = 5120$.
Whence $n = 64$ or -80,
and $n + 16 = 80$ or -64.
The negative roots are rejected.

8. Let x and $\frac{1}{x}$ represent the number and its reciprocal.
Then $x + \frac{1}{x} = \frac{41}{20}$,
or $x^2 - \frac{41x}{20} = -1$.
Whence $x = \frac{5}{4}$ or $\frac{4}{5}$.
Then $\frac{1}{x} = \frac{4}{5}$ or $\frac{5}{4}$.

9. Let $n =$ the altitude in rods.
Then $n + 51 =$ the base in rods.
Therefore
$$\frac{n(n + 51)}{2} = \frac{45}{8} \cdot 160,$$

or $n^2 + 51n = 1800.$

Whence $n = 24$ or $-75,$

and $n + 51 = 75$ or $-24.$

The negative values are rejected.

10. Let $s =$ the side of one in rods.

Then $s + 20 =$ the side of the other in rods

Therefore

$s^2 + (s + 20)^2 = 62.5 \cdot 160,$

or $s^2 + 20s = 4800.$

Whence $s = 60$ or $-80,$

and $s + 20 = 80$ or $-60.$

The negative values are rejected.

11. Let $s =$ the side of the square and the breadth of the rectangle in inches.

Then $s + 7 =$ the side of the rectangle.

Therefore $s(s + 7) + 18 = 2s^2,$

or $s^2 - 7s = 18.$

Whence $s = 9$ or $-2.$

The negative value is rejected.

12. Let $l =$ the shorter leg in feet.

Then $l + 31 =$ the longer leg.

Therefore

$l^2 + (l + 31)^2 = (41)^2,$

or $l^2 + 31l = 360.$

Whence $l = 9$ or $-40,$

and $l + 31 = 40$ or $-9.$

The negative roots are rejected.

13. Let $3x$ and $4x$ represent the legs.

Then

$9x^2 + 16x^2 = 400,$

or $x^2 = 16.$

Whence $x = 4$ or $-4.$

Then $3x = 12,$

and $4x = 16.$

14. Let x and $2x$ represent the other leg and the hypotenuse.

Then $x^2 + 81 = 4x^2,$

or $27 = x^2.$

Whence $x = \pm 3\sqrt{3}.$

But $3\sqrt{3} = 5.19+.$

Then $x = 5.19+,$

and $2x = 10.39+.$

15. Let $l =$ the other leg.

Then

$l^2 + (5\sqrt{3})^2 = 100,$

or $l^2 = 25.$

Whence $l = \pm 5.$

Since one leg is ½ the hypotenuse, one angle is 30°, one 60°, and the third 90°.

16. Let $x =$ the side of the square in feet.

Then $x^2 = 4x.$

Whence $x = 4$ or $0.$

17. Let $x =$ the side of the square in inches.

Then $\left(\frac{x}{12}\right)^2 = 4x,$

or $x^2 - 576x = 0.$

Whence $x = 576$ or $0,$

and $\frac{576}{12} = 48$ feet.

18. Let $s =$ the side of the square in inches.

Then $s^2 = 4\left(\frac{s}{12}\right),$

or $3s^2 = s.$

Whence $s = \frac{1}{3}$ or $0.$

19. Let x, $x + 2$, and $x + 4$ represent the dimensions and the longest straight line respectively in inches.

Then
$$x^2 + (x + 2)^2 = (x + 4)^2,$$
or $$x^2 - 4x = 12.$$
Whence $x = 6$ or -2.
Whence $x = 6$,
$$x + 2 = 8,$$
and $$x + 4 = 10.$$

20. Let x, $2x$, and $3x$ represent the dimensions in inches.

Then $2 \cdot 2x^2 + 2 \cdot 3x^2 + 2 \cdot 6x^2 = 792,$
or $$22x^2 = 792.$$
Whence $x = \pm 6$.
Then $x = 6$,
$$2x = 12,$$
and $$3x = 18.$$

21. Let x and $x + 1$ represent the edges in yards.

Then
$$(x + 1)^3 - x^3 = 61,$$
or $$x^2 + x - 20 = 0.$$
Whence $x = -5$ or $+4$.
Therefore $x = 4$,
and $$x + 1 = 5.$$

22. Let r and $r + 5$ represent the rates in miles per hour.

Then
$$\frac{280}{r} - \frac{280}{r + 5} = 1,$$
or $$r^2 + 5r = 1400.$$
Whence $r = 35$ or -40.
Therefore $r = 35$,
and $$r + 5 = 40.$$

23. Let r and $r + 4$ represent the rates in miles per hour.

Then
$$\frac{80}{r} + \frac{80}{r + 4} = 9,$$
or $$9r^2 - 124r = 320.$$
Whence $r = 16$ or $-2\frac{2}{9}$.
Therefore $r = 16$,
and $$r + 4 = 20.$$

24. Let x and $x + 2$ represent the width and the length in inches respectively.

Then
$$2(x - 2)(x) = x(x + 2),$$
or $$x^2 - 6x = 0.$$
Whence $x = 6$ or 0.
Then $x = 6$,
and $$x + 2 = 8.$$

25. Let x and $x - 40$ represent the number of acres bought and sold respectively.

Then $\dfrac{16{,}000}{x} + 20 = \dfrac{16{,}000}{x - 40},$
or $x^2 - 40x - 32{,}000 = 0.$
Whence $x = 200$ or -160,
and $$x - 40 = 160.$$

26. Let $x =$ the original cost of an orange in cents.

Then $\dfrac{50}{x} - 5 = \dfrac{50}{x + \frac{10}{12}},$
or $x^2 + \dfrac{5x}{6} - \dfrac{25}{3} = 0.$
Whence $x = 2\frac{1}{2}$ or $-3\frac{1}{3}$.

27. Let x and $x + 2$ represent the time in hours required by each respectively.

Then
$$\frac{1}{x} + \frac{1}{x + 2} = \frac{4}{3},$$
or $$2x^2 + x = 3.$$

Whence $x = 1$ or $-\frac{3}{2}$.
Therefore $x = 1$,
and $x + 2 = 3$.

28. Let r and $r + 7$ represent the rates in miles per hour respectively.

Then

$$\frac{200}{r} - \frac{200}{r+7} = \frac{7}{4},$$

or $r^2 + 7r = 800$.
Whence $r = 25$ or -32.
Therefore $r = 25$,
and $r + 7 = 32$.

29. Let $x =$ the number of miles B goes,
and $x + 17 =$ the number of miles A goes.

Then

$$x^2 + (x + 17)^2 = (53)^2,$$

or $x^2 + 17x = 1260$.
Whence $x = 28$ or -45.

Therefore $\frac{x}{5} = \frac{28}{5}$, B's rate in miles per hour;

and $\frac{x+17}{5} = \frac{45}{5} = 9$, A's rate in miles per hour.

30. Let $x =$ the distance between them at the end of 5 hours.
Then $x - 9 =$ A's distance at the end of 5 hours,
and $x - 8 =$ B's distance at the end of 5 hours.
Then $(x-8)^2 + (x-9)^2 = x^2$,
or $x^2 - 34x + 145 = 0$.
Whence $x = 5$ or 29.
The value 5 for x is rejected.

Hence $$\text{A's rate} = \frac{29-9}{5} = 4 \text{ miles per hour,}$$

and $$\text{B's rate} = \frac{29-8}{5} = 4\tfrac{1}{5} \text{ miles per hour.}$$

31. Let $s =$ the height in feet.

Then $$\frac{32 \cdot (15)^2}{2} = s,$$

or $3600 = s$.

32. Let $x =$ the time in seconds for falling,
and $6\frac{1}{2} - x =$ the time in seconds for the sound to return.
Then $16x^2 =$ the distance fallen,
and $(6\frac{1}{2} - x)1152 =$ the distance the sound returns.
Hence $16x^2 = (\frac{13}{2} - x)1152$,
or $x^2 + 72x - 468 = 0$.
Whence $x = 6$ or -78.

Then $$\text{height} = \frac{32 \cdot 6^2}{2} = 576 \text{ feet.}$$

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1. $55 \cdot 3 \div 11 = 15$, the number of revolutions.

2. $132 \cdot 3 \div (4 \cdot \frac{22}{7}) = 31\frac{1}{2}$, the number of revolutions.

3. Let $x =$ the circumference of the fore wheel in feet.

Then $x + 2 =$ the circumference of the rear wheel in feet.

Therefore $$\frac{420}{x} - \frac{420}{x+2} = 5,$$

or $$x^2 + 2x = 168.$$

Whence $$x = 12 \text{ or } -14,$$

and $$x + 2 = 14.$$

4. Let $x =$ the circumference of the fore wheel in feet.

Then $x + 2 =$ the circumference of the rear wheel in feet.

Therefore $$\frac{300}{x} - \frac{300}{x+2} = 5,$$

or $$x^2 + 2x = 120.$$

Whence $$x = 10 \text{ or } -12,$$

and $$x + 2 = 12.$$

Again $$10 \div \frac{2 \cdot 22}{7} = 1\frac{10}{11}, \text{ radius of the fore wheel.}$$

$$12 \div \frac{2 \cdot 22}{7} = 1\frac{13}{22}, \text{ radius of the rear wheel.}$$

5. Let x and $x - 2$ represent the circumference of the rear and the fore wheels respectively in feet.

Then $$\frac{60}{x} + \frac{60}{x-2} = 11,$$

or $$11x^2 - 142x + 120 = 0.$$

Whence $$x = 12 \text{ or } \frac{10}{11},$$

and $$x - 2 = 10 \text{ or } -\frac{12}{11}.$$

Using the first values, $12 \div \frac{22}{7} = 3\frac{9}{11}$, diameter of rear wheel in feet.

and $10 \div \frac{22}{7} = 3\frac{2}{11}$, diameter of fore wheel in feet.

6. Let $r =$ the radius of the larger circle in inches.

Then $r - 7 =$ the radius of the smaller circle in inches.

Therefore $$\frac{22}{7} r^2 - \frac{22}{7}(r - 7)^2 = 770,$$

or $$44r = 924.$$

Whence $$r = 21,$$

and $$r - 7 = 14.$$

7. $AB(18 - AB) = 4 \cdot 20,$

or $\overline{AB}^2 - 18\,\overline{AB} + 80 = 0.$

Whence $AB = 8 \text{ or } 10.$

8. $AB(5 - AB) = 4 \cdot \frac{3}{2},$

or $\overline{AB}^2 - 5\,\overline{AB} + 6 = 0.$

Whence $AB = 2 \text{ or } 3 \text{ feet},$

and $BC = 3 \text{ or } 2 \text{ feet}.$

9. $81 = BC(30 - BC),$

or $\overline{BC}^2 - 30\,\overline{BC} + 81 = 0.$

Whence $BC = 27 \text{ or } 3,$

and $BD = 3 \text{ or } 27.$

10. $400 = 2\,BD \cdot BD,$

or $\overline{BD}^2 = 200.$

Whence $BD = 10\sqrt{2}.$

11. $\dfrac{12}{AP} = \dfrac{AP}{12 - AP},$

or $\overline{AP}^2 - 12\,\overline{AP} - 144 = 0.$

$$AP = -6 \pm 6\sqrt{5}.$$

The negative answer has a meaning in the dividing of a line externally by a point.

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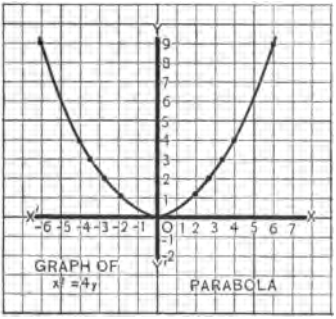
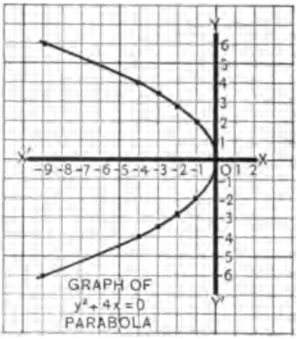
1.

x	0	± 2	± 2.8	± 3.4	± 4	± 6
y	0	1	2	3	4	9

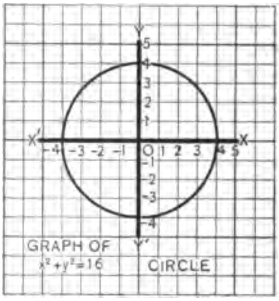


2.

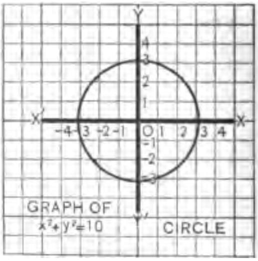
x	0	-1	-2	-3	-4	-9
y	0	± 2	± 2.8	± 3.4	± 4	± 6



3. $r = \sqrt{16} = 4.$

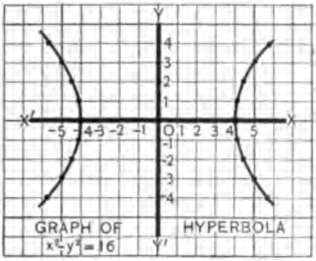


4. $r = \sqrt{10} = 3.1+.$



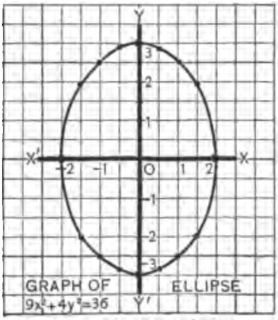
5.

x	± 4	± 4.1	± 4.5	± 5	± 5.6	± 4.1	± 4.5	± 5	± 5.6
y	0	1	2	3	4	− 1	− 2	− 3	− 4



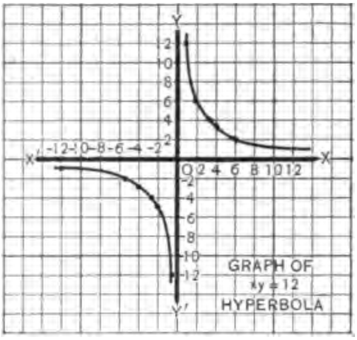
6.

x	0	± 2	− 1	$-\frac{3}{2}$	$-\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{1}{2}$
y	± 3	0	± 2.6	± 1.9	± 2.8	± 2.6	± 1.9	± 2.8



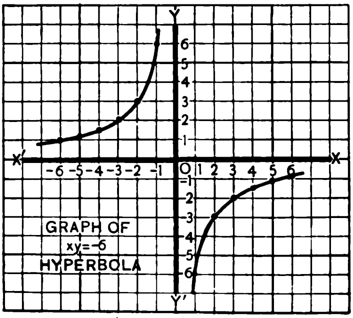
7.

x	± 12	± 6	± 5	± 4	± 2	± 1
y	± 1	± 2	$\pm \frac{12}{5}$	± 3	± 6	± 12



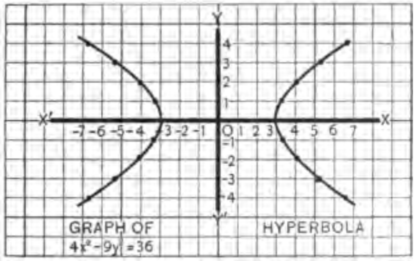
8.

x	± 6	± 5	± 4	± 3	± 2	± 1
y	∓ 1	$\mp \frac{7}{6}$	$\mp \frac{3}{2}$	∓ 2	∓ 3	∓ 6



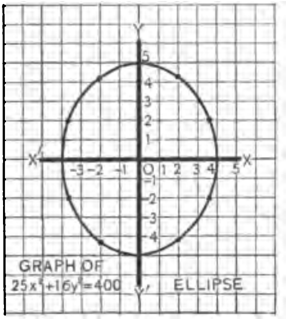
9.

x	± 6.7	± 6.7	± 5.4	± 5.4	± 4.2	± 4.2	± 3.3	± 3.3	± 3
y	+ 4	− 4	+ 3	− 3	+ 2	− 2	+ 1	− 1	0



10.

y	± 5	2	− 2	3.6	− 3.6	0
x	0	± 4.3	± 4.3	± 2	± 2	± 4



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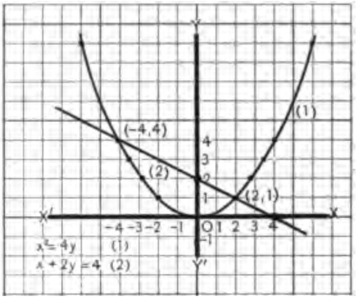
1. $x^2 = 4y.$

x	± 2	± 2.8	± 3.4	± 4	± 4.9
y	1	2	3	4	6

$x + 2y = 4.$

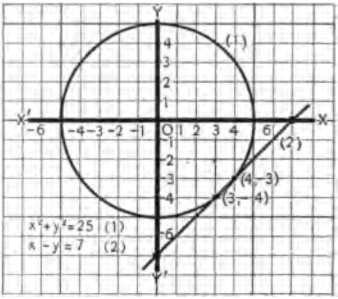
x	0	4
y	2	0



2. $x^2 + y^2 = 25,$
$r = 5.$

$x - y = 7.$

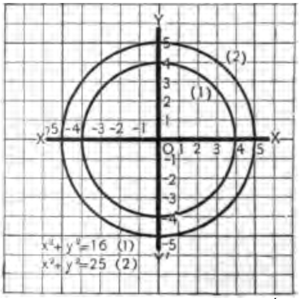
x	0	7
y	-7	0



3. $x^2 + y^2 = 16,$
$r = 4.$

$x^2 + y^2 = 25,$
$r = 5.$

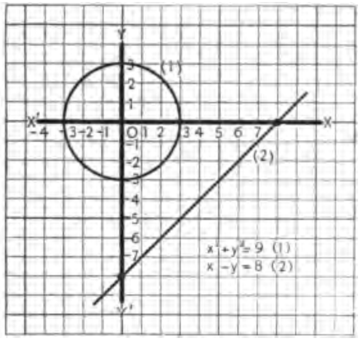
The system is inconsistent.



4.

$$x^2 + y^2 = 9,$$
$$r = 3.$$
$$x - y = 8.$$

x	0	8
y	-8	0

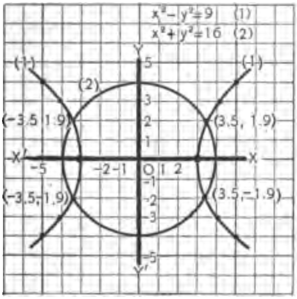


There are no sets of real roots, for the graphs do not intersect.

5.

$$x^2 + y^2 = 16,$$
$$r = 4.$$
$$x^2 - y^2 = 9.$$

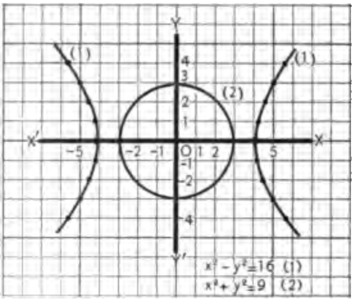
x	± 3	± 3.6	± 3.6	± 4.2	± 4.2	± 5	± 5
y	0	2	-2	3	-3	4	-4



6.

$$x^2 + y^2 = 9,$$
$$r = 3.$$
$$x^2 - y^2 = 16.$$

x	± 4	± 4.2	± 4.2	± 4.4	± 4.4	± 5	± 5	± 5.6	± 5.6
y	0	1	-1	2	-2	3	-3	4	-4

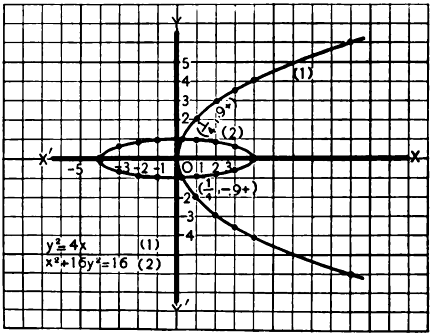


The system has no set of real roots.

7. $y^2 = 4x$, given in text on page 280.

$$x^2 + 16y^2 = 16.$$

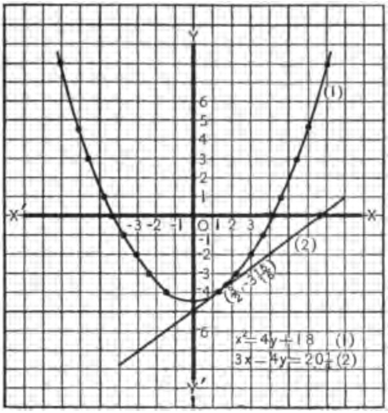
x	4	3	2	1	0	-1	-2	-3	-4
y	0	$\pm .6$	$\pm .8$	$\pm .9$	± 1	$\pm .9$	$\pm .8$	$\pm .6$	0



8. $x^2 = 4y + 18$, given in text on page 285.

$$3x - 4y = 20\tfrac{1}{4}.$$

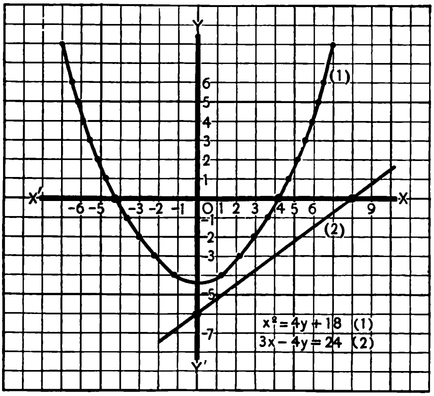
x	0	$6\frac{3}{4}$
y	$-5\frac{1}{16}$	0



9. $x^2 = 4y + 18$, given in text on page 285.

$$3x - 4y = 24.$$

x	0	8
y	-6	0



The system has no set of real roots.

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1. $x + y = 8,$ (1)
$x^2 + y^2 = 34.$ (2)

Substituting $8 - y$ for x from (1) in (2),

$y^2 + (8 - y)^2 = 34.$ (3)

Solving (3), $x = 3$ or $5,$
and $y = 5$ or $3.$

2. $2m + n = 14,$ (1)
$m^2 + 3mn = 49.$ (2)

Substituting $14 - 2m$ for n from (1) in (2),

$m^2 + 3m(14 - 2m) = 49.$ (3)

Solving (3), $m = 7$ or $\frac{7}{5},$
and $n = 0$ or $\frac{56}{5}.$

3. $2m^2 + n^2 = 22,$ (1)
$m - n\sqrt{5} = 0.$ (2)

Substituting $n\sqrt{5}$ for m from (2) in (1),

$10n^2 + n^2 = 22.$ (3)

Solving (3), $n = +\sqrt{2}$ or $-\sqrt{2},$
and $m = +\sqrt{10}$ or $-\sqrt{10}.$

4. $4s + 5t = 6,$ (1)
$st = -2.$ (2)

Substituting $\frac{6 - 5t}{4}$ for s from (1) in (2),

$\frac{t(6 - 5t)}{4} = -2.$ (3)

Solving (3), $t = 2$ or $-\frac{4}{5},$
and $s = -1$ or $\frac{5}{2}.$

5. $xy + 12 = 0,$ (1)
$4x - 3y = 30.$ (2)

Substituting $\frac{3y + 30}{4}$ for x from (2) in (1),

$\frac{3y^2 + 30y}{4} + 12 = 0.$ (3)

Solving (3), $y = -2$ or $-8,$
and $x = 6$ or $\frac{3}{2}.$

6. $x\sqrt{3} + 5y\sqrt{3} = -72,$ (1)
$xy = -15.$ (2)

Substituting $-24\sqrt{3} - 5y$ for x from (1) in (2),

$-24y\sqrt{3} - 5y^2 = -15.$ (3)

Solving (3) by formula,

$y = \frac{\sqrt{3}}{5}$ or $-5\sqrt{3},$
and $x = -25\sqrt{3}$ or $\sqrt{3}.$

7. $3R_1 + 2R_2 = 5,$ (1)
$R_1R_2 - 6R_1 = -3.$ (2)

Substituting $\frac{5 - 2R_2}{3}$ for R_1 from (1) in (2),

$2R_2^2 - 17R_2 + 21 = 0.$ (3)

Solving (3), $R_2 = 7$ or $\frac{3}{2},$
and $R_1 = -3$ or $\frac{2}{3}.$

8. $xy + y^2 - 5 = 0,$ (1)
$xy + 20 = 0.$ (2)

(1) – (2), $y^2 = 25.$
Whence $y = +5$ or $-5,$
and $x = -4$ or $+4.$

9. $h^2 + k^2 + k = 10,$ (1)
$h + k + 1 = 0.$ (2)

Substituting $-k - 1$ for h from (2) in (1),

$2k^2 + 3k - 9 = 0.$ (3)

Solving (3), $k = \frac{3}{2}$ or $-3,$
and $h = -\frac{5}{2}$ or $2.$

10. $m^2 + 3mn + n^2 = 22,$ (1)
$2m = n.$ (2)

Substituting $2m$ for n from (2) in (1),

$11m^2 = 22.$ (3)

Solving (3), $m = +\sqrt{2}$ or $-\sqrt{2},$
and $n = 2\sqrt{2}$ or $-2\sqrt{2}.$

11. $x^2 + y^2 + 2x + 3y = 10,$ (1)
$x - 5 = y.$ (2)

Substituting $x-5$ for y from (2) in (1),

$$2x^2-5x=0. \qquad (3)$$

Solving (3), $x=0$ or $\frac{5}{2}$,
and $y=-5$ or $-\frac{5}{2}$.

12. $y-x\sqrt{8}=0,$ (1)
$y^2+x^3=9x.$ (2)

Substituting $x\sqrt{8}$ for y from (1) in (2), $8x^2+x^3=9x.$ (3)

Transposing and factoring the left member,

$$x(x+9)(x-1)=0.$$

Whence $x=0,\ -9,$ or $1,$
and $y=0,\ -18\sqrt{2},$ and $+2\sqrt{2}.$

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Example 2.
$A,\ x=3,\ y=2;$
$B,\ x=2,\ y=3;$
$C,\ x=-2,\ y=-3;$
$D,\ x=-3,\ y=-2.$

1. $x-y=1,$ (1)
$xy=12.$ (2)

$(1)^2+(2)\cdot 4,$

$$x^2+2xy+y^2=49,$$

or $\begin{cases} x+y=+7, & (3)\\ x+y=-7. & (4)\end{cases}$

$[(1)+(3)]\div 2,$ $x=4,$
and $y=3.$
$[(1)+(4)]\div 2,$ $x=-3,$
and $y=-4.$

2. $x+2y=4,$ (1)
$xy+6=0.$ (2)

$(1)^2-(2)\cdot 8,$

$$x^2-4xy+4y^2=64,$$

or $\begin{cases} x-2y=8, & (3)\\ x-2y=-8. & (4)\end{cases}$

$[(1)+(3)]\div 2,$ $x=6,$
and $y=-1.$
$[(1)+(4)]\div 2,$ $x=-2,$
and $y=3.$

3. $x^2+4y^2=41,$ (1)
$xy+10=0.$ (2)

$(1)+4\cdot(2),$

$$x^2+4xy+4y^2=1,$$

or $\begin{cases} x+2y=1, & (3)\\ x+2y=-1. & (4)\end{cases}$

$(1)-4(2),$

$$x^2-4xy+4y^2=81,$$

or $\begin{cases} x-2y=9, & (5)\\ x-2y=-9. & (6)\end{cases}$

$[(3)+(5)]\div 2,$ $x=5,$
and $y=-2.$
$[(3)+(6)]\div 2,$ $x=-4,$
and $y=\frac{5}{2}.$
$[(4)+(5)]\div 2,$ $x=4,$
and $y=-\frac{5}{2}.$
$[(4)+(6)]\div 2,$ $x=-5,$
and $y=2.$

4. $3x-y=12,$ (1)
$9x^2+y^2=72.$ (2)
$(1)^2-(2),$ $-6xy=72.$ (3)

$(2)-(3),$

$$9x^2+6xy+y^2=0,$$

or $\begin{cases} 3x+y=0, & (4)\\ 3x+y=0. & (5)\end{cases}$

From (1) and (4), $x=2,$
and $y=-6.$

5. $x^2-3xy+4y^2=11,$ (1)
$xy=10.$ (2)

$(1)-(2),$

$$x^2-4xy+4y^2=1,$$

or $\begin{cases} x-2y=1, & (3)\\ x-2y=-1. & (4)\end{cases}$

$(1)+(2)\cdot 7,$

$$x^2+4xy+4y^2=81,$$

or $\begin{cases} x+2y=9, & (5)\\ x+2y=-9. & (6'\end{cases}$

From (3) and (5), $x = 5$,
and $y = 2$.

From (3) and (6), $x = -4$,
and $y = -\frac{5}{2}$.

From (4) and (5), $x = 4$,
and $y = \frac{5}{2}$.

From (4) and (6), $x = -5$,
and $y = -2$.

6. $x^2 + y^2 = 25$, (1)
$x^2 + 2xy + y^2 = 49$. (2)
(2) − (1), $2xy = 24$. (3)
(1) − (3),
$x^2 - 2xy + y^2 = 1$,

or $\begin{cases} x - y = +1, & (4) \\ x - y = -1. & (5) \end{cases}$

From (2), $\begin{cases} x + y = +7, & (6) \\ x + y = -7. & (7) \end{cases}$

From (4) and (6), $x = 4$,
and $y = 3$.

From (4) and (7), $x = -3$,
and $y = -4$.

From (5) and (6), $x = 3$,
and $y = 4$.

From (5) and (7), $x = -4$,
and $y = -3$.

7. $x^2 + y^2 = 25$,
$x^2 - 2xy + y^2 = 49$.

Work as in Ex. (6).
$x = 4, -4, 3, -3$,
$y = -3, 3, -4, 4$.

8. $x^2 + y^2 = 15$, (1)
$x + y = 3\sqrt{3}$. (2)
$(2)^2 - (1)$, $2xy = 12$. (3)
(1) − (3),
$x^2 - 2xy + y^2 = 3$,

or $\begin{cases} x - y = +\sqrt{3}, & (4) \\ x - y = -\sqrt{3}. & (5) \end{cases}$

From (2) and (4), $x = 2\sqrt{3}$,
and $y = \sqrt{3}$.

From (2) and (5), $x = \sqrt{3}$,
and $y = 2\sqrt{3}$.

9. $x^2 - xy = 4$, (1)
$y^2 - xy = -3$. (2)
(1) + (2),
$x^2 - 2xy + y^2 = 1$,

or $\begin{cases} x - y = 1, & (3) \\ x - y = -1. & (4) \end{cases}$

Substituting from (3) in (1), and solving,
$x = 4$,
and $y = 3$.

Substituting from (4) in (1), and solving,
$x = -4$,
and $y = -3$.

10. $x^2 = y^2 + 5$, (1)
$y^2 + xy = 10$. (2)
(1) + 2 (2),
$x^2 + 2xy + y^2 = 25$,

or $\begin{cases} x + y = 5, & (3) \\ x + y = -5. & (4) \end{cases}$

Substituting from (3) in (2), and solving,
$y = 2$,
and $x = 3$.

Substituting from (4) in (2), and solving,
$y = -2$, and $x = -3$.

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1. $x^2 - y^2 = 16$, (1)
$x + y = 8$. (2)
(1) ÷ (2), $x - y = 2$. (3)
Solving (2) and (3), $x = 5$,
and $y = 3$.

2. $\frac{1}{x^2}-\frac{1}{y^2}=5,$ (1)

$\frac{1}{x}-\frac{1}{y}=1.$ (2)

(1) ÷ (2), $\frac{1}{x}+\frac{1}{y}=5.$ (3)

Solving (2) and (3), $x=\frac{1}{3}$,
and $y=\frac{1}{2}$.

3. $R^2h-50=0,$
or $R^2h=50,$ (1)
$Rh=10.$ (2)
(1) ÷ (2), $R=5.$
Then $h=2.$

4. $h^2k-100=0,$
or $h^2k=100.$ (1)
$k^2h+80=0,$
or $k^2h=-80.$ (2)
(1) ÷ (2), $\frac{h}{k}=-\frac{5}{4},$
or $h=\frac{-5k}{4}.$ (3)

Substituting from (3) in (1),

$$\frac{25k^3}{16}-100=0.$$

Whence $k=4.$
Then $h=-5.$

5. $P(1+r)^2=112.36,$ (1)
$P+Pr=106.$ (2)
(1) ÷ (2), $1+r=1.06,$
or $r=\frac{6}{100}.$
Then $P=100.$

6. $\frac{gt^2}{2}=.0256,$ (1)
$gt=1.28.$ (2)
[(1) ÷ (2)] · 2, $t=.04.$
Then $g=32.$

7. $x^2y^2+6=5xy,$

or $\begin{cases}xy-3=0, & (1)\\ xy-2=0. & (2)\end{cases}$

$xy+4=6,$
or $xy-2=0.$ (3)

The system $\begin{cases}(1),\ xy-3=0\\(3),\ xy-2=0\end{cases}$ is inconsistent.

The system $\begin{cases}(2),\ xy-2=0\\(3),\ xy-2=0\end{cases}$ has an infinite number of sets of roots.

There are therefore an infinite number of sets of roots for the original system.

8. $x^4=y^4+175,$
or $x^4-y^4=175.$ (1)
$x^2=y^2+7,$
or $x^2-y^2=7.$ (2)
(1) ÷ (2), $x^2+y^2=25.$ (3)
[(2) + (3)] ÷ 2, $x^2=16,$
or $x=+4$ or $-4.$

Substituting 4 for x in (2), and solving,
$y=+3$ or $-3.$

Substituting -4 for x in (2), and solving, $y=+3$ or $-3.$

Then $\begin{cases}x=4,\ -4,\ -4,\ +4.\\ y=3,\ -3,\ +3,\ -3.\end{cases}$

9. $x^2-xy-6y^2=16,$ (1)
$x-3y=1.$ (2)
(1) ÷ (2), $x+2y=16.$ (3)
[(3) − (2)] ÷ 5, $y=3.$
Then $x=10.$

10. $$x^2 + 7 = 4xy + 5y^2,$$

or $$(x - 5y)(x + y) = -7. \quad (1)$$

$$x^2 = 1 + y^2,$$

or $$(x - y)(x + y) = 1. \quad (2)$$

(1) ÷ (2), $$\frac{x - 5y}{x - y} = -7,$$

or $$x = \frac{3y}{2}. \quad (3)$$

Substituting from (3) in (2), and solving,

$$y = \tfrac{2}{5}\sqrt{5} \text{ or } -\tfrac{2}{5}\sqrt{5}.$$

Then $x = \frac{3}{5}\sqrt{5}$ or $-\frac{3}{5}\sqrt{5}$.

11. $$x^3 + y^3 = 12, \quad (1)$$

$$x + y = 2. \quad (2)$$

(1) ÷ (2), $x^2 - xy + y^2 = 6. \quad (3)$

$[(2)^2 - (3)] \div 3$, $xy = -\frac{2}{3}. \quad (4)$

(3) − (4),

$$x^2 - 2xy + y^2 = \tfrac{20}{3},$$

or $$\begin{cases} x - y = +\sqrt{\tfrac{20}{3}}, & (5) \\ x - y = -\sqrt{\tfrac{20}{3}}. & (6) \end{cases}$$

From (2) and (6), $x = 1 + \dfrac{\sqrt{15}}{3}$,

and $$y = 1 - \frac{\sqrt{15}}{3}.$$

From (2) and (5), $x = 1 - \dfrac{\sqrt{15}}{3}$,

and $$y = 1 + \frac{\sqrt{15}}{3}.$$

12. $$x^2 - xy + y^2 = 6, \quad (1)$$

$$x^3 + y^3 = 12. \quad (2)$$

(2) ÷ (1), $x + y = 2. \quad (3)$

The solution of (1) and (3) is like that of (2) and (3) in Ex. 11, giving the same sets of roots.

13. $$x + y = 2, \quad (1)$$

$$x^2 - xy + y^2 = 6. \quad (2)$$

The solution of (1) and (2) is like that of (2) and (3) in Ex. 11, and gives the same sets of roots.

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1. Let g = the greater number,
and l = the less.

Then $$g - l = 3, \quad (1)$$

and $$g^2 - l^2 = 45. \quad (2)$$

(2) ÷ (1), $g + l = 15. \quad (3)$

Solving the system $\begin{cases}(1)\\(3)\end{cases}$,

$$g = 9,$$

and $$l = 6.$$

2. Let g = the greater number,
and l = the less.

Then $$g + l = 18, \quad (1)$$

and $$g^2 + l^2 = 170. \quad (2)$$

$(1)^2 - (2)$, $2gl = 154. \quad (3)$

(2) − (3),

$$g^2 - 2gl + l^2 = 16,$$

or $$\begin{cases} g - l = 4, & (4) \\ g - l = -4. & (5) \end{cases}$$

From (1) and (4),

$$g = 11,$$

and $$l = 7.$$

From (1) and (5),

$$g = 7,$$

and $$l = 11.$$

The second set is rejected.

3. Let m = the one number,
and n = the other.

Then $$mn = 108, \quad (1)$$

and $$\frac{n}{m} = \frac{3}{4}. \quad (2)$$

Substituting from (2) in (1), and solving,

$$n = 9 \text{ or } -9.$$

Then $$m = 12 \text{ or } -12.$$

4. Let a = one leg in feet,
and b = the other leg in feet.

Then $\frac{a \cdot b}{2} = 84,$

or $ab = 168,$ (1)

and $a^2 + b^2 = (25)^2.$ (2)

$(2) + 2 \cdot (1),$

$$a^2 + 2ab + b^2 = 961,$$

or $\begin{cases} a + b = +31, & (3) \\ a + b = -31. & (4) \end{cases}$

$(2) - 2 \cdot (1),$

$$a^2 - 2ab + b^2 = 289,$$

or $\begin{cases} a - b = +17, & (5) \\ a - b = -17. & (6) \end{cases}$

Solving $\begin{cases} (3) \\ (5) \end{cases}$, $\begin{cases} (4) \\ (5) \end{cases}$, $\begin{cases} (3) \\ (6) \end{cases}$, and $\begin{cases} (4) \\ (6) \end{cases}$, and rejecting negative roots, we have

$a = 24,$
$b = 7;$

or $a = 7,$
$b = 24.$

5. Let l = the length in rods,
and w = the width in rods.

Then $l - w = 39,$ (1)

and $lw = 1600.$ (2)

Substituting $w + 39$ for l from (1) in (2),

$$w^2 + 39w = 1600.$$

Whence $w = 25$ or $-64.$

Rejecting $w = -64,$

$$l = 64.$$

6. Let x = the side of one square in feet,
and y = the side of the other square in feet.

Then $x^2 - y^2 = 208,$ (1)

and $4x - 4y = 32.$ (2)

$(1) \div [(2) \div 4],$

$x + y = 26.$ (3)

$[(2) + 4(3)] \div 8,$ $x = 17.$

Then $y = 9.$

7. Let x and y represent the dimensions in rods.

Then $xy = 360,$ (1)

and $x^2 + y^2 = 1681.$ (2)

$(1) \cdot 2 + (2),$

$$x^2 + 2xy + y^2 = 2401,$$

or $\begin{cases} x + y = 49, & (3) \\ x + y = -49. & (4) \end{cases}$

$(2) - (1) \cdot 2,$

$$x^2 - 2xy + y^2 = 961,$$

or $\begin{cases} x - y = 31, & (5) \\ x - y = -31. & (6) \end{cases}$

Solving the systems $\begin{cases} (3) \\ (5) \end{cases}$, $\begin{cases} (3) \\ (6) \end{cases}$, $\begin{cases} (4) \\ (5) \end{cases}$, and $\begin{cases} (4) \\ (6) \end{cases}$, and rejecting negative roots, we have

$x = 40$ or $9,$

and $y = 9$ or $40.$

Therefore the perimeter is 98 rods.

8. Let l = the length in feet,
and w = the width in feet.

Then $2l + 2w = 92,$ (1)

and $lw = 504.$ (2)

Solving (1) and (2),

$l = 28$ or $18,$

and $w = 18$ or $28.$

9. Let x = one number,
and y = the other number.

Then $\sqrt{xy} = \sqrt{40},$

or $xy = 40,$ (1)

and $x^2 + y^2 = 89.$ (2)

Solving, we have

$\begin{cases} x + y = 13, & (3) \\ x + y = -13. & (4) \end{cases}$

$\begin{cases} x - y = 3, & (5) \\ x - y = -3. & (6) \end{cases}$

$x = 8$ or $-8,$

and $y = 5$ or $-5.$

10. Let $\frac{x}{y}$ = the fraction.

Then $$\frac{x}{y}=\frac{2}{3},$$

or $$3x=2y. \qquad (1)$$

$$\frac{x^2-50}{y^2-50}=\frac{2}{7},$$

or $$7x^2-2y^2=250. \qquad (2)$$

Solving (1) and (2) by substitution,

$$x=10,$$

and $$y=15.$$

Hence $$\frac{x}{y}=\frac{10}{15}.$$

11. Let b = the base in inches,

and a = the altitude in inches.

Then $$b-a=5, \qquad (1)$$

and $$\frac{b\cdot a}{2}=\frac{25}{24}\cdot 144,$$

or $$ab=300. \qquad (2)$$

Solving (1) and (2) by substitution,

$$b=20,$$

and $$a=15.$$

12. Let x and y represent the edges in inches.

Then $$x^3-y^3=1647, \qquad (1)$$

and $$x-y=3. \qquad (2)$$

(1) ÷ (2),

$$x^2+xy+y^2=549. \qquad (3)$$

Solving (2) and (3) by substitution,

$$x=15 \text{ or } -12,$$

$$y=12 \text{ or } -15.$$

The negative values are rejected.

13. Let r_1 and r_2 represent the radii in inches.

Then $$r_1+r_2=27, \qquad (1)$$

and $$r_1^2-r_2^2=81. \qquad (2)$$

Solving (1) and (2) by substitution,

$$r_1=15,$$

and $$r_2=12.$$

14. Let l and w represent the length and width respectively.

Then $$2l+2w=3c, \qquad (1)$$

and $$lw=\frac{c^2}{2}. \qquad (2)$$

Solving (1) and (2) by substitution,

$$l=c \text{ or } \frac{c}{2},$$

and $$w=\frac{c}{2} \text{ or } c.$$

15. Let x and y represent the legs.

Then $$\frac{xy}{2}=2a^2-2b^2,$$

or $$xy=4a^2-4b^2. \qquad (1)$$

$$\sqrt{x^2+y^2}=2\sqrt{2a^2+2b^2},$$

or $$x^2+y^2=8a^2+8b^2. \qquad (2)$$

Solving (1) and (2) as before,

$$x=2a-2b \text{ or } 2a+2b,$$

and $$y=2a+2b \text{ or } 2a-2b.$$

16. Let a, b, and c represent the legs and the hypotenuse respectively.

Then $$a+b+c=40, \qquad (1)$$

$$a^2+b^2=c^2, \qquad (2)$$

and $$\frac{ab}{2}=60. \qquad (3)$$

Substituting $40-a-b$ for c from (1) in (2),

$$40a+40b=800+ab. \qquad (4)$$

From (3), $$ab=120.$$

Substituting 120 for ab in (4),

$$a+b=23. \qquad (5)$$

(1) – (5), $$c=17.$$

Solving $$\begin{cases} ab=120, \\ a+b=23, \end{cases}$$

$$a=8 \text{ or } 15,$$

and $$b=15 \text{ or } 8.$$

17. Let x and y represent the integers.

Then $x - y = 3,$ (1)

and $x^2 + y^2 = 115.$ (2)

By substituting from (1) into (2),

$$(y + 3)^2 + y^2 = 115,$$

or $y^2 + 3y = 53.$

Then $y = \dfrac{-3 \pm \sqrt{231}}{2}$, not a positive integer.

18. Let $10x + y =$ the number.

Then

$(10x + y)(x + y) = 576,$

or

$10x^2 + 11xy + y^2 = 576.$ (1)

$10x + y + 3x + 3y = 10y + x,$

or $y = 2x.$ (2)

Solving (1) and (2) by substitution,

$x = 4$ or $-4,$

and $y = 8,$

or $10x + y = 48.$

19. Let $P =$ the principal in dollars,

and $\dfrac{r}{100} =$ the rate.

Then $\dfrac{P \cdot r}{100} = 48,$

or $Pr = 4800.$ (1)

$$(P + 200)\left(\frac{r - 1}{100}\right) = 50,$$

or $Pr + 200r - P = 5200.$ (2)

(2) − (1),

$200r - P = 400.$ (3)

Solving (1) and (3) by substitution,

$r = 6$ or $-4,$

and $P = 800.$

20. Let x and y represent the rates of A and B in miles per hour respectively.

Then

$$(x)^2 + (y + 3)^2 = 17^2,$$

or $x^2 + y^2 + 6y = 280.$ (1)

$$\left(\frac{7x}{2}\right)^2 + \left(\frac{7y}{2} + 3\right)^2 = 53^2,$$

or $49x^2 + 49y^2 + 84y = 11{,}200.$ (2)

$[(2) - (1) \cdot 49] \div -210,$

$y = 12.$

Then $x = 8.$

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1. Solving

$$\begin{cases} x^2 + y^2 = 36, & (1) \\ (10 - x)^2 + y^2 = 64. & (2) \end{cases}$$

$[(2) - (1)] \div 20,$ $x = 3\frac{3}{5},$

and $y = 4\frac{4}{5}.$

2. Let a be the altitude, b the portion of the base below 5, and $13 - b$ the portion below 12, in a figure similar to that of Ex. 1 in text.

Then $a^2 + b^2 = 25,$

and $(13 - b)^2 + a^2 = 144.$

Whence $a = 4\frac{8}{13},$

and area $= 30.$

3. Let a be the altitude, b the portion of the base below 13, and $14 - b$ the portion below 15.

Then $a^2 + b^2 = 169,$

and $a^2 + (14 - b)^2 = 225.$

Whence $a = 12,$

and area $= 84.$

4. Let a be the altitude, b the portion of the base below 10, and $9 - b$ the portion below 17.

Then $a^2 + b^2 = 100,$

and $(9 - b)^2 + a^2 = 289.$

Whence $a = 8,$

and area $= 36.$

5. Let a be the altitude, b the portion of the base below 13, and $11 - b$ the portion below 20.

Then $a^2 + b^2 = 169,$

and $a^2 + (11 - b)^2 = 400.$

Whence $a = 12,$

and area $= 66.$

6. $x^2 + y^2 = 100,$ (1)

$(12 - x)^2 + y^2 = 100.$ (2)

Solving, $y = 8.$

7. Using a figure similar to that of Ex. 6 in text,

$$x^2 + y^2 = 36,$$
$$(10 - x)^2 + y^2 = 64.$$

Solving, $y = 4\frac{4}{5}.$

8. Using a figure similar to that of Ex. 6 in text,

$$x^2 + y^2 = 625,$$
$$(6 - x)^2 + y^2 = 841.$$

Solving, $y = 20.$

$$\text{Area} = \left(\frac{10 + 16}{2}\right)20 = 260.$$

9. Using a figure similar to that of Ex. 6 in text,

$$x^2 + y^2 = 100,$$
$$(21 - x)^2 + y^2 = 289.$$

Solving, $y = 8.$

$$\text{Area} = \left(\frac{20 + 41}{2}\right)8 = 244.$$

10. Let y represent the altitude as in Ex. 6, x the portion below 17, and $12 - x$ the portion below 25.

Then $x^2 + y^2 = 289,$

and $(12 - x)^2 + y^2 = 625.$

Whence $y = 15.$

Then $\left(\frac{12 + 24}{2}\right)15 = 270.$

11. Let $x =$ the altitude.

Then 29, 21, and x bound a right triangle, and

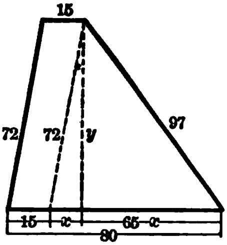
$$x^2 + 441 = 841,$$

or $x = 20.$

$$\text{Area} = \frac{9 + 30}{2} \cdot 20 = 390.$$

12. From the figure

$$x^2 + y^2 = (72)^2, \quad (1)$$
$$(65 - x)^2 + y^2 = (97)^2. \quad (2)$$



Solving (1) and (2),

$y = 72,$

and $x = 0.$

$$\text{Area} = \frac{(15 + 80)\,72}{2} = 3420.$$

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1. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$

2. $4^{-3} = \frac{1}{4^3} = \frac{1}{64}.$

3. $2^{-4} \cdot 3^0 = \frac{1}{2^4} \cdot 1 = \frac{1}{16}.$

4. $2^{-2} \cdot 3^{-4} = \frac{1}{2^2 \cdot 3^4} = \frac{1}{324}.$

5. $7 \cdot 7^0 \cdot 0 = 0.$

6. $\left(\frac{1}{2}\right)^{-3} = \frac{1}{(\frac{1}{2})^3} = \frac{1}{\frac{1}{8}} = 8.$

7. $\left(\frac{2}{3}\right)^{-2}\cdot 4^0 = \frac{1}{(\frac{2}{3})^2} = \frac{1}{\frac{4}{9}} = \frac{9}{4}.$

8. $\left(\frac{4}{5}\right)^{-3}\left(\frac{10}{3}\right)^{-2} = \frac{1}{(\frac{4}{5})^3(\frac{10}{3})^2}$
$= \frac{1}{\frac{64\cdot 100}{125\cdot 9}} = \frac{1}{\frac{256}{45}} = \frac{45}{256}.$

9. $\frac{2}{3^{-2}} = 2\cdot 3^2 = 18.$

10. $\frac{3}{3^0} = \frac{3}{1} = 3.$

11. $\frac{12}{4^{-1}} = 12\cdot 4^1 = 48.$

12. $5\cdot 2^0 - (5\cdot 2)^0 = 5 - 1 = 4.$

13. $\frac{4^{-2}\cdot 3^{-2}}{6^{-2}} = \frac{6^2}{4^2\cdot 3^2} = \frac{2^2}{4^2} = \frac{1}{4}.$

14. $(m - n)^0 = 1$ if $m \neq n$.

15. $32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}} = \frac{1}{2^2} = \frac{1}{4}.$

16. $0^5\cdot 5^0 = 0.$

17. $4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{2}.$

18. $8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}.$

19. $16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{2}.$

20. $8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{2^2} = \frac{1}{4}.$

21. $16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{4^3} = \frac{1}{64}.$

22. $25^{1.5} = 25^{\frac{3}{2}} = 5^3 = 125.$

23. $0^3\cdot 0^{\frac{3}{4}} = 0.$

24. $(-8)^{-\frac{2}{3}} = \frac{1}{(-8)^{\frac{2}{3}}} = \frac{1}{(-2)^2} = \frac{1}{4}.$

25. $(-64)^{-\frac{2}{3}} = \frac{1}{(-64)^{\frac{2}{3}}} = \frac{1}{(-4)^2} = \frac{1}{16}.$

26. $(-32)^{\frac{1}{5}} = -2.$

27. $(32)^{-.4} = \frac{1}{(32)^{\frac{2}{5}}} = \frac{1}{2^2} = \frac{1}{4}.$

28. $(-125)^{-\frac{2}{3}} = \frac{1}{(-125)^{\frac{2}{3}}} = \frac{1}{(-5)^2} = \frac{1}{25}.$

29. $\sqrt[3]{27^{-2}} = \sqrt[3]{3^{-6}} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$

30. $\sqrt[3]{8^{-2}} = \sqrt[3]{2^{-6}} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}.$

31. $(\sqrt[3]{-8})^2 = (-2)^2 = 4.$

32. $\left(\frac{1}{2}\right)^{-4}\cdot\left(\frac{1}{3}\right)^{-3}\cdot\left(\frac{1}{2}\right)^0 = \frac{1}{(\frac{1}{2})^4(\frac{1}{3})^3} = \frac{1}{\frac{1}{16\cdot 27}} = 432.$

33. $\left(\frac{1}{6}\right)^{-2} = \frac{1}{(\frac{1}{6})^2} = \frac{1}{\frac{1}{36}} = 36.$

34. $(.04)^{\frac{3}{2}} = (.2)^3 = .008.$

35. $(.027)^{-\frac{2}{3}} = \dfrac{1}{(.027)^{\frac{2}{3}}} = \dfrac{1}{(.3)^2} = \dfrac{1}{.09} = \dfrac{100}{9}.$

36. $(.064)^{-\frac{1}{3}} = \dfrac{1}{(.064)^{\frac{1}{3}}} = \dfrac{1}{.4} = \dfrac{5}{2}.$

37. $(.00032)^{\frac{2}{5}} = (.2)^2 = .04.$

38. $\dfrac{\sqrt[3]{9^{-3}} \cdot \sqrt[2]{9^{-2}}}{3^{-4}} = \dfrac{9^{-1} \cdot 9^{-1}}{3^{-4}} = \dfrac{3^4}{9 \cdot 9} = 1.$

39. $\dfrac{2^{-1}}{2^{-2} - 2^{-3}} = \dfrac{\frac{1}{2}}{\dfrac{1}{2^2} - \dfrac{1}{2^3}} = \dfrac{\frac{1}{2}}{\frac{1}{8}} = 4.$

40. $\dfrac{3^{-2} - 2^{-2}}{3^{-1} - 2^{-1}} = \dfrac{\dfrac{1}{3^2} - \dfrac{1}{2^2}}{\frac{1}{3} - \frac{1}{2}} = \dfrac{1}{3} + \dfrac{1}{2} = \dfrac{5}{6}.$

41. $\dfrac{2^{-1} + 3^{-1}}{2^{-3} + 3^{-3}} = \dfrac{\frac{1}{2} + \frac{1}{3}}{\dfrac{1}{2^3} + \dfrac{1}{3^3}} = \dfrac{1}{\dfrac{1}{2^2} - \dfrac{1}{2} \cdot \dfrac{1}{3} + \dfrac{1}{3^2}} = \dfrac{1}{\frac{7}{36}} = \dfrac{36}{7}.$

42. $\dfrac{3^{-3} - 2^{-3}}{3^{-1} - 2^{-1}} = \dfrac{\dfrac{1}{3^3} - \dfrac{1}{2^3}}{\frac{1}{3} - \frac{1}{2}} = \dfrac{1}{3^2} + \dfrac{1}{3} \cdot \dfrac{1}{2} + \dfrac{1}{2^2} = \dfrac{19}{36}.$

43. $m^{-3} = \dfrac{1}{m^3}.$

44. $2a^{-3} = \dfrac{2}{a^3}.$

45. $3ab^{-2} = \dfrac{3a}{b^2}.$

46. $7x^2y^{-2} = \dfrac{7x^2}{y^2}.$

47. $x^{-1}y^{-2}z = \dfrac{z}{xy^2}.$

48. $4a^3b^{-2}c^2 = \dfrac{4a^3c^2}{b^2}.$

49. $\dfrac{3}{a^{-2}} = 3a^2.$

50. $\dfrac{4x}{y^{-3}} = 4xy^3.$

51. $\dfrac{4c^0}{xy^{-2}} = \dfrac{4y^2}{x}.$

52. $\dfrac{4a^{-2}b^0}{y^{-5}} = \dfrac{4y^5}{a^2}.$

53. $\dfrac{5^{-1}(ab)^0}{10^{-2}b^2} = \dfrac{10^2}{5b^2} = \dfrac{20}{b^2}.$

54. $\dfrac{3a^3b^{-2}c}{4a^{-2}b^0} = \dfrac{3a^5c}{4b^2}.$

55. $\dfrac{12x^2y^{-1}}{2yx^{-1}} = \dfrac{6x^3}{y^2}.$

56. $\dfrac{10^{-1}a}{bc^3} = \dfrac{a}{10bc^3}.$

57. $\dfrac{4a^{-2}bc^{-3}}{6a^{-2}b^{-3}c^0} = \dfrac{2b^4}{3c^3}.$

58. $\dfrac{4^{-3} \cdot r^{-6} \cdot s^6}{s^{-2}r^{-2}t^3} = \dfrac{s^8}{4^3 \cdot r^4 \cdot t^3} = \dfrac{s^8}{64r^4t^3}.$

59. $\dfrac{2s^{-1}}{m^na^{-b}} = \dfrac{2a^b}{m^ns}.$

60. $\dfrac{5x^2y^a}{x^{-m}y^{-4}} = 5x^{(m+2)}y^{(a+4)}.$

61. $$\frac{2}{a^{-2}-b^{-2}}=\frac{2}{\dfrac{1}{a^2}-\dfrac{1}{b^2}}=\frac{2}{\dfrac{b^2-a^2}{a^2b^2}}=\frac{2\,a^2b^2}{b^2-a^2}.$$

62. $$\frac{3}{a^{-1}+b^{-1}}=\frac{3}{\dfrac{1}{a}+\dfrac{1}{b}}=\frac{3}{\dfrac{b+a}{ab}}=\frac{3\,ab}{a+b}.$$

63. $$\frac{a}{a^{-2}-b^{-2}}=\frac{a}{\dfrac{1}{a^2}-\dfrac{1}{b^2}}=\frac{a}{\dfrac{b^2-a^2}{a^2b^2}}=\frac{a^3b^2}{b^2-a^2}.$$

64. $$\frac{5\,se^2}{s^{-2}+e^{-2}}=\frac{5\,se^2}{\dfrac{1}{s^2}+\dfrac{1}{e^2}}=\frac{5\,se^2}{\dfrac{e^2+s^2}{s^2e^2}}=\frac{5\,s^3e^4}{e^2+s^2}.$$

65. $$\frac{a^{-2}}{a^{-2}+b^{-2}}=\frac{1}{a^2\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)}=\frac{1}{\dfrac{a^2+b^2}{b^2}}=\frac{b^2}{a^2+b^2}.$$

66. $$\frac{a^{-3}b^{-3}}{a^{-3}+b^{-3}}=\frac{1}{a^3b^3\left(\dfrac{1}{a^3}+\dfrac{1}{b^3}\right)}=\frac{1}{b^3+a^3}.$$

67. $$\frac{a^{-2}-b^{-2}}{a^{-1}+b^{-1}}=\frac{\dfrac{1}{a^2}-\dfrac{1}{b^2}}{\dfrac{1}{a}+\dfrac{1}{b}}=\frac{1}{a}-\frac{1}{b}=\frac{b-a}{ab}.$$

68. $$\frac{a^{-4}-b^{-4}}{a^{-2}-b^{-2}}=\frac{\dfrac{1}{a^4}-\dfrac{1}{b^4}}{\dfrac{1}{a^2}-\dfrac{1}{b^2}}=\frac{1}{a^2}+\frac{1}{b^2}=\frac{b^2+a^2}{a^2b^2}.$$

69. $$\frac{a^{-1}+b^{-1}}{a^{-3}+b^{-3}}=\frac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a^3}+\dfrac{1}{b^3}}=\frac{1}{\dfrac{1}{a^2}-\dfrac{1}{ab}+\dfrac{1}{b^2}}=\frac{1}{\dfrac{b^2-ab+a^2}{a^2b^2}}=\frac{a^2b^2}{b^2-ab+a^2}.$$

70. $$\frac{a^{-3}-27^{-1}}{a^{-1}-3^{-1}}=a^{-2}+a^{-1}3^{-1}+3^{-2}=\frac{1}{a^2}+\frac{1}{3\,a}+\frac{1}{9}=\frac{a^2+3\,a+9}{9\,a^2}.$$

71. $\dfrac{2\,xy}{z^2}=2\,xyz^{-2}.$ **72.** $\dfrac{4\,a^3}{b^3}=4\,a^3b^{-3}.$ **73.** $\dfrac{3\,x}{a^{-2}b^4}=3\,a^2b^{-4}x.$

74. $\dfrac{4\,sc^{-3}}{2^{-1}s^{-2}}=2^3\cdot s^3c^{-3}=8\,s^3c^{-3}.$ **76.** $\dfrac{7\,x^{-1}y^2}{2^{-1}y^3}=14\,x^{-1}y^{-1}.$

75. $\dfrac{12\,a^2b^3}{4\,xy^2}=3\,a^2b^3x^{-1}y^{-2}.$ **77.** $\dfrac{5\,ac^{-2}}{c(x-y)^2}=5\,ac^{-3}(x-y)^{-2}.$

78. $\dfrac{7\,m^{-3}n^{\frac{1}{2}}}{m^{\frac{2}{3}}(m-n)^0} = 7\,m^{-\frac{11}{3}}n^{\frac{1}{2}}.$

79. $\dfrac{1}{5\,a^2(c+d)^{-3}} = 5^{-1}a^{-2}(c+d)^3.$

80. $\dfrac{a\,(x-y)^{-2}}{bcx\,(x-y)} = ab^{-1}c^{-1}x^{-1}\,(x-y)^{-3}.$

81. $\dfrac{42\,m^{-n}n^{2m}}{56\,m^{-2n}n^{-3n}} = 3\cdot 4^{-1}m^n n^{2m+3n}.$

82. $\dfrac{r^{-1}s^2}{r^{-2}s^3(s-r)^3} = rs^{-1}(s-r)^{-3}.$

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1. $x^4\cdot x^{-2} = x^2.$

2. $x^4\cdot x^{\frac{1}{2}} = x^{\frac{9}{2}}.$

3. $x^{\frac{1}{3}}\cdot x^{\frac{1}{2}} = x^{\frac{5}{6}}.$

4. $x^{\frac{2}{3}}\cdot x^{\frac{3}{2}} = x^{\frac{13}{6}}.$

5. $x^{\frac{1}{2}}\cdot x^{\frac{1}{3}}\cdot x^0 = x^{\frac{1}{2}+\frac{1}{3}} = x^{\frac{5}{6}}.$

6. $x^{\frac{3}{4}}\cdot x^{\frac{4}{3}} = x^{\frac{25}{12}}.$

7. $7\,x^0\cdot x^{\frac{2}{3}}\cdot x^{\frac{3}{2}} = 7\,x^{\frac{13}{6}}.$

8. $\sqrt{x}\cdot x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = x^{\frac{5}{6}}.$

9. $\sqrt[3]{x}\cdot x^{\frac{1}{4}} = x^{\frac{1}{3}+\frac{1}{4}} = x^{\frac{7}{12}}.$

10. $(7\,x)^0\cdot 7\,x^0\cdot x^{\frac{1}{2}}\cdot xy = 7\,x^{\frac{3}{2}}y.$

11. $\sqrt[3]{x}\cdot\sqrt[4]{x^3} = x^{\frac{1}{3}+\frac{3}{4}} = x^{\frac{13}{12}}.$

12. $\sqrt{x^{-4}}\cdot\sqrt[3]{x^{-6}} = \dfrac{1}{x^2}\cdot\dfrac{1}{x^2} = \dfrac{1}{x^4}.$

13. $a\sqrt{ax}\cdot a^{\frac{1}{2}}\sqrt{ax^{-1}}$
$= aa^{\frac{1}{2}}x^{\frac{1}{2}}a^{\frac{1}{2}}a^{\frac{1}{2}}x^{-\frac{1}{2}} = a^{\frac{5}{2}}.$

14. $c^2\sqrt[3]{a^3x^{-2}}\cdot c\sqrt[3]{a^3x^{-2}}$
$= c^2ax^{-\frac{2}{3}}\cdot c\cdot ax^{-\frac{2}{3}} = \dfrac{c^3a^2}{x^{\frac{4}{3}}}.$

15. $e^x\cdot e^{-x} = e^0 = 1.$

16. $e^x\cdot e^{2x} = e^{3x}.$

17. $e^{a-3}\cdot e^3 = e^a.$

18. $e^{a-3}\cdot e^{2a+1} = e^{3a-2}.$

19. $e^{3-2a}\cdot e^{2+3a} = e^{5+a}.$

20. $e^0\cdot e^{3a-1}\cdot e^{3-2a} = e^{a+2}.$

21. $x\cdot x^a\cdot x^b\cdot x^0\cdot x^{2b-3a}$
$= x^{3b-2a+1}.$

22. $x^{a-b}\cdot y^a\cdot x^b\cdot y^{2-a} = x^ay^2.$

23. $(2^3)^3 = 2^9 = 512.$

24. $(2^3)^{-2} = 2^{-6} = \frac{1}{64}.$

25. $(2^{-3})^{-2} = 2^6 = 64.$

26. $\left[\left(\dfrac{3}{2}\right)^{-3}\right]^2 = \left(\dfrac{3}{2}\right)^{-6} = \dfrac{1}{\frac{729}{64}} = \dfrac{64}{729}.$

27. $(x^2)^3 = x^6.$

28. $(x^{\frac{1}{2}})^4 = x^2.$

29. $(x^{\frac{1}{3}})^{\frac{1}{2}} = x^{\frac{1}{6}}.$

30. $(x^2)^{-2} = x^{-4} = \dfrac{1}{x^4}.$

31. $(x^{-3})^{-2} = x^6.$

32. $(x^{-\frac{1}{2}})^{-\frac{1}{3}} = x^{\frac{1}{6}}.$

33. $(3\,x^{-2})^3 = 27\,x^{-6} = \dfrac{27}{x^6}.$

34. $(5\,a^2)^{-3} = \dfrac{1}{(5\,a^2)^3} = \dfrac{1}{125\,a^6}$

35. $(c^{-2}d)^2 = c^{-4}d^2 = \dfrac{d^2}{c^4}.$

36. $(5^0\cdot 2^6\cdot 3^3)^{\frac{1}{3}} = 2^2\cdot 3 = 12.$

37. $(25\,a^4b^6)^{-\frac{1}{2}} = 5^{-1}a^{-2}b^{-3} = \dfrac{1}{5\,a^2b^3}.$

38. $[(2\,x^5)^0\cdot 8\cdot 4^{\frac{1}{2}}]^{\frac{1}{2}} = 8^{\frac{1}{2}}\cdot 2^{\frac{1}{2}} = 4.$

39. $(a^2b)^3\,(a+b) = a^6b^3\,(a+b) = a^7b^3 + a^6b^4.$

40. $(a^2)^{3x} \cdot a^{3x} = a^{6x} \cdot a^{3x} = a^{9x}.$

41. $(a^3)^{2x}(a^2)^{3x} = a^{6x} \cdot a^{6x} = a^{12x}.$

42. $(a^{x+1})^2 \cdot (a^{1-x})^2 = a^{2x+2} \cdot a^{2-2x} = a^4.$

43. $(a^3)^{x+y}(a^2)^{y-x} = a^{3x+3y} \cdot a^{2y-2x} = a^{x+5y}.$

44. $(a^2b)^x \cdot (b^2a^3)^{2x} = a^{2x}b^x \cdot b^{4x} \cdot a^{6x} = a^{8x}b^{5x}.$

45. $(x^2 - x^{-2})\,x^3 = x^5 - x.$

46. $(x^2 + xy^{-2})\,x^{-2} = 1 + x^{-1}y^{-2} = 1 + \dfrac{1}{xy^2}.$

47. $(x^4 - a^4)\,x^{-2}a^{-2} = x^2a^{-2} - x^{-2}a^2 = \dfrac{x^2}{a^2} - \dfrac{a^2}{x^2}.$

48. $(x^2 - 5\,ax + 6\,a^2)\,a^{\frac{2}{3}}x^{-\frac{2}{3}} = x^{\frac{4}{3}}a^{\frac{2}{3}} - 5\,a^{\frac{5}{3}}x^{\frac{1}{3}} + 6\,a^{\frac{8}{3}}x^{-\frac{2}{3}}$
$= x^{\frac{4}{3}}a^{\frac{2}{3}} - 5\,a^{\frac{5}{3}}x^{\frac{1}{3}} + 6\,a^{\frac{8}{3}}x^{-\frac{2}{3}}.$

49. $(x^{\frac{1}{3}} + y^{\frac{1}{3}})\,x^{\frac{2}{3}}y^{\frac{1}{3}} = xy^{\frac{1}{3}} + x^{\frac{2}{3}}y^{\frac{2}{3}}.$

50. $(a^{\frac{1}{2}} - x^{\frac{1}{2}})\,(a^{\frac{1}{2}} + x^{\frac{1}{2}}) = a - x.$

51. $(x^{\frac{1}{3}} + y^{\frac{1}{3}})\,(x^{\frac{1}{3}} - y^{\frac{1}{3}}) = x^{\frac{2}{3}} - y^{\frac{2}{3}}.$

52. $(a^{-2} + 3)\,(a^{-2} - 5) = a^{-4} - 2\,a^{-2} - 15 = \dfrac{1}{a^4} - \dfrac{2}{a^2} - 15.$

53. $(a^{-1}a)^2 = 1^2 = 1.$

54. $(a^3 - 2\,a^{-2})^3 = a^9 - 3\,(a^3)^2(2\,a^{-2}) + 3\,(a^3)\,(2\,a^{-2})^2 - (2\,a^{-2})^3$
$= a^9 - 6\,a^4 + 12\,a^{-1} - 8\,a^{-6}$
$= a^9 - 6\,a^4 + \dfrac{12}{a} - \dfrac{8}{a^6}.$

55. $(a^{-1} - 2\,a + 3\,a^{-2})^2 = a^{-2} + 4\,a^2 + 9\,a^{-4} - 4 + 6\,a^{-3} - 12\,a^{-1}$
$= \dfrac{1}{a^2} + 4\,a^2 + \dfrac{9}{a^4} - 4 + \dfrac{6}{a^3} - \dfrac{12}{a}.$

56. $(e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x} = e^{2x} + 2 + \dfrac{1}{e^{2x}}.$

57. $(a^{-1} + b^{-2})\left(\dfrac{1}{a} - \dfrac{1}{b_2}\right) = \left(\dfrac{1}{a} + \dfrac{1}{b_2}\right)\left(\dfrac{1}{a} - \dfrac{1}{b_2}\right) = \dfrac{1}{a^2} - \dfrac{1}{b_4}.$

58. $(e^{2x} - 2 + e^{-2x})^2 = e^{4x} + 4 + e^{-4x} - 4\,e^{2x} + 2 - 4\,e^{-2x}$
$= e^{4x} + 6 + \dfrac{1}{e^{4x}} - 4\,e^{2x} - \dfrac{4}{e^{2x}}.$

59. $(3\,a^{-\frac{1}{2}} + 2\,a^{\frac{2}{3}})^2 = 9\,a^{-1} + 12\,a^{\frac{1}{6}} + 4\,a^{\frac{4}{3}}$
$= \dfrac{9}{a} + 12\,a^{\frac{1}{6}} + 4\,a^{\frac{4}{3}}.$

60. $(a^{\frac{4}{5}} - 2\,a^{\frac{2}{5}}x + 4\,x^2)\,(a^{\frac{2}{5}} + 2\,x) = a^{\frac{6}{5}} + 8\,x^3.$

61. $(x^{\frac{1}{3}} + 2\,y^{\frac{1}{3}})\,(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}}y^{\frac{1}{3}} + 4\,y^{\frac{2}{3}}) = x + 8\,y.$

62. $(a - a^{\frac{1}{2}}b^{\frac{1}{2}} + b)(a + a^{\frac{1}{2}}b^{\frac{1}{2}} + b) = a^2 + ab + b^2.$

63. $(a^{\frac{1}{2}}+a^{\frac{1}{4}}b^{\frac{1}{4}}+b^{\frac{1}{2}})(a^{\frac{1}{2}}-a^{\frac{1}{4}}b^{\frac{1}{4}}+b^{\frac{1}{2}})=a+a^{\frac{1}{2}}b^{\frac{1}{2}}+b.$

64. $(x-x^{\frac{1}{2}}y^{-\frac{1}{2}}+y^{-1})(x+x^{\frac{1}{2}}y^{-\frac{1}{2}}+y^{-1})=x^2+xy^{-1}+y^{-2}$

$$=x^2+\frac{x}{y}+\frac{1}{y^2}.$$

65. $(\sqrt[3]{a^5}-3\sqrt{a})(\sqrt[3]{a^5}-3\sqrt{a})=\sqrt[3]{a^{10}}-6\sqrt[3]{a^5}\cdot\sqrt{a}+9a$

$$=a^{\frac{10}{3}}-6a^{\frac{13}{6}}+9a.$$

66. $(5\sqrt{c^{-5}}\sqrt[3]{d^{-3}}-a\sqrt[5]{c^{-2}})^3=(5c^{-\frac{5}{2}}d^{-1}-ac^{-\frac{2}{5}})^3$

$$=\frac{125}{c^{\frac{15}{2}}d^3}-\frac{75a}{c^{\frac{27}{5}}d^2}+\frac{15a^2}{c^{\frac{33}{10}}d}-\frac{a^3}{c^{\frac{6}{5}}}.$$

67. $\left(\sqrt{a}+\frac{\sqrt{b}}{c}\right)\left(\sqrt{a}+\frac{\sqrt{b}}{c}\right)^2=\left(a^{\frac{1}{2}}+\frac{b^{\frac{1}{2}}}{c}\right)^3=a^{\frac{3}{2}}+\frac{3ab^{\frac{1}{2}}}{c}+\frac{3a^{\frac{1}{2}}b}{c^2}+\frac{b^{\frac{3}{2}}}{c^3}.$

68.

$$\begin{array}{lllll}
\multicolumn{5}{l}{m^{n-1}-5m^{n-2}a^n+25a^{2n}}\\
\multicolumn{5}{l}{m+5a^n}\\ \hline
m^n & -5m^{n-1}a^n & +25ma^{2n} & & \\
 & 5m^{n-1}a^n & & -25m^{n-2}a^{2n} & +125a^{3n}\\ \hline
m^n & & +25ma^{2n} & -25m^{n-2}a^{2n} & +125a^{3n}
\end{array}$$

69.

$$\begin{array}{lllllll}
\multicolumn{7}{l}{r^{\frac{5}{2}}-r^2+r^{\frac{3}{2}}-r+r^{\frac{1}{2}}-1}\\
\multicolumn{7}{l}{r^{\frac{1}{2}}+1}\\ \hline
r^3 & -r^{\frac{5}{2}} & +r^2 & -r^{\frac{3}{2}} & +r & -r^{\frac{1}{2}} & \\
 & r^{\frac{5}{2}} & -r^2 & +r^{\frac{3}{2}} & -r & +r^{\frac{1}{2}} & -1\\ \hline
r^3 & & & & & & -1
\end{array}$$

70.

$$\begin{array}{llllll}
\multicolumn{6}{l}{16a^{\frac{4}{5}}+8a^{\frac{3}{5}}b^{\frac{1}{2}}+4a^{\frac{2}{5}}b+2a^{\frac{1}{5}}b^{\frac{3}{2}}+b^2}\\
\multicolumn{6}{l}{2a^{\frac{1}{5}}-b^{\frac{1}{2}}}\\ \hline
32a & +16a^{\frac{4}{5}}b^{\frac{1}{2}} & +8a^{\frac{3}{5}}b & +4a^{\frac{2}{5}}b^{\frac{3}{2}} & +2a^{\frac{1}{5}}b^2 & \\
 & -16a^{\frac{4}{5}}b^{\frac{1}{2}} & -8a^{\frac{3}{5}}b & -4a^{\frac{2}{5}}b^{\frac{3}{2}} & -2a^{\frac{1}{5}}b^2 & -b^{\frac{5}{2}}\\ \hline
32a & & & & & -b^{\frac{5}{2}}
\end{array}$$

71.

$$\begin{array}{llll}
\multicolumn{4}{l}{25x^{-4}+15x^{-2}y^{-8}+9y^{-16}}\\
\multicolumn{4}{l}{5x^{-2}-3y^{-8}}\\ \hline
125x^{-6} & +75x^{-4}y^{-8} & +45x^{-2}y^{-16} & \\
 & -75x^{-4}y^{-8} & -45x^{-2}y^{-16} & -27y^{-24}\\ \hline
125x^{-6} & & & -27y^{-24}=\dfrac{125}{x^6}-\dfrac{27}{y^{24}}.
\end{array}$$

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1. $x^4 \div x^6 = x^{-2} = \dfrac{1}{x^2}.$

2. $x^3 \div x^{\frac{1}{3}} = x^{\frac{8}{3}}.$

3. $x^{\frac{1}{2}} \div x^2 = x^{-\frac{3}{2}} = \dfrac{1}{x^{\frac{3}{2}}}.$

4. $\dfrac{ax^{\frac{2}{3}}}{a^{\frac{1}{2}}x^{\frac{1}{2}}} = a^{\frac{1}{2}}x^{\frac{1}{6}}.$

5. $\dfrac{ax - a^2x^3}{a^2x^{\frac{1}{2}}} = \dfrac{x^{\frac{1}{2}}}{a} - x^{\frac{5}{2}}.$

6. $(x^a - 2x^{2a-1} + 3x^{3a-2}) \div x^{2a-1} = x^{1-a} - 2 + 3x^{a-1}.$

7. $(6a^{3+4n} - 9a^{n-2} + 5a^{2-n}) \div 3a^{n-2} = 2a^{5+3n} - 3 + \frac{5}{3}a^{4-2n}.$

8. $(x - y) \div (x^{\frac{1}{2}} - y^{\frac{1}{2}}) = x^{\frac{1}{2}} + y^{\frac{1}{2}}.$

9. $(x + y) \div (x^{\frac{1}{3}} + y^{\frac{1}{3}}) = x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}}.$

10.
$$\begin{array}{l|l}
x - 8y & x^{\frac{1}{3}} - 2y^{\frac{1}{3}} \\ \cline{2-2}
\underline{x - 2x^{\frac{2}{3}}y^{\frac{1}{3}}} & \underline{x^{\frac{2}{3}} + 2x^{\frac{1}{3}}y^{\frac{1}{3}} + 4y^{\frac{2}{3}}} \\
\quad 2x^{\frac{2}{3}}y^{\frac{1}{3}} & \\
\quad \underline{2x^{\frac{2}{3}}y^{\frac{1}{3}} - 4x^{\frac{1}{3}}y^{\frac{2}{3}}} & \\
\qquad\qquad 4x^{\frac{1}{3}}y^{\frac{2}{3}} - 8y & \\
\qquad\qquad \underline{4x^{\frac{1}{3}}y^{\frac{2}{3}} - 8y} &
\end{array}$$

11.
$$\begin{array}{l|l}
16x^2 - 81y^2 & 2x^{\frac{1}{2}} + 3y^{\frac{1}{2}} \\ \cline{2-2}
\underline{16x^2 + 24x^{\frac{3}{2}}y^{\frac{1}{2}}} & \underline{8x^{\frac{3}{2}} - 12xy^{\frac{1}{2}} + 18x^{\frac{1}{2}}y - 27y^{\frac{3}{2}}} \\
\quad -24x^{\frac{3}{2}}y^{\frac{1}{2}} & \\
\quad \underline{-24x^{\frac{3}{2}}y^{\frac{1}{2}} - 36xy} & \\
\qquad\qquad 36xy & \\
\qquad\qquad \underline{36xy + 54x^{\frac{1}{2}}y^{\frac{3}{2}}} & \\
\qquad\qquad\qquad -54x^{\frac{1}{2}}y^{\frac{3}{2}} - 81y^2 & \\
\qquad\qquad\qquad \underline{-54x^{\frac{1}{2}}y^{\frac{3}{2}} - 81y^2} &
\end{array}$$

12.
$$\begin{array}{l|l}
a^3 - b^2 & a^{\frac{1}{2}} + b^{\frac{1}{3}} \\ \cline{2-2}
\underline{a^3 + a^{\frac{5}{2}}b^{\frac{1}{3}}} & \underline{a^{\frac{5}{2}} - a^2b^{\frac{1}{3}} + a^{\frac{3}{2}}b^{\frac{2}{3}} - ab + a^{\frac{1}{2}}b^{\frac{4}{3}} - b^{\frac{5}{3}}} \\
\quad -a^{\frac{5}{2}}b^{\frac{1}{3}} & \\
\quad \underline{-a^{\frac{5}{2}}b^{\frac{1}{3}} - a^2b^{\frac{2}{3}}} & \\
\qquad\qquad a^2b^{\frac{2}{3}} & \\
\qquad\qquad \underline{a^2b^{\frac{2}{3}} + a^{\frac{3}{2}}b} & \\
\qquad\qquad\quad -a^{\frac{3}{2}}b & \\
\qquad\qquad\quad \underline{-a^{\frac{3}{2}}b - ab^{\frac{4}{3}}} &
\end{array}
\qquad
\begin{array}{l}
ab^{\frac{4}{3}} - b^2 \\
\underline{ab^{\frac{4}{3}} + a^{\frac{1}{2}}b^{\frac{5}{3}}\quad} \\
\quad -a^{\frac{1}{2}}b^{\frac{5}{3}} - b^2 \\
\quad \underline{-a^{\frac{1}{2}}b^{\frac{5}{3}} - b^2}
\end{array}$$

13.
$$\begin{array}{l|l}
a^2 + ab^{-1} + b^{-2} & a - a^{\frac{1}{2}}b^{-\frac{1}{2}} + b^{-1} \\ \cline{2-2}
\underline{a^2 - a^{\frac{3}{2}}b^{-\frac{1}{2}}} + ab^{-1} & a + a^{\frac{1}{2}}b^{-\frac{1}{2}} + b^{-1} \\ \cline{2-2}
\quad a^{\frac{3}{2}}b^{-\frac{1}{2}} & \\
\quad \underline{a^{\frac{3}{2}}b^{-\frac{1}{2}} - ab^{-1} + a^{\frac{1}{2}}b^{-\frac{3}{2}}} & \\
\qquad\qquad ab^{-1} - a^{\frac{1}{2}}b^{-\frac{3}{2}} + b^{-2} & \\
\qquad\qquad \underline{ab^{-1} - a^{\frac{1}{2}}b^{-\frac{3}{2}} + b^{-2}} &
\end{array}$$

14.
$$\begin{array}{l|l}
e^{2x} - 2 + e^{-2x} & e^x - e^{-x} \\ \cline{2-2}
\underline{e^{2x} - 1} & e^x - e^{-x} \\ \cline{2-2}
\quad -1 + e^{-2x} & \\
\quad \underline{-1 + e^{-2x}} &
\end{array}$$

15.
$$\begin{array}{l|l}
e^{5x} + 3e^x + \frac{3}{e^x} + \frac{1}{e^{3x}} & e^x + e^{-x} \\ \cline{2-2}
\underline{e^{3x} + \quad e^x} & e^{2x} + 2 + \frac{1}{e^{2x}} \\ \cline{2-2}
\qquad 2e^x + \frac{3}{e^x} & \\
\qquad \underline{2e^x + \frac{2}{e^x}} & \\
\qquad\qquad \frac{1}{e^x} + \frac{1}{e^{3x}} & \\
\qquad\qquad \underline{\frac{1}{e^x} + \frac{1}{e^{3x}}} &
\end{array}$$

16.
$$\begin{array}{l|l}
e^{4x} - 4e^{2x} + 6 - 4e^{-2x} + e^{-4x} & e^x - e^{-x} \\ \cline{2-2}
\underline{e^{4x} - \quad e^{2x}} & e^{3x} - 3e^x + 3e^{-x} - e^{-3x} \\ \cline{2-2}
\quad -3e^{2x} + 6 & \\
\quad \underline{-3e^{2x} + 3} & \\
\qquad\qquad 3 - 4e^{-2x} & \\
\qquad\qquad \underline{3 - 3e^{-2x}} & \\
\qquad\qquad\quad - \quad e^{-2x} + e^{-4x} & \\
\qquad\qquad\quad \underline{- \quad e^{-2x} + e^{-4x}} &
\end{array}$$

17.
$$\begin{array}{l|l}
a + a^{\frac{3}{5}}b^{\frac{2}{3}} + 2a^{\frac{2}{5}}b^{\frac{1}{3}} + 2b & a^{\frac{3}{5}} + 2b^{\frac{1}{3}} \\ \cline{2-2}
\underline{a \qquad\qquad + 2a^{\frac{2}{5}}b^{\frac{1}{3}}} & a^{\frac{2}{5}} + b^{\frac{2}{3}} \\ \cline{2-2}
\quad a^{\frac{3}{5}}b^{\frac{2}{3}} \qquad\qquad + 2b & \\
\quad \underline{a^{\frac{3}{5}}b^{\frac{2}{3}} \qquad\qquad + 2b} &
\end{array}$$

18. $$\begin{array}{l|l} m^4 + 7m^2 + 8 - 7m^{-2} + m^{-4} & m^2 + 5 - m^{-2} \\ \cline{2-2} \underline{m^4 + 5m^2 - 1} & \underline{m^2 + 2 - m^{-2}} \\ \quad 2m^2 + 9 - 7m^{-2} & \\ \quad \underline{2m^2 + 10 - 2m^{-2}} & \\ \qquad -1 - 5m^{-2} + m^{-4} & \\ \qquad \underline{-1 - 5m^{-2} + m^{-4}} & \end{array}$$

19. $$\begin{array}{l|l} 2x^{2n-1} - x^{3n-2} + 9x^{5n-4} & 2x^{n-1} + 3x^{2n-2} \\ \cline{2-2} \underline{2x^{2n-1} + 3x^{3n-2}} & \underline{x^n - 2x^{2n-1} + 3x^{3n-2}} \\ \quad -4x^{3n-2} + 9x^{5n-4} & \\ \quad \underline{-4x^{3n-2} - 6x^{4n-3}} & \\ \qquad 6x^{4n-3} + 9x^{5n-4} & \\ \qquad \underline{6x^{4n-3} + 9x^{5n-4}} & \end{array}$$

20. $$\begin{array}{l|l} x^{-\frac{5}{2}}y - 2x^{-\frac{3}{2}}y^{\frac{1}{2}} - 8y^{\frac{1}{2}} + 16x & x^{-\frac{1}{2}}y^{-\frac{1}{2}} - 8x^2y^{-1} \\ \cline{2-2} \underline{x^{-\frac{5}{2}}y \qquad\qquad - 8y^{\frac{1}{2}}} & \underline{x^{-2}y^{\frac{3}{2}} - 2x^{-1}y} \\ \quad -2x^{-\frac{3}{2}}y^{\frac{1}{2}} \qquad + 16x & \\ \quad \underline{-2x^{-\frac{3}{2}}y^{\frac{1}{2}} \qquad + 16x} & \end{array}$$

21. $$\begin{array}{l|l} -16a^{\frac{19}{5}}b^{\frac{3}{4}} + 40ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}} & -4a^{\frac{12}{5}}b^{-\frac{5}{8}} + 5a^{-\frac{2}{5}}b^{-\frac{3}{8}} \\ \cline{2-2} \underline{-16a^{\frac{19}{5}}b^{\frac{3}{4}} + 20ab} & \underline{4a^{\frac{7}{5}}b^{\frac{11}{8}} - 5a^{-\frac{7}{5}}b^{\frac{13}{8}}} \\ \qquad 20ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}} & \\ \qquad \underline{20ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}}} & \end{array}$$

22. $$\begin{array}{l|l} x^{3a} + 11x^{2a} + 38x^a + 33 - 28x^{-a} + 2x^{-2a} & x^a + 4 - 2x^{-a} \\ \cline{2-2} \underline{x^{3a} + 4x^{2a} - 2x^a} & \underline{x^{2a} + 7x^a + 12 - x^{-a}} \\ \quad 7x^{2a} + 40x^a + 33 & \\ \quad \underline{7x^{2a} + 28x^a - 14} & \\ \qquad 12x^a + 47 - 28x^{-a} & \\ \qquad \underline{12x^a + 48 - 24x^{-a}} & \\ \qquad\quad -1 - 4x^{-a} + 2x^{-2a} & \\ \qquad\quad \underline{-1 - 4x^{-a} + 2x^{-2a}} & \end{array}$$

23. $$\begin{array}{l|l} 25m^3 - 4mn^{\frac{4}{3}} + 4mn^2 - mn^{\frac{8}{3}} & 5m^{\frac{3}{2}} - 2n^{\frac{2}{3}}m^{\frac{1}{2}} + m^{\frac{1}{2}}n^{\frac{4}{3}} \\ \cline{2-2} \underline{25m^3 - 10m^2n^{\frac{2}{3}} + 5m^2n^{\frac{4}{3}}} & \underline{5m^{\frac{3}{2}} + 2m^{\frac{1}{2}}n^{\frac{2}{3}} - m^{\frac{1}{2}}n^{\frac{4}{3}}} \\ \quad +10m^2n^{\frac{2}{3}} - 5m^2n^{\frac{4}{3}} - 4mn^{\frac{4}{3}} + 4mn^2 & \\ \quad \underline{+10m^2n^{\frac{2}{3}} \qquad\qquad - 4mn^{\frac{4}{3}} + 2mn^2} & \\ \qquad -5m^2n^{\frac{4}{3}} \qquad\qquad + 2mn^2 - mn^{\frac{8}{3}} & \\ \qquad \underline{-5m^2n^{\frac{4}{3}}} \qquad\qquad \underline{+ 2mn^2 - mn^{\frac{8}{3}}} & \end{array}$$

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1. $\sqrt{x-6}=5.$

Squaring, $x-6=25.$

$$x=31.$$

2.
$$\begin{aligned}\sqrt{x+5}+3&=6.\\ \sqrt{x+5}&=3.\\ x+5&=9.\\ x&=4.\end{aligned}$$

3.
$$\begin{aligned}4\sqrt{x+9}+5&=17.\\ 4\sqrt{x+9}&=12.\\ \sqrt{x+9}&=3.\\ x+9&=9.\\ x&=0.\end{aligned}$$

4.
$$\begin{aligned}\sqrt[3]{2x+2}&=2.\\ 2x+2&=8.\\ 2x&=6.\\ x&=3.\end{aligned}$$

5.
$$\begin{aligned}\sqrt[3]{24x-19}+4&=1.\\ \sqrt[3]{24x-19}&=-3.\\ 24x-19&=-27.\\ 24x&=-8.\\ x&=-\tfrac{1}{3}.\end{aligned}$$

6.
$$\begin{aligned}2\sqrt[3]{26x-5}+13&=3.\\ 2\sqrt[3]{26x-5}&=-10.\\ \sqrt[3]{26x-5}&=-5.\\ 26x-5&=-125.\\ 26x&=-120.\\ x&=-\tfrac{60}{13}.\end{aligned}$$

7.
$$\begin{aligned}5x\sqrt{2x}&=15\sqrt{8x}.\\ x\sqrt{2x}&=3\sqrt{8x}.\\ 2x^3&=72x.\\ x^3-36x&=0.\\ x(x^2-36)&=0.\\ x&=0,\ 6,\text{ or }-6.\end{aligned}$$

8.
$$\begin{aligned}\sqrt{4x}&=\sqrt{x}+3.\\ 2\sqrt{x}&=\sqrt{x}+3.\\ \sqrt{x}&=3.\\ x&=9.\end{aligned}$$

9.
$$\begin{aligned}\sqrt{3x^2-2}+x&=8.\\ \sqrt{3x^2-2}&=8-x.\\ 3x^2-2&=64-16x+x^2.\\ (x+11)(x-3)&=0.\\ x&=3\text{ or }-11.\end{aligned}$$

10.
$$\begin{aligned}\sqrt[3]{2x+7}-\sqrt[3]{10+3x}&=0.\\ \sqrt[3]{2x+7}&=\sqrt[3]{10+3x}.\\ 2x+7&=10+3x.\\ x&=-3.\end{aligned}$$

11.
$$\begin{aligned}\sqrt[3]{x^2-x+2}+3&=5.\\ \sqrt[3]{x^2-x+2}&=2.\\ x^2-x+2&=8.\\ (x-3)(x+2)&=0.\\ x&=3\text{ or }-2.\end{aligned}$$

12.
$$\begin{aligned}\sqrt[3]{85-5x}+\sqrt[3]{10x-45}&=0.\\ \sqrt[3]{85-5x}&=-\sqrt[3]{10x-45}.\\ 85-5x&=-10x+45.\\ 5x&=-40,\text{ or}\\ x&=-8.\end{aligned}$$

13.
$$\begin{aligned}\sqrt{2x+3}&=\sqrt[4]{15-x^2+2x-6}.\\ 4x^2+12x+9&=15-x^2+2x-6.\\ x&=-2\text{ or }0.\end{aligned}$$

The value -2 for x is rejected.

14.
$$\begin{aligned}\sqrt{2+x}+\sqrt{10-3x}&=4.\\ \sqrt{10-3x}&=4-\sqrt{2+x}.\end{aligned}$$

$$10 - 3x = 16 - 8\sqrt{2 + x} + 2 + x.$$

$$-8 - 4x = -8\sqrt{2 + x}.$$

Dividing,
$$x + 2 = 2\sqrt{2 + x}.$$

$$x^2 + 4x + 4 = 8 + 4x.$$

$$x = 2 \text{ or } -2.$$

15.
$$\sqrt{x - 2} + \sqrt{4x + 1} = 3.$$

$$\sqrt{4x + 1} = 3 - \sqrt{x - 2}.$$

$$4x + 1 = 9 - 6\sqrt{x - 2} + x - 2.$$

$$3x - 6 = -6\sqrt{x - 2}.$$

Dividing,
$$x - 2 = -2\sqrt{x - 2}.$$

$$x^2 - 4x + 4 = 4x - 8.$$

$$(x - 6)(x - 2) = 0.$$

$$x = 6 \text{ or } 2.$$

The value 6 for x is rejected.

16.
$$\sqrt{2x + 7} = \sqrt{x} + 2.$$

$$2x + 7 = x + 4\sqrt{x} + 4.$$

$$x + 3 = 4\sqrt{x}.$$

$$x^2 + 6x + 9 = 16x.$$

$$x = 9 \text{ or } 1.$$

17.
$$\sqrt{x + 2} = \sqrt{x} + 2.$$

$$x + 2 = x + 4\sqrt{x} + 4.$$

$$-2 = 4\sqrt{x}.$$

$$4 = 16x.$$

$$x = \tfrac{1}{4}.$$

The value $\frac{1}{4}$ for x does not satisfy.

19.
$$\frac{\sqrt{5x + 2}}{\sqrt{2x - 4}} = \sqrt{\frac{7x - 2}{3x - 8}}.$$

$$\frac{5x + 2}{2x - 4} = \frac{7x - 2}{3x - 8}.$$

$$x = 6 \text{ or } -4.$$

20.
$$\frac{(x + 4)^{\frac{1}{2}}}{3} = \frac{3}{(x + 4)^{\frac{1}{2}}}.$$

Clearing, $x + 4 = 9.$

$$x = 5.$$

18.
$$\frac{\sqrt{x + 9}}{\sqrt{x + 2}} = \frac{4}{3}. \qquad \frac{x + 9}{x + 2} = \frac{16}{9}.$$

$$x = 7.$$

21.
$$\frac{2\sqrt{a}}{\sqrt{x - a}} - \frac{\sqrt{x + 4a}}{3\sqrt{a}} = 0.$$

$$\frac{2\sqrt{a}}{\sqrt{x - a}} = \frac{\sqrt{x + 4a}}{3\sqrt{a}}.$$

$$\frac{4a}{x - a} = \frac{x + 4a}{9a}.$$

$$(x + 8a)(x - 5a) = 0.$$

$$x = 5a \text{ or } -8a.$$

The value $-8a$ for x is rejected.

22.
$$\frac{9x+8}{(8x+1)^{\frac{1}{2}}} = (10x+19)^{\frac{1}{2}}.$$

Clearing,
$$9x+8 = (80x^2+162x+19)^{\frac{1}{2}}.$$
$$81x^2+144x+64 = 80x^2+162x+19.$$
$$x^2-18x+45 = 0.$$
$$x = 3 \text{ or } 15.$$

23.
$$\frac{5}{\sqrt{2x}+1} = \frac{1}{9-\sqrt{8x}}.$$

Clearing,
$$45-10\sqrt{2x} = \sqrt{2x}+1.$$
$$4 = \sqrt{2x}.$$
$$2x = 16,$$
or
$$x = 8.$$

24.
$$\left(\frac{1}{2}+x\right)^{\frac{1}{2}} = \frac{1+2x}{(\frac{1}{2}+x)^{\frac{1}{2}}}.$$

Clearing, $\frac{1}{2}+x = 1+2x.$
$$x = -\tfrac{1}{2}.$$

The value $-\frac{1}{2}$ for x does not satisfy, for the check involves division by zero, which is excluded.

25.
$$(8x)^{\frac{1}{2}} - (8x-15)^{\frac{1}{2}} = \frac{7}{(8x-15)^{\frac{1}{2}}}.$$

Clearing,
$$(64x^2-120x)^{\frac{1}{2}} - (8x-15) = 7.$$
$$2(16x^2-30x)^{\frac{1}{2}} - 8x+15 = 7.$$
$$(16x^2-30x)^{\frac{1}{2}} = 4x-4.$$
$$16x^2-30x = 16x^2-32x+16.$$
$$2x = 16,$$
or
$$x = 8.$$

26.
$$(x-1)^{\frac{1}{2}} + (3x+1)^{\frac{1}{2}} = 6.$$
$$(3x+1)^{\frac{1}{2}} = 6-(x-1)^{\frac{1}{2}}.$$
$$3x+1 = 36-12\sqrt{x-1}+x-1.$$
$$2x-34 = -12\sqrt{x-1}.$$
$$x-17 = -6\sqrt{x-1}.$$
$$x^2-34x+289 = 36x-36.$$
$$x^2-70x+325 = 0.$$
$$x = 5 \text{ or } 65.$$

The value 65 for x is rejected, as it does not satisfy the equation.

27.
$$\sqrt{x+3}+\sqrt{x-4} = \sqrt{4x-3}.$$

Squaring,
$$x+3+2\sqrt{x^2-x-12}+x-4 = 4x-3.$$
$$2\sqrt{x^2-x-12} = 2x-2.$$
$$x^2-x-12 = x^2-2x+1.$$
$$x = 13.$$

28. $\sqrt{2x-1}+\sqrt{x+3}-\sqrt{3x+2}=0.$

$$\sqrt{2x-1}+\sqrt{x+3}=\sqrt{3x+2}.$$
$$2x-1+2\sqrt{2x^2+5x-3}+x+3=3x+2.$$
$$2\sqrt{2x^2+5x-3}=0.$$
$$2x^2+5x-3=0.$$
$$x=\tfrac{1}{2}\text{ or }-3.$$

29. $\sqrt{2x-1}+\sqrt{x+3}+\sqrt{3x+2}=0.$

$$\sqrt{3x+2}=-\sqrt{2x-1}-\sqrt{x+3}.$$
$$3x+2=2x-1+2\sqrt{2x^2+5x-3}+x+3.$$
$$0=2\sqrt{2x^2+5x-3}.$$
$$2x^2+5x-3=0.$$
$$x=\tfrac{1}{2}\text{ or }-3.$$

Neither value satisfies.

30.
$$\sqrt{3x+9-\sqrt{8x^2+x+6}}-3=0.$$
$$\sqrt{3x+9-\sqrt{8x^2+x+6}}=3.$$
$$3x+9-\sqrt{8x^2+x+6}=9.$$
$$\sqrt{8x^2+x+6}=3x.$$
$$8x^2+x+6=9x^2.$$
$$x=3\text{ or }-2.$$

The value -2 for x does not satisfy.

31.
$$\sqrt{9-\sqrt{30-\sqrt{20+\sqrt{10x-25}}}}-2=0.$$
$$\sqrt{9-\sqrt{30-\sqrt{20+\sqrt{10x-25}}}}=2.$$
$$9-\sqrt{30-\sqrt{20+\sqrt{10x-25}}}=4.$$
$$-\sqrt{30-\sqrt{20+\sqrt{10x-25}}}=-5.$$
$$30-\sqrt{20+\sqrt{10x-25}}=25.$$
$$-\sqrt{20+\sqrt{10x-25}}=-5.$$
$$20+\sqrt{10x-25}=25.$$
$$\sqrt{10x-25}=5.$$
$$10x-25=25.$$
$$x=5.$$

32. $x+2\sqrt{x}-8=0.$

$$x-8=-2\sqrt{x}.$$
$$x^2-16x+64=4x.$$
$$x=4\text{ or }16.$$

The value 16 for x is rejected.

33. $2x-7x^{\frac{1}{2}}+3=0.$

$$2x+3=7x^{\frac{1}{2}}.$$
$$4x^2+12x+9=49x.$$
$$x=9\text{ or}$$

34.
$$E = \frac{mv^2}{2}.$$
$$v^2 = \frac{2E}{m}.$$
$$v = \pm \frac{1}{m}\sqrt{2Em}.$$

35.
$$t = \pi\sqrt{\frac{l}{g}}.$$
$$t^2 = \frac{\pi^2 l}{g}.$$
$$g = \frac{\pi^2 l}{t^2},$$
and
$$l = \frac{gt^2}{\pi^2}.$$

36.
$$A = \pi R^2.$$
$$R = \pm\sqrt{\frac{A}{\pi}}$$
$$= \pm\frac{1}{\pi}\sqrt{A\pi}.$$

37.
$$V = \frac{\pi R^2 H}{3}.$$
$$R = \pm\sqrt{\frac{3V}{\pi H}}$$
$$= \pm\frac{1}{\pi H}\sqrt{3\pi HV}.$$

38.
$$\frac{R^2}{4} + S^2 = R^2.$$
$$R^2 = \frac{4S^2}{3}.$$
$$R = \pm\frac{2S}{3}\sqrt{3}.$$

39.
$$\sqrt{R^2 - \left(\frac{R\sqrt{3}}{2}\right)^2} = A.$$
$$R^2 - \frac{3R^2}{4} = A^2.$$
$$R = \pm 2A.$$

40.
$$\left(\frac{R}{2}\sqrt{2-\sqrt{2}}\right)^2 = R^2 - a^2.$$
$$R^2(2-\sqrt{2}) = 4R^2 - 4a^2.$$
$$R = \pm a\sqrt{4-2\sqrt{2}},.$$
and
$$a = \pm\frac{R\sqrt{2+\sqrt{2}}}{2}.$$

41.
$$v = \sqrt{2gR}.$$
$$v = \sqrt{2\left(\frac{32}{5280}\right)4000}$$
= 7 miles, approximately.

42.
$$.976\,v^2x - gx^2 = 0.$$
Substituting,
$$9760\,x - 32\,x^2 = 0.$$
$$x = 0 \text{ or } 305.$$
Therefore the distance was 305 feet.

43.
$$.976\,v^2x - gx^2 = 0.$$
Substituting,
$$390.4\,v^2 - 32 \cdot 160{,}000 = 0.$$
$$v = \frac{400}{3.49}$$
$$= 114.5.$$
Therefore the velocity was 114.5 feet per second.

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2.
$$\frac{x_1}{x_2} = \frac{y_1}{y_2}.$$
Substituting,
$$\frac{6}{10} = \frac{15}{y_2}.$$
$$6\,y_2 = 150.$$
$$y_2 = 25.$$

3.
$$\frac{x_1}{x_2} = \frac{y_1}{y_2}.$$
Substituting,
$$\frac{h}{k} = \frac{m}{y_2}.$$
$$hy_2 = km.$$
$$y_2 = \frac{km}{h}.$$

5. $$x_1 : x_2 = \frac{1}{y_1} : \frac{1}{y_2}.$$

Substituting, $$4 : x_2 = \tfrac{1}{100} : \tfrac{1}{10}.$$

$$\frac{4}{10} = \frac{x_2}{100}.$$

$$x_2 = 40.$$

6. $$y_1 : y_2 = \frac{1}{z_1} : \frac{1}{z_2}.$$

Substituting, $$h : y_2 = \frac{1}{k} : \frac{1}{m}.$$

$$\frac{h}{m} = \frac{y_2}{k}.$$

$$y_2 = \frac{hk}{m}.$$

8. $$\frac{x_1}{x_2} = \frac{y_1 z_1}{y_2 z_2}.$$

Substituting, $$\frac{3}{x_2} = \frac{4 \cdot 5}{20 \cdot 2}.$$

$$\frac{3}{x_2} = \frac{1}{2}.$$

$$x_2 = 6.$$

9. $$x_1 : x_2 = \frac{y_1}{z_1} : \frac{y_2}{z_2}.$$

Substituting, $$\frac{x_1}{10} = \frac{2}{6} : \frac{4}{9}.$$

$$\frac{x_1}{10} = \frac{3}{4}.$$

$$x_1 = 7\tfrac{1}{2}.$$

10. $$\frac{d_1}{d_2} = \frac{t_1^2}{t_2^2}.$$

Substituting, $$\frac{64}{d_2} = \frac{4}{16}.$$

$$64 = \frac{d_2}{4}.$$

$$d_2 = 256.$$

11. $$\frac{V_1}{V_2} = \frac{T_1}{P_1} : \frac{T_2}{P_2}.$$

Substituting, $$\frac{80}{45} = \frac{400}{15} : \frac{450}{P_2}.$$

$$\frac{80 \cdot 450}{P_2} = \frac{45 \cdot 400}{15}.$$

$$P_2 = 30.$$

12. (*a*) $$\frac{W_1}{W_2} = \frac{D_1}{D_2}.$$

$$\frac{100}{W_2} = \frac{4\cancel{000}}{3\cancel{000}}.$$

$$W_2 = 75.$$

Therefore the weight is 75 pounds.

(*b*) $$\frac{100}{W_2} = \frac{4\cancel{000}}{2\cancel{000}}.$$

$$W_2 = 50.$$

Therefore the weight is 50 pounds.

(*c*) $$\frac{W_2}{100} = \frac{0}{4000}.$$

Therefore $W_2 = 0$.

13. Let R_1 and R_2 represent the number of revolutions and D_1 and D_2 the diameters.

$$R_1 : R_2 = \frac{1}{D_1} : \frac{1}{D_2}.$$

$$360 : R_2 = \tfrac{3}{14} : \tfrac{1}{5}.$$

$$72 = \frac{3\,R_2}{14}.$$

Whence $R_2 = 336$.

14. Let t_1 and t_2 represent the times and D_1 and D_2 the distances.

$$\frac{t_1}{t_2} = \frac{D_1}{D_2}.$$

$$\frac{4\frac{2}{5}}{2} = \frac{5280}{D_2}.$$

$$\tfrac{22}{5}\,D_2 = 10{,}560,$$

or $$D_2 = 2400 \text{ feet}.$$

15. $$\tfrac{24}{20} = \tfrac{36}{30}.$$

$$V \propto \frac{1}{P},$$

or $$\frac{V_1}{V_2} = \frac{1}{P_1} : \frac{1}{P}$$

$$\frac{24}{V_2}=\frac{48}{30},$$

or $$\frac{1}{V_2}=\frac{2}{30}.$$

Solving, $V_2 = 15$ cubic inches.

16. Let D_1 and D_2 represent the distances, and t_1 and t_2 the times.

Then $D_1 : D_2 = t_1^2 : t_2^2$.

$16 : D_2 = 1 : 36$.

$36 \cdot 16 = D_2$, or $D_2 = 576$.

17. Let I_1 and I_2 represent the intensities, and D_1 and D_2 the distances.

$$\frac{I_1}{I_2}=\frac{1}{D_1^2}:\frac{1}{D_2^2}.$$

$$\frac{I_1}{I_2}=\frac{1}{16}:\frac{1}{36}.$$

$$\frac{I_1}{36}=\frac{I_2}{16}.\quad I_1=\tfrac{9}{4}I_2.$$

The first intensity is $\frac{9}{4}$ of the second.

18. Let I_1 and I_2 represent the intensities, and D_1 and D_2 the distances.

$$\frac{I_1}{I_2}=\frac{1}{D_1^2}:\frac{1}{D_2^2}.$$

$$\frac{1}{4}=\frac{1}{81}:\frac{1}{D_2^2}.$$

$$\frac{1}{4}=\frac{D_2^2}{81}.$$

$$D_2^2=\tfrac{81}{4}.$$

Whence $D_2 = 4\frac{1}{2}$.

Therefore it must be placed $4\frac{1}{2}$ feet from the light.

$$\frac{I}{2I}=\frac{1}{81}:\frac{1}{D^2}.$$

Whence $D = \frac{9}{2}\sqrt{2}$.

Therefore it must be placed $\frac{9}{2}\sqrt{2}$ feet from the light.

19. Let W_1 and W_2 represent the weights, and D_1 and D_2 the distances.

(*a*) $$W_1 : W_2 = \frac{1}{D_1^2}:\frac{1}{D_2^2}.$$

$$\frac{100}{W_2}=\frac{1}{4\cancel{0}\cancel{0}\cancel{0}^2}:\frac{1}{5\cancel{0}\cancel{0}\cancel{0}^2}.$$

$$\frac{100}{25}=\frac{W_2}{16}.$$

Whence $W_2 = 64$.

Therefore it weighs 64 pounds.

(*b*) $$\frac{100}{W_2}=\frac{1}{4\cancel{0}\cancel{0}\cancel{0}^2}:\frac{1}{6\cancel{0}\cancel{0}\cancel{0}^2}.$$

$$\frac{100}{36}=\frac{W_2}{16},$$

or $W_2 = 44\frac{4}{9}$.

Therefore it weighs $44\frac{4}{9}$ pounds.

(*c*) $$\frac{100}{W_2}=\frac{1}{4000^2}:\frac{1}{8000^2}.$$

$$\tfrac{100}{4}=W_2,$$

or $W_2 = 25$.

Therefore it weighs 25 pounds.

20. Let A_1 and A_2 represent the areas, and r_1 and r_2 the radii respectively.

Then $$\frac{A_1}{A_2}=\frac{r_1^2}{r_2^2}.$$

In this case $$\frac{154}{594}=\frac{7^2}{r_2^2}.$$

Whence $r_2 = 3\sqrt{21}$ inches.

21. Let W_1 and W_2 represent the weights, and R_1 and R_2 the radii respectively.

Then $$\frac{W_1}{W_2}=\frac{R_1^3}{R_2^3}.$$

$$\frac{6}{W_2}=\frac{2^3}{6^3}$$

$$=\tfrac{1}{27}.$$

Whence $W_2 = 162$.

Therefore it weighs 162 pounds.

22. Let t_1 and t_2 represent the times, and l_1 and l_2 the lengths respectively.

Then $$\frac{t_1}{t_2}=\frac{\sqrt{l_1}}{\sqrt{l_2}}.$$
$$\frac{1}{t_2}=\frac{\sqrt{100}}{\sqrt{64}}$$
$$=\tfrac{10}{8}.$$

Solving, $t_2=\frac{4}{5}$.

Therefore it requires $\frac{4}{5}$ of a second.

23. As in Ex. 22, $\frac{1}{2}=\frac{\sqrt{100}}{\sqrt{l_2}}$.

Squaring, $$\frac{1}{4}=\frac{100}{l_2}.$$

Whence $l_2=400$.

Therefore its length is 400 centimeters.

24. Let p_1 and p_2 represent the pressures, a_1 and a_2 the areas, and v_1 and v_2 the velocities respectively.

Then $$p_1:p_2=a_1v_1^2:a_2v_2^2.$$
$$.9:18=1\cdot 225:9\,v_2^2.$$
$$8.1\,v_2^2=225\cdot 18,$$
or $$v_2=10\sqrt{5}.$$

Therefore the velocity is $10\sqrt{5}$ miles per hour.

25. The weights of spheres of the same material vary as the cubes of their diameters.

The pull of a weight on a lever arm varies inversely as the distance from the fulcrum.

The pressure of sunlight on a planet varies directly as the square of the planet's diameter, and inversely as the square of the planet's distance from the sun.

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1. $3\sqrt{-1}+4\sqrt{-1}-2\sqrt{-1}=5\sqrt{-1}.$

2. $\sqrt{-4}=2\sqrt{-1}.$

3. $\sqrt{-25}=5\sqrt{-1}.$

4. $5\sqrt{-1}+\sqrt{-9}=5\sqrt{-1}+3\sqrt{-1}=8\sqrt{-1}.$

5. $\sqrt{-4}+\sqrt{-16}=2\sqrt{-1}+4\sqrt{-1}=6\sqrt{-1}.$

6. $5\sqrt{-36x^2}-2\sqrt{-49x^2}=30x\sqrt{-1}-14x\sqrt{-1}=16x\sqrt{-1}.$

7. $\sqrt{-18}+\sqrt{-8}=3\sqrt{-2}+2\sqrt{-2}=5\sqrt{-2}.$

8. $(-12)^{\frac{1}{2}}+(-27)^{\frac{1}{2}}=2\sqrt{-3}+3\sqrt{-3}=5\sqrt{-3}.$

9. $3+2\sqrt{-1}+5-6\sqrt{-1}=8-4\sqrt{-1}.$

10. $$5\sqrt{-x^2}-7a-3\sqrt{-x^2}=5x\sqrt{-1}-7a-3x\sqrt{-1}$$
$$=2x\sqrt{-1}-7a.$$

11. $4-8\sqrt{-1}+16-3\sqrt{-4}=20-8\sqrt{-1}-6\sqrt{-1}=20-14\sqrt{-1}.$

12. $$6-2\sqrt{-64x^2}-3\sqrt{-25x^2}+8=14-16x\sqrt{-1}-15x\sqrt{-1}$$
$$=14-31x\sqrt{-1}.$$

13. $18 - 3(-1)^{\frac{1}{2}} + 6(-2)^{\frac{1}{2}} + (-100)^{\frac{1}{2}} + 4$
$= 22 - 3\sqrt{-1} + 6\sqrt{-2} + 10\sqrt{-1}$
$= 22 + 7\sqrt{-1} + 6\sqrt{-2}.$

14. $5\sqrt{-3} + 3\sqrt{-2} - \sqrt{-27} + 2\sqrt{-8}$
$= 5\sqrt{-3} + 3\sqrt{-2} - 3\sqrt{-3} + 4\sqrt{-2}$
$= 2\sqrt{-3} + 7\sqrt{-2}.$

15. $6\sqrt{-4a^4} - 7a^2\sqrt{-9} + 3\sqrt{-6} - 5\sqrt{-24}$
$= 12a^2\sqrt{-1} - 21a^2\sqrt{-1} + 3\sqrt{-6} - 10\sqrt{-6}$
$= -9a^2\sqrt{-1} - 7\sqrt{-6}.$

16. $(12 - 6\sqrt{-9}) - (15 + 2\sqrt{-36}) = 12 - 18\sqrt{-1} - 15 - 12\sqrt{-1}$
$= -3 - 30\sqrt{-1}.$

17. $3a - 2x - (2a\sqrt{-a^2} - 5ra^2\sqrt{-1})$
$= 3a - 2x - 2a^2\sqrt{-1} + 5ra^2\sqrt{-1}$
$= 3a - 2x - a^2(2 - 5r)\sqrt{-1}.$

18. $(x - iy) - (n - iv) = x - iy - n + iv = x - n - (y - v)i.$

19. $\sqrt{-10} = \sqrt{10} \cdot \sqrt{-1}.$

20. $\sqrt{-6} = \sqrt{6} \cdot \sqrt{-1}.$

21. $2\sqrt{-3} = 2\sqrt{3} \cdot \sqrt{-1}.$

22. $\sqrt{-a} = \sqrt{a} \cdot \sqrt{-1}.$

23. $a\sqrt{-b} = a\sqrt{b} \cdot \sqrt{-1}.$

24. $\sqrt{-a-b} = \sqrt{-1(a+b)}$
$= \sqrt{a+b} \cdot \sqrt{-1}.$

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1. $(\sqrt{-1})^5 = (\sqrt{-1})^2(\sqrt{-1})^2(\sqrt{-1}) = (-1)(-1)\sqrt{-1} = \sqrt{-1}.$

2. $(\sqrt{-1})^6 = (\sqrt{-1})^5(\sqrt{-1}) = \sqrt{-1} \cdot \sqrt{-1} = -1.$

3. $(\sqrt{-1})^7 = (\sqrt{-1})^6(\sqrt{-1}) = -1\sqrt{-1}.$

4. $(\sqrt{-1})^8 = (\sqrt{-1})^6(\sqrt{-1})^2 = (-1)(-1) = 1.$

5. $2\sqrt{-1} \cdot 3\sqrt{-1} = 6(\sqrt{-1})(\sqrt{-1}) = -6.$

6. $\sqrt{-9} \cdot \sqrt{-16} = 3 \cdot \sqrt{-1} \cdot 4 \cdot \sqrt{-1} = 12(-1) = -12.$

7. $(\sqrt{-5})(-\sqrt{-6}) = \sqrt{5} \cdot \sqrt{-1}(-\sqrt{6})\sqrt{-1} = (-\sqrt{30})(-1) = \sqrt{30}.$

8. $\sqrt{-25} \cdot \sqrt{3} = 5\sqrt{-1} \cdot \sqrt{3} = 5\sqrt{-3}.$

9. $2\sqrt{-3} \cdot 3\sqrt{-2} = 2\sqrt{3} \cdot \sqrt{-1} \cdot 3\sqrt{2} \cdot \sqrt{-1} = 6\sqrt{6}(-1) = -6\sqrt{6}.$

10. $\sqrt{-m} \cdot \sqrt{-n} = \sqrt{m} \cdot \sqrt{-1} \cdot \sqrt{n} \cdot \sqrt{-1} = \sqrt{mn}(-1) = -\sqrt{mn}.$

11. $(4\sqrt{-5})(-3\sqrt{-6}) = 4\sqrt{5} \cdot \sqrt{-1}(-3)\sqrt{6} \cdot \sqrt{-1}$
$= -12\sqrt{30}(-1) = 12\sqrt{30}.$

12. $\sqrt{a+b} \cdot \sqrt{-a-b} = \sqrt{a+b} \cdot \sqrt{a+b} \cdot \sqrt{-1} = (a+b)\sqrt{-1}.$

13. $(2 + \sqrt{-1})(2 - \sqrt{-1}) = 4 - (-1) = 4 + 1 = 5.$

14. $(3+\sqrt{-2})(3-\sqrt{-2})=(3+\sqrt{2}\cdot\sqrt{-1})(3-\sqrt{2}\cdot\sqrt{-1})$
$$=9-2(-1)=9+2=11.$$

15. $(4-2\sqrt{3}\,i)(4+2\sqrt{3}\,i)=16-12(-1)=16+12=28.$

16. $(3+\sqrt{-1})(6-\sqrt{-2})=(3+\sqrt{-1})(6-\sqrt{2}\cdot\sqrt{-1})$
$$=18+6\sqrt{-1}-3\sqrt{2}\cdot\sqrt{-1}-\sqrt{2}(-1)$$
$$=18+6\sqrt{-1}-3\sqrt{-2}+\sqrt{2}.$$

17. $(4-2\,i)(3-2\sqrt{3}\,i)=12-6\,i-8\,i\sqrt{3}+4\sqrt{3}(-1)$
$$=12-6\,i-8\,i\sqrt{3}-4\sqrt{3}.$$

18. $(a+ib)(c+id)=ac+ibc+adi+bdi^2=ac+ibc+adi-bd.$

19. $(a+ib)^2=a^2+2\,abi+i^2b^2=a^2+2\,abi-b^2.$

20. $(a+bi)(a-bi)=a^2-b^2i^2=a^2+b^2.$

21. $\left(-\frac{1}{2}+\frac{1}{2}\sqrt{-3}\right)^2=\left(\frac{-1+\sqrt{3}\cdot\sqrt{-1}}{2}\right)^2=\frac{1-2\sqrt{3}\cdot\sqrt{-1}+3(-1)}{4}$
$$=\frac{1-2\sqrt{-3}-3}{4}=\frac{-1-\sqrt{-3}}{2}.$$

22. $\left(-\frac{1}{2}-\frac{1}{2}\sqrt{-3}\right)^2=\left(\frac{-1-1\sqrt{3}\cdot\sqrt{-1}}{2}\right)^2$
$$=\frac{1+2\sqrt{3}\cdot\sqrt{-1}+3(-1)}{4}$$
$$=\frac{-2+2\sqrt{-3}}{4}=\frac{\sqrt{-3}-1}{2}.$$

23. $(x-iy)^2-(x+iy)^2=(x^2-2\,xyi+i^2y^2)-(x^2+2\,xyi+i^2y^2)$
$$=x^2-2\,xyi+y^2-x^2+2\,xyi-y^2$$
$$=-4\,xyi.$$

24. $\left(-\frac{1}{2}+\frac{1}{2}\sqrt{-3}\right)^3-\left(-\frac{1}{2}-\frac{1}{2}\sqrt{-3}\right)^3$
$$=\left(\frac{-1+\sqrt{3}\cdot\sqrt{-1}}{2}\right)^3-\left(\frac{-1-\sqrt{3}\cdot\sqrt{-1}}{2}\right)^3$$
$$=\tfrac{1}{8}[(-1)^3+3(-1)^2\sqrt{3}\sqrt{-1}+3(-1)(\sqrt{3}\sqrt{-1})^2+(\sqrt{3}\sqrt{-1})^3]-\tfrac{1}{8}[(-1)^3+3(-1)^2(-\sqrt{3}\sqrt{-1})+3(-1)(-\sqrt{3}\sqrt{-1})^2+(-\sqrt{3}\sqrt{-1})^3]$$
$$=\frac{-1+3\sqrt{3}\sqrt{-1}+9-3\sqrt{3}\sqrt{-1}}{8}-\frac{-1-3\sqrt{3}\sqrt{-1}+9+3\sqrt{3}\sqrt{-1}}{8}=0.$$

25. $(a+i\sqrt{1-x^2})(a-i\sqrt{1-x^2})=a^2-i^2(1-x^2)=a^2+1-x^2$

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1. $$\sqrt{-8} \div \sqrt{-2} = \frac{\sqrt{-8}}{\sqrt{-2}} = \frac{\sqrt{-8}\cdot\sqrt{-2}}{\sqrt{-2}\cdot\sqrt{-2}} = \frac{\sqrt{8}\cdot\sqrt{-1}\cdot\sqrt{2}\cdot\sqrt{-1}}{\sqrt{2}\cdot\sqrt{-1}\cdot\sqrt{2}\cdot\sqrt{-1}}$$
$$= \frac{-4}{-2} = 2.$$

2. $$\sqrt{-6} \div \sqrt{-2} = \frac{\sqrt{-6}}{\sqrt{-2}} = \frac{\sqrt{-6}\cdot\sqrt{-2}}{\sqrt{-2}\cdot\sqrt{-2}} = \frac{\sqrt{6}\cdot\sqrt{-1}\cdot\sqrt{2}\cdot\sqrt{-1}}{\sqrt{2}\cdot\sqrt{-1}\cdot\sqrt{2}\cdot\sqrt{-1}}$$
$$= \frac{-\sqrt{12}}{-2} = \sqrt{3}.$$

3. $$2\sqrt{-3} \div 3\sqrt{-1} = \frac{2\sqrt{-3}}{3\sqrt{-1}} = \frac{2\sqrt{3}\cdot\sqrt{-1}\cdot\sqrt{-1}}{3\sqrt{-3}\cdot\sqrt{-1}} = \frac{-2\sqrt{3}}{-3}$$
$$= \tfrac{2}{3}\sqrt{3}.$$

4. $$\sqrt[4]{-1} \div \sqrt[4]{-4} = \frac{\sqrt[4]{-1}}{\sqrt[4]{-4}} = \frac{\sqrt[4]{-1}\sqrt[4]{(-1)^3}}{\sqrt[4]{4}\,\sqrt[4]{-1}\,\sqrt[4]{(-1)^3}} = \frac{-1}{-\sqrt[4]{4}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

5. $$\sqrt{6} \div \sqrt{-2} = \frac{\sqrt{6}}{\sqrt{-2}} = \frac{\sqrt{6}\cdot\sqrt{-2}}{\sqrt{-2}\cdot\sqrt{-2}} = \frac{\sqrt{6}\cdot\sqrt{2}\cdot\sqrt{-1}}{\sqrt{2}\cdot\sqrt{-1}\cdot\sqrt{2}\cdot\sqrt{-1}}$$
$$= \frac{\sqrt{12}\cdot\sqrt{-1}}{-2} = -\sqrt{3}\cdot\sqrt{-1} = -\sqrt{-3}.$$

6. $$(-25)^{\frac{1}{4}} \div (-81)^{\frac{1}{4}} = \frac{\sqrt[4]{-25}}{\sqrt[4]{-81}} = \frac{\sqrt[4]{-25}\cdot\sqrt[4]{(-1)^3}}{\sqrt[4]{-81}\cdot\sqrt[4]{(-1)^3}}$$
$$= \frac{\sqrt[4]{25}\cdot\sqrt[4]{-1}\cdot\sqrt[4]{-1}\cdot\sqrt[4]{-1}\cdot\sqrt{-1}}{\sqrt[4]{81}\cdot\sqrt[4]{-1}\cdot\sqrt[4]{-1}\cdot\sqrt[4]{-1}\cdot\sqrt[4]{-1}} = \frac{-\sqrt{5}}{-3} = \frac{1}{3}\sqrt{5}.$$

7. $$\sqrt{ax} \div \sqrt{-x} = \frac{\sqrt{ax}}{\sqrt{-x}} = \frac{\sqrt{ax}\cdot\sqrt{-1}}{\sqrt{-x}\cdot\sqrt{-1}} = \frac{\sqrt{ax}\cdot\sqrt{-1}}{\sqrt{x}\cdot\sqrt{-1}\cdot\sqrt{-1}}$$
$$= \frac{\sqrt{a}\cdot\sqrt{-1}}{-1} = -\sqrt{-a}.$$

8. $$\sqrt{-a} \div \sqrt{-b} = \frac{\sqrt{-a}}{\sqrt{-b}} = \frac{\sqrt{-a}\cdot\sqrt{-b}}{\sqrt{-b}\cdot\sqrt{-b}} = \frac{\sqrt{a}\cdot\sqrt{-1}\cdot\sqrt{b}\cdot\sqrt{-1}}{\sqrt{b}\cdot\sqrt{-1}\cdot\sqrt{b}\cdot\sqrt{-1}}$$
$$= \frac{-\sqrt{ab}}{-b} = \frac{\sqrt{ab}}{b}.$$

9. $$(-5ax)^{\frac{1}{2}} \div (-2x)^{\frac{1}{2}} = \frac{\sqrt{-5ax}}{\sqrt{-2x}} = \frac{\sqrt{-5ax}\cdot\sqrt{-2}}{\sqrt{-2x}\cdot\sqrt{-2}}$$
$$= \frac{\sqrt{5ax}\cdot\sqrt{-1}\cdot\sqrt{2}\cdot\sqrt{-1}}{\sqrt{2x}\cdot\sqrt{-1}\cdot\sqrt{2}\cdot\sqrt{-1}} = \frac{-\sqrt{10a}}{-2} = \frac{1}{2}\sqrt{10a}.$$

10. $[(-a^6)^{\frac{1}{4}} - (-a^2)^{\frac{1}{4}}] \div (-a)^{\frac{1}{4}} = \dfrac{\sqrt[4]{-a^6} - \sqrt[4]{-a^2}}{\sqrt[4]{-a}}$

$$= \frac{(\sqrt[4]{-a^6} - \sqrt[4]{-a^2})\sqrt[4]{(-a)^3}}{\sqrt[4]{-a}\sqrt[4]{(-a)^3}}$$

$$= \frac{(\sqrt[4]{a^5}\sqrt[4]{-a} - \sqrt[4]{a}\sqrt[4]{-a})\sqrt[4]{(-a)^3}}{\sqrt[4]{-a}\sqrt[4]{(-a)^3}}$$

$$= \frac{a^2\sqrt[4]{a} - a\sqrt[4]{a}}{a} = a\sqrt[4]{a} - \sqrt[4]{a} = (a-1)\sqrt[4]{a}.$$

11. $2 \div (1 - \sqrt{-1}) = \dfrac{2}{1 - \sqrt{-1}} = \dfrac{2(1 + \sqrt{-1})}{(1 - \sqrt{-1})(1 + \sqrt{-1})}$

$$= \frac{2 + 2\sqrt{-1}}{1 + 1} = 1 + \sqrt{-1}.$$

12. $3 \div (2 - \sqrt{-2}) = \dfrac{3}{2 - \sqrt{-2}} = \dfrac{3(2 + \sqrt{-2})}{(2 - \sqrt{-2})(2 + \sqrt{-2})}$

$$= \frac{6 + 3\sqrt{-2}}{4 + 2} = 1 + \frac{1}{2}\sqrt{-2}.$$

13. $2\sqrt{-1} \div (\sqrt{-1} + 3) = \dfrac{2\sqrt{-1}}{\sqrt{-1} + 3} = \dfrac{2\sqrt{-1}(\sqrt{-1} - 3)}{(\sqrt{-1} + 3)(\sqrt{-1} - 3)}$

$$= \frac{-2 - 6\sqrt{-1}}{-1 - 9} = \frac{+1 + 3\sqrt{-1}}{5}.$$

14. $3\sqrt{-2} \div (2\sqrt{-3} + 2) = \dfrac{3\sqrt{-2}}{2\sqrt{-3} + 2} = \dfrac{3\sqrt{-2}(2\sqrt{-3} - 2)}{(2\sqrt{-3} + 2)(2\sqrt{-3} - 2)}$

$$= \frac{-6\sqrt{6} - 6\sqrt{-2}}{-12 - 4} = \frac{3\sqrt{6} + 3\sqrt{-2}}{8}.$$

15. $\dfrac{-1 + \sqrt{-3}}{-1 - \sqrt{-3}} = \dfrac{(-1 + \sqrt{-3})(-1 + \sqrt{-3})}{(-1 - \sqrt{-3})(-1 + \sqrt{-3})} = \dfrac{1 - 2\sqrt{-3} - 3}{1 + 3}$

$$= -\tfrac{1}{2} - \tfrac{1}{2}\sqrt{-3}.$$

16. $\dfrac{1 + i}{1 - i} = \dfrac{(1 + i)(1 + i)}{(1 - i)(1 + i)} = \dfrac{1 + 2i - 1}{1 + 1} = i.$

17. $\dfrac{a}{a + bi} = \dfrac{a(a - bi)}{(a + bi)(a - bi)} = \dfrac{a^2 - abi}{a^2 + b^2}.$

18. $\dfrac{a + ib}{c + id} = \dfrac{(a + ib)(c - id)}{(c + id)(c - id)} = \dfrac{ac + bci - adi - i^2bd}{c^2 - i^2d^2}$

$$= \frac{ac + bci - adi + bd}{c^2 + d^2}.$$

19. $(2 \pm \sqrt{-3})^2 - 4(2 \pm \sqrt{-3}) + 7 = 0$, or $0 = 0$. Yes.

20. $(2 + 3\sqrt{-1}) + (2 - 3\sqrt{-1}) = 4$, a real number. They are.
And $(2 + 3\sqrt{-1})(2 - 3\sqrt{-1}) = 13$, a real number. They are.

21. Let $a + b\sqrt{-1}$ and $a - b\sqrt{-1}$ = the conjugate complex numbers.

$$a + b\sqrt{-1} + a - b\sqrt{-1} = 2a, \text{ a real number.}$$

$$\begin{aligned}(a + b\sqrt{-1})(a - b\sqrt{-1}) &= [a^2 - (-1)b^2] \\ &= a^2 + b^2, \text{ a real number.}\end{aligned}$$

22. $$\frac{a + b\sqrt{-1}}{a - b\sqrt{-1}} = \frac{(a + b\sqrt{-1})(a + b\sqrt{-1})}{(a - b\sqrt{-1})(a + b\sqrt{-1})} = \frac{a^2 + 2ab\sqrt{-1} - b^2}{a^2 + b^2}, \text{ a complex number.}$$

23. Lines 1 and 2 are obvious identities and are therefore correct. Line 3 assumes that division of imaginaries is performed by direct division of radicand by radicand, which is not true. In line 3, the first member $\frac{\sqrt{-1}}{\sqrt{1}} = \sqrt{-1}$ and the second member $\frac{\sqrt{1}}{\sqrt{-1}} = \frac{\sqrt{1}\sqrt{-1}}{\sqrt{-1}\sqrt{-1}} = \frac{\sqrt{-1}}{-1} = -\sqrt{-1}$. But $\sqrt{-1} \neq -\sqrt{-1}$. For the correct and general method see paragraph 2, section 119 of the text.

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1. $$x^2 + 4x + 12 = 0.$$

Solving, $$x = \frac{-4 \pm \sqrt{16 - 48}}{2} = \frac{-4 \pm 4\sqrt{-2}}{2} = -2 \pm 2\sqrt{-2}.$$

2. $$x^2 - 6x + 36 = 0.$$

Solving, $$x = \frac{6 \pm \sqrt{36 - 144}}{2} = \frac{6 \pm 6\sqrt{-3}}{2} = 3 \pm 3\sqrt{-3}.$$

3. $$x^2 + 5x + 7 = 0.$$

Solving, $$x = \frac{-5 \pm \sqrt{25 - 28}}{2} = \frac{-5 \pm \sqrt{-3}}{2}.$$

4. $$x^2 - 3x + 10 = 0.$$

Solving, $$x = \frac{3 \pm \sqrt{9 - 40}}{2} = \frac{3 \pm \sqrt{-31}}{2}.$$

5. $$2x^2 + 6x + 5 = 0.$$

Solving, $$x = \frac{-6 \pm \sqrt{36 - 40}}{4} = \frac{-6 \pm 2\sqrt{-1}}{4} = \frac{-3 \pm \sqrt{-1}}{2}.$$

6. $$3x^2 - 7x + 6 = 0.$$

Solving, $$x = \frac{7 \pm \sqrt{49 - 72}}{6} = \frac{7 \pm \sqrt{-23}}{6}.$$

7.

$$x^3 = 1.$$

$$x^3 - 1 = 0.$$

Hence $$(x-1)(x^2+x+1) = 0.$$

Whence $$x = 1.$$

$$x^2 + x + 1 = 0.$$

Solving, $$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}.$$

8.

$$x^3 = 8.$$

$$x^3 - 8 = 0.$$

$$(x-2)(x^2+2x+4) = 0.$$

$$x = 2.$$

$$x^2 + 2x + 4 = 0.$$

Solving, $$x = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm 2\sqrt{-3}}{2} = -1 \pm \sqrt{-3}.$$

9.

$$x^3 = -27.$$

$$x^3 + 27 = 0.$$

$$(x+3)(x^2-3x+9) = 0.$$

$$x = -3.$$

$$x^2 - 3x + 9 = 0.$$

Solving, $$x = \frac{3 \pm \sqrt{9-36}}{2} = \frac{3 \pm 3\sqrt{-3}}{2}.$$

10.

$$x^4 = 1.$$

$$x^4 - 1 = 0.$$

$$(x^2+1)(x+1)(x-1) = 0.$$

$$x = 1 \text{ or } -1.$$

$$x = \pm\sqrt{-1}.$$

11.

$$x^4 = 16.$$

$$x^4 - 16 = 0.$$

$$(x^2+4)(x+2)(x-2) = 0.$$

$$x = \pm 2.$$

$$x = \pm 2\sqrt{-1}.$$

12.

$$x^6 = 1.$$

$$x^6 - 1 = 0.$$

$$(x^3+1)(x^3-1) = 0.$$

$$(x+1)(x^2-x+1)(x-1)(x^2-x+1) = 0.$$

$$x = \pm 1.$$

$$x^2 - x + 1 = 0.$$

Solving, $x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{-3}}{2},$

and $x^2 + x + 1 = 0.$

Solving, $x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}.$

13.
$$x^6 = 64.$$
$$x^6 - 64 = 0.$$
$$(x^3 + 8)(x^6 - 8) = 0.$$
$$(x + 2)(x^2 - 2x + 4)(x - 2)(x^2 - 2x + 4) = 0.$$
$$x = \pm 2.$$
$$x^2 - 2x + 4 = 0.$$

Solving, $x = \frac{2 \pm \sqrt{4-16}}{2} = \frac{2 \pm 2\sqrt{-3}}{2} = 1 \pm \sqrt{-3},$

and $x^2 + 2x + 4 = 0.$

Solving, $x = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm 2\sqrt{-3}}{2} = -1 \pm \sqrt{-3}.$

14. Two, three, four, six.

15. Any real number may have n nth roots.

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2. Let x equal the number of minute spaces that the minute hand travels from its position at 7 o'clock until it overtakes the hour hand. Obviously the minute hand travels 35 minute spaces more than the hour hand.

Then $x = (x - 35)\,12.$

Whence $x = 38\tfrac{2}{11}.$

3. Let x equal the number of minute spaces that the minute hand travels from its position at 2 o'clock until it is in the required position. For the first of the required positions the minute hand must gain 10 minute spaces over the hour hand; for the second position 40 spaces.

Then $x = (x - 10)\,12.$

Whence $x = 10\tfrac{10}{11},$

and $x = (x - 40)\,12.$

Whence $x = 43\tfrac{7}{11}.$

4. Let x equal the number of minute spaces passed over by the minute hand in going from its position at 6 o'clock to the required position. For the first it must gain 40 spaces and for the second 20 spaces.

Then $x = (x - 40)\,12.$

Whence $x = 43\frac{7}{11}$,

and $x = (x - 20)\,12$.

Whence $x = 21\frac{9}{11}$.

5. Let x equal the number of minute spaces passed over by the minute hand in going from its position at 5 o'clock to the required position. Now if the hands are at right angles, one is 15 minute spaces from the other. For the first position the minute hand must gain 10 spaces and for the second 40 spaces.

Then $x = (x - 10)\,12$.

Whence $x = 10\frac{10}{11}$,

and $x = (x - 40)\,12$.

Whence $x = 43\frac{7}{11}$.

7. Let x equal the number of days from April 28, 1909, until the next conjunction.

Then, as in Exercise 6, $\dfrac{x}{225} = \dfrac{x}{365} + 1$.

Whence $x = 586+$.

Therefore the required date is December 5, 1910.

8. Let x equal the number of days from opposition to opposition.

Then $\dfrac{x}{365} = \dfrac{x}{4332} + 1$.

Whence $x = 398+$.

Therefore the required date is April 2, 1910.

9. Let x equal the number of days from opposition to opposition.

Then $\dfrac{x}{365} = \dfrac{x}{10{,}759} + 1$.

Whence $x = 377+$.

Therefore the required date is April 15, 1910.

10. Let x equal the number of hours from the start until they are again together.

Then $\dfrac{x}{2\frac{1}{2}} = \dfrac{x}{3} + 1$.

Whence $x = 15$.

11. Reducing $2\frac{3}{4}$, $3\frac{1}{3}$, and $4\frac{2}{5}$ to a common denominator gives $\frac{165}{60}$, $\frac{200}{60}$, and $\frac{264}{60}$.

The L.C.M. of the numerators is 6600.

The L.C.M. of the fractions is $\frac{6600}{60} = 110$.

Hence the three are together 110 hours after the start.

12. Yes. $15 \div 2\frac{1}{2} = 6$, and $15 \div 3 = 5$. Yes.

13. Yes; Yes.

14. $\frac{11}{4}, \frac{10}{3}, \frac{22}{5}, \frac{\text{L.C.M. of numerators}}{\text{G.C.D. of denominators}} = \frac{110}{1}$.

15. Rule. Divide the L.C.M. of the numerators by the G.C.D. of the denominators.

L.C.M. of $\frac{7}{6}$, $\frac{7}{3}$, and $\frac{28}{9}$ is $\frac{28}{3}$ or $9\frac{1}{3}$.

16. L.C.M. of $\frac{a}{b}$ and $\frac{c}{d} = \frac{ac}{1} = ac$.

L.C.M. of $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f} = \frac{ace}{1} = ace$.

17. Let x equal the number of ounces needed.

Then $$\frac{56}{56 + x} = \frac{70}{100}.$$

Whence $$x = 24.$$

18. $$\frac{\frac{16 \cdot 410}{100} + x}{410 + x} = \frac{18}{100}.$$

$$6560 + 100x = 7380 + 18x.$$

Whence $$x = 10 \text{ pounds}.$$

19. Let m equal the number of gallons of milk and w the number of gallons of water.

Then $$m + w = 30, \qquad (1)$$

and $$\frac{\frac{20m}{100}}{30} = \frac{16}{100}. \qquad (2)$$

Solving (1) and (2), $$m = 24,$$

and $$w = 6.$$

20. Let x equal the number of gallons of 90% alcohol.

Then $$\frac{\frac{90}{100}x + \frac{95}{100} \cdot 10}{x + 10} = \frac{92}{100}.$$

Whence $$x = 15.$$

21. Let x equal the number of gallons of 95% alcohol and y the number of gallons of 15% alcohol.

Then $\dfrac{.95x + .15y}{10} = \dfrac{45}{100}$,

and $x + y = 10$.

Whence $x = 3\frac{3}{4}$,

and $y = 6\frac{1}{4}$.

22. Let x equal the per cent of purity of the first, and y the per cent of purity of the second.

Then $\dfrac{8x + 12y}{20} = \dfrac{84}{100}$,

and $\dfrac{3x + 2y}{5} = \dfrac{86}{100}$.

Whence $x = 90\%$,

and $y = 80\%$.

23. Let e equal the diameter of the earth in miles, and m the diameter of the moon in miles.

Then $e = \dfrac{11m}{3}$,

and $e - m = 5760$.

Whence $e = 7920$,

and $m = 2160$.

24. Let s equal the diameter of the sun in miles, and e the diameter of the earth in miles.

Then $s = 3220 + 109e$,

and $s + e = 874{,}420$.

Whence $s = 866{,}500$,

and $e = 7920$.

25. Let s equal the sun's distance from the earth in miles, and m the moon's distance from the earth in miles.

Then $s = 387\frac{1}{2}m$,

and $\dfrac{s}{186{,}000} = \dfrac{m}{186{,}000} + 498\frac{22}{31}$.

Whence $s = 93{,}000{,}000$,

and $m = 240{,}000$.

26. Let m equal the distance in miles from Mars to the sun, and e the distance in miles from the earth to the sun.

Then $m + e = 234{,}500{,}000$,

and $m - e = 48{,}500{,}000$.

Whence $m = 141{,}500{,}000$,

and $e = 93{,}000{,}000$.

27. Let j equal the diameter of Jupiter in miles, and e the diameter of the earth in miles.

Then $$j = \frac{120\,e}{11},$$
and $$j + e = 94{,}320.$$
Whence $$j = 86{,}400,$$
and $$e = 7920.$$

28. Let $x =$ the depth in inches.
Then $x + 1 =$ the width in inches,
and $x + 2 =$ the length in inches.
Then $2x(x+1) + 2x(x+2) + 2(x+1)(x+2) = 292.$
Whence $6x^2 + 12x + 4 = 292.$
Then $x = 6,$
$x + 1 = 7,$
and $x + 2 = 8.$

29. Let x equal the side of the square in inches. Then $x - 6$ equals both the length and the width of the box. The depth is 3 inches.
Therefore $3(x-6)(x-6) = 300.$
Whence $x = 16$ or $-4.$
The area of the piece of tin is 256 square inches.

30. The depth of the box is x inches.
The length of the box is $12 - 2x$ inches.
The width of the box is $8 - 2x$ inches.
The volume equals $x(12 - 2x)(8 - 2x) = 4x^3 - 40x^2 + 96x.$

31. $$12x^2 - 80x + 96 = 0.$$
$$3x^2 - 20x + 24 = 0.$$
Solving by formula, $$x = \frac{20 \pm \sqrt{400 - 286}}{6}.$$
Whence $x = 5.09+$ or $1.56+.$
Since 5.09 is more than half the width, the only answer is 1.56+.

32. Let w equal the width of the box in inches and d the depth of the box in inches.
The volume $= 8\,dw.$
The surface $= 8(2d + 2w) + 2\,dw.$
Therefore $8\,dw = 192,$
and $16d + 16w + 2\,dw = 208.$
Whence $d = 4,$
and $w = 6.$

33. Let x equal the number of miles the head of the army has moved when the messenger reaches it. Then while the army moves x miles the messenger travels $x + 25$ miles.

Now while the messenger returns x miles the army moves forward $25 - x$ miles. Hence we have the proportion

$$\frac{x}{25 + x} = \frac{25 - x}{x}.$$

Whence $x = \frac{25}{2}\sqrt{2}$ miles, the negative answer being impossible.

But the messenger travels forward $25 + x$ miles and back x miles, a total of $2 + 2x$ miles. This equals $25 + 25\sqrt{2} = 60.3+$ miles.

ANNOUNCEMENTS

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